Chapter 4 : Functions

Content Outline

Include :

- concepts of function, domain and range
- use of notations such as $\mathrm{f}(x) = x^2 + 5$, $\mathrm{f} : x \mapsto x^2 + 5$, $\mathrm{f}^{-1}(x)$, $\mathrm{fg}(x)$ and $\mathrm{f}^2(x)$
- finding inverse functions and composite functions
- conditions for the existence of inverse functions and composite functions
- domain restriction to obtain an inverse function
- relationship between a function and its inverse

Exclude the use of the relation $(\mathrm{fg})^{-1} = \mathrm{g}^{-1}\mathrm{f}^{-1}$, and restriction of domain to obtain a composite function.

Reference

- http://www.h2maths.site
- https://www.mathisfun.com/sets/function.html
  [Website on properties of function , composite functions]
- Heinemann Higher Mathematics, Heinemann. Call No: CLA 510.712
- MEI Pure Mathematics 2, 2nd ed, Hoddler & Stoughton. Call No: HAN 510
- Modular Maths Pure Mathematics 2, Hoddler & Stoughton. Call No: SYK 510
- DoBrilliantly ASMaths, Collins. Call No: GRA 510

Prerequisites

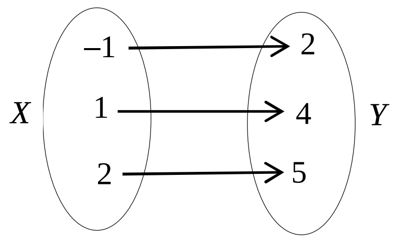
- Sketch graphs $y = \mathrm{f}(x)$ of the following types: linear, quadratic, cubic, reciprocal, exponential, logarithmic, sine, cosine, and tangent.

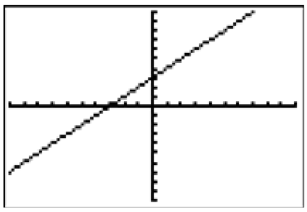
Introduction

Before we introduce functions, we need to understand what a **relation** is.
In layman's term, a **relation** is an equation connecting a variable x with another variable y.

- A **relation** is a set of ordered pairs.
- The first elements in the ordered pairs, the x-values, form the **domain** (Input values).

Function is a <u>relation</u> between a set of inputs and a set of outputs with the property that each input maps to exactly one output.

<table>
<tr><th colspan="2">Relation</th></tr>
<tr><th>Function</th><th>Not a Function</th></tr>
<tr>
<td>E.g. $y=x+3$
<table>
<tr><td>x</td><td>-1</td><td>1</td><td>2</td></tr>
<tr><td>y</td><td>2</td><td>4</td><td>5</td></tr>
</table>



</td>
<td>E.g. $y^2=x$
<table>
<tr><td>x</td><td>0</td><td>4</td><td>9</td></tr>
<tr><td>y</td><td>0</td><td>2 or -2</td><td>3 or -3</td></tr>
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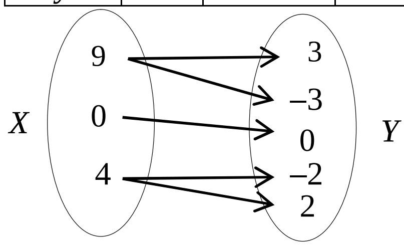

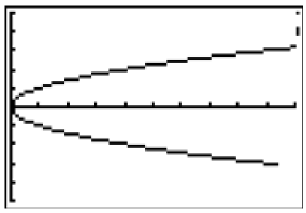
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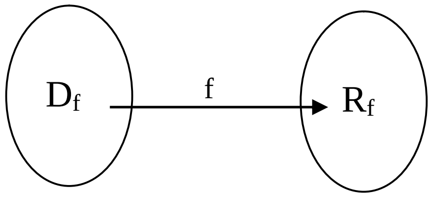
1. Functions

A **function** f : $X \to Y$ is a **rule** (or relation) which associates each element in X (known as the domain) with a <u>unique</u> element in Y (known as the range), i.e. a function is a rule y = f(x) such that every value of $x \in X$ has exactly one value of $y \in Y$.

In diagrammatic form, a function can be visualised as follows:

D$_\text{f}$ is the notation for domain of f.



R$_\text{f}$ is the notation for range of f.

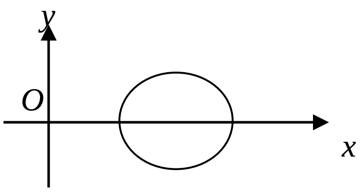
- The set of input values forms the **domain**.
- The corresponding set of output values forms the **range**.

1.1 To determine whether a given rule is a function

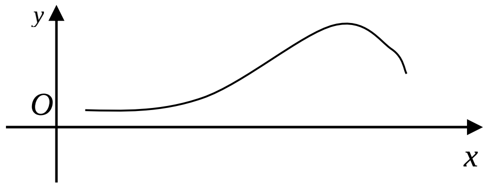
A **function** f : $X \to Y$ is a **rule** (or relation) which associates each element in *X* (the domain) with a unique element in *Y* (the range). This means that for the graph of a function, there will be only one *y*-value corresponding to every *x*-value. So a vertical line drawn at any *x*-value in the domain should cut the graph of the function at exactly one point.

Hence, if there exists **a vertical line (*x* = *k*)** which cuts the graph of the relation at more than one point (which means an *x*-value has more than one *y*-value), then the relation does not represent a function.

Example 1
Determine which of the following represents the graph of a function and state your reason(s):
(a)



(b)



Solution:
1(a) **There exists** a vertical line, $x = k$ which cuts the graph at two points, hence it is not a function.
1(b) **Any** vertical line will cut the graph at most once. Hence this graph represents a function.

Mini Exercise

Write the following in inequalities and interval notations.

Description of solution set	**Inequalities**	**Interval notation**
values from 1 (inclusive) to infinity	$x \geq 1$	
values from negative infinity to 1 (exclusive)	$x < 1$	
values from 1 (inclusive) to 5 (exclusive)	$1 \leq x < 5$	
all real values excluding 1	$x < 1$ or $x > 1$	or $\mathbb{R} \setminus \{1\}$

Note the use of interval notation :
(a,b) to represent the interval $\{x \in \mathbb{R} : a < x < b\}$,
$[a,b]$ to represent the interval $\{x \in \mathbb{R} : a \leq x \leq b\}$,
$(a,b]$ to represent the interval $\{x \in \mathbb{R} : a < x \leq b\}$,
$[a,b)$ to represent the interval $\{x \in \mathbb{R} : a \leq x < b\}$,
(a,∞) to represent the interval $\{x \in \mathbb{R} : x > a\}$,
$[a,\infty)$ to represent the interval $\{x \in \mathbb{R} : x \geq a\}$,
$(-\infty,b)$ to represent the interval $\{x \in \mathbb{R} : x < b\}$,
$(-\infty,b]$ to represent the interval $\{x \in \mathbb{R} : x \leq b\}$.
$\mathbb{R}$ represents the set of real numbers.
$\mathbb{R}^+$ represents the set of positive real numbers.
$\mathbb{R}_0^+$ represents the set of positive real numbers including 0. [or $\mathbb{R}^+ \cup \{0\}$]

Example 2:

The function f is defined by f: $x \mapsto x^2$, $x > 2$. State the rule and the domain of f.

Solution:

The rule is

The domain of f =

Note:

There are various ways of writing a function as shown below:

(a) $\mathrm{p}(x) = \mathrm{e}^x$, $x \le 0$

(b) $\mathrm{f} : x \mapsto x^2$, $x \in \mathbb{R}$

(c) $\mathrm{g} : x \mapsto x^3$, $x > 0$

(d) $\mathrm{q}(x) = \begin{cases} 1 - x, & \text{for } 0 \le x \le 1 \\ 1 + x, & \text{for } -1 \le x < 0 \end{cases}$

In most cases, the name of the function is denoted by lower case alphabet (f, g, p, q etc).
All functions are defined by a rule and a domain.

> **Note:**
> Two functions with the **same rule** but with **different domains** are considered as **different functions.**

Example 3

Given f : $x \mapsto (x + 1)^2$, $x \in \mathbb{R}$. Determine whether g or h is the same as f.
Give reasons for your answers.

(a) $\mathrm{g}(u) = u^2 + 2u + 1$, $u \in \mathbb{R}$ (b) $\mathrm{h}(x) = (x + 1)^2$, $x \in \mathbb{R}^+$

Solution :

1.2 Use of GC to find range

The graph of a function is a way of visualising the relationship between y and x. The **graph** provides a simple way to **see the range** of a function. We can sketch the graph using the GC.

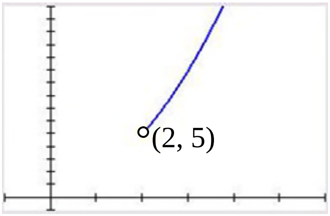
Example 4

Find the range of the function $\mathrm{g}: x \mapsto x^2 + 1,\ x > 2$.

Note to students:

First , sketch the graph of $y = x^2 + 1$, $x > 2$. [Refer to Annex 5.1]
State the end point (2, 5).
From the graph, determine the set of possible values of y that the function can take.

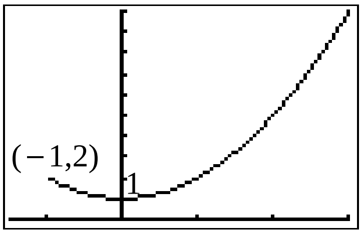
Solution:



Example 5

Find the range of the function f: $x \mapsto x^2 + 1,\ x \geq -1$.

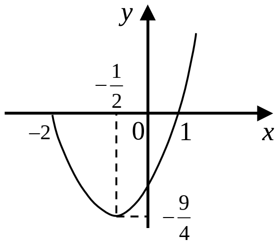
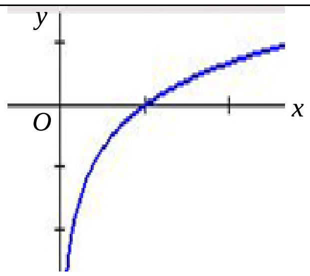
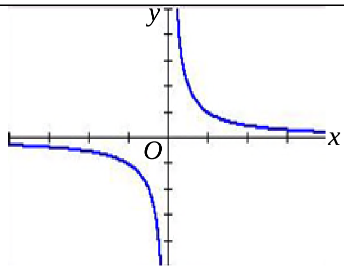
Solution:



Hence, $\mathrm{R_f}$ =

<u>Mini Exercise</u>

Complete the following table of exercises:

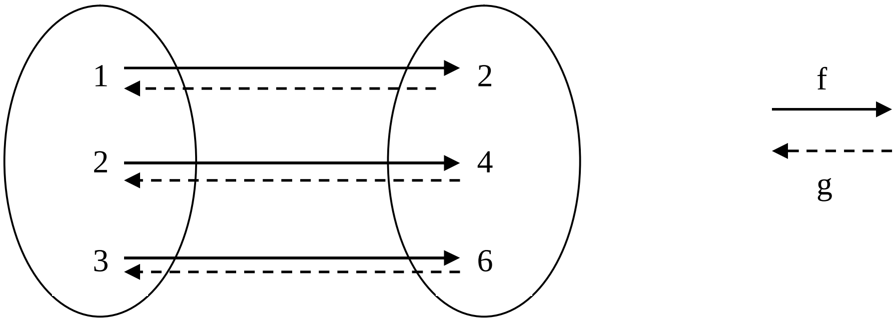
No.	**Types of Functions**	**Sketches**	**Domain / Range of functions**
1.	$\mathrm{g}(x)=(x-1)(x+2), x \geq -2$		
2.	$\mathrm{h}(x)=\ln x, x>0$		
3.	$\mathrm{q}(x)=\frac{1}{x}, x \in \mathbb{R} \setminus \{0\}$		

2. Inverse functions

Not every function has an inverse. A function has to be **one-one** for its inverse to exist. The idea of one-one function will be further addressed in 2.1.

Example 6

Consider the function f: $x \to 2x$ with $x \in \{1, 2, 3\}$. Then, f(1) = 2, f(2) = 4, f(3) = 6.
Find the rule and domain for the function g such that g(2) = 1, g(4) = 2, g(6) = 3.



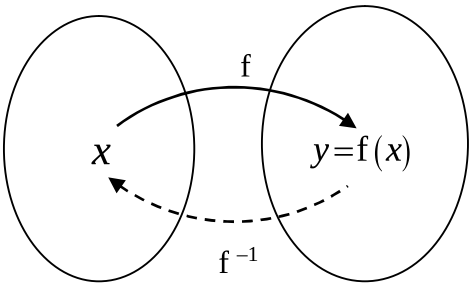
Solution:

Function represented by ◄- - - - is g, with rule

NOTE:

(i) Function g is known as the **inverse** of f, and is usually represented by $\mathbf{f}^{-1}$ (instead of g). It "reverses" the effect of the original function f. Another way of saying this is "the inverse takes the *y*-values and recovers the *x*-values."

In the form of a diagram, the inverse can be visualized as follows:



In general,	$y = \mathrm{f}(x) \Leftrightarrow x = \mathrm{f}^{-1}(y)$

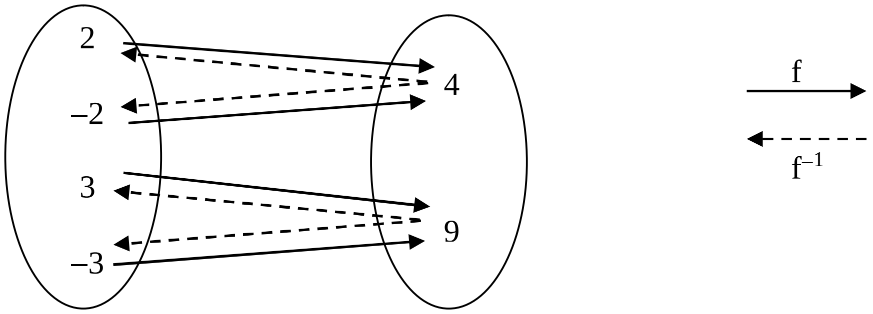
(ii) Notice that the domain of f^{-1} is the same as the range of f, and the range of f^{-1} is the same as the domain of f.

Therefore,	$D_{\mathrm{f}^{-1}} = R_{\mathrm{f}}$	$R_{\mathrm{f}^{-1}} = D_{\mathrm{f}}$

(iii) Bear in mind that $\mathrm{f}^{-1}(x) \neq \dfrac{1}{\mathrm{f}(x)}$.

2.1 Existence of Inverse Function

If we try to find the "inverse" for $f(x) = x^2$, we end up with the following "rule": $x = \pm\sqrt{y}$. Recall that a function relates each "x" (each input value) with **exactly one** output "y" via a rule. Does this condition apply when we consider $x = \pm\sqrt{y}$?



In fact, the inverse function f^{-1} **is defined if and only if f is a one-one function**.

Definition of one-one function

A function for which every element of the range of the function corresponds to exactly one element of the domain is a one-one function

To determine whether a function is one-one

(1) Use Graph to show that the function f is one-one or NOT one-one

Based on the definition of one-one function, we notice that **if a function is one-one, then any horizontal line $y = k$ will cut the graph at most once** (a single value of y has a single value of x related to it).

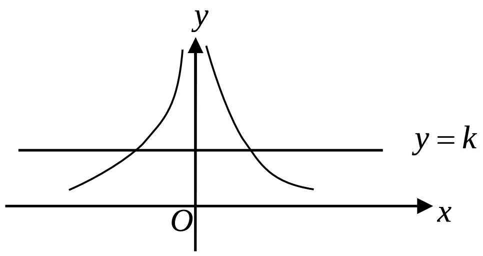
If there exists **a horizontal line $y = k$** which cuts the graph more than once (meaning at more than one point), then the function is **<u>NOT</u>** a one-one function.

(2) Use a counter example to show that the function f is NOT one-one

Find two values of x, say a and b and show that $f(a) = f(b)$.

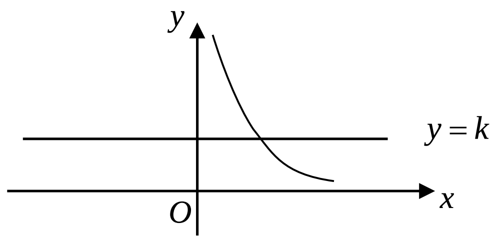
<u>Example 7</u> (The solution presented in this example is expected for examination)
Determine if each of the functions is one-one.

(i) $f: x \mapsto \frac{1}{x^2},\ x \in \mathbb{R}\setminus\{0\}$



There exists a horizontal line $y = k$ that cuts the graph of f <u>more than once</u>, f is <u>not a one-one</u> function.

(ii) $g: x \mapsto \frac{1}{x^2},\ x > 0$



Any horizontal line $y = k$ cuts the graph of g <u>at most once</u>, g is <u>one-one</u> function.

Note: Show the horizontal line $y = k$ on your sketch.

Alternatively, for (i) since $f(-1) = f(1) = 1$, f is <u>not a one-one</u> function.

2.2 To Find the Inverse Rule and Domain

We use the symbol f^{-1} to represent the **inverse** of the function f. Recall from Example 6 that:

$$\boxed{\mathrm{D}_{\mathrm{f}^{-1}} = \mathrm{R}_{\mathrm{f}} \qquad \mathrm{R}_{\mathrm{f}^{-1}} = \mathrm{D}_{\mathrm{f}}}$$

In general, the steps for finding inverse function of f are

Step 1 Write $y = \mathrm{f}(x)$, and $x \in \mathrm{D}_{\mathrm{f}}$.

Step 2 Manipulate to make x as the subject

Step 3 Replace y with x

Step 4 Write down the domain of f^{-1} in terms of x (Obtain from the range of f).

Example 8

Find the inverse of the function $\mathrm{f} : x \mapsto 1 + \mathrm{e}^{1-x}$, $x \in \mathbb{R}$.

Solution:

Step 1 Let $y = \mathrm{f}(x) = 1 + \mathrm{e}^{1-x}$, $x \in \mathbb{R}$

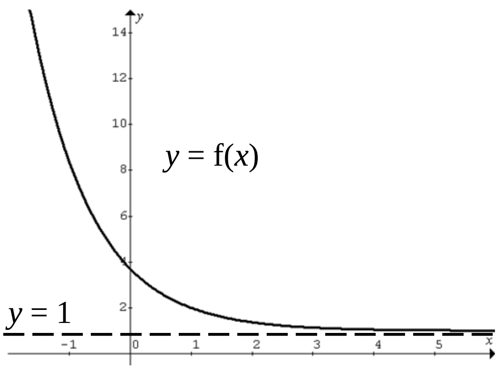
Step 2 Then, $y - 1 = \mathrm{e}^{1-x}$

$\ln(y-1) = 1 - x$

$x = 1 - \ln(y-1)$

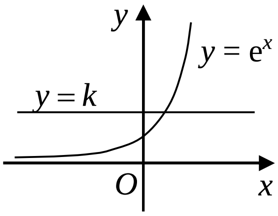
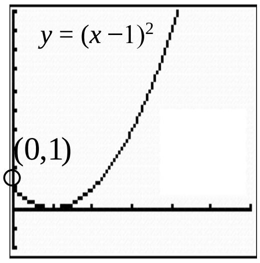
Step 3 $\mathrm{f}^{-1}(x) = 1 - \ln(x-1)$

Step 4 $\mathrm{D}_{\mathrm{f}^{-1}} = \mathrm{R}_{\mathrm{f}} =$



Note: Domain must always be indicated when writing down a function.

<u>Example 9</u>
Complete the table below:

Function	**Graph**	**Is this function one-one?**	**Does the inverse function exist? If yes, find the inverse.**
(a) $\mathrm{p}(x)=\mathrm{e}^x,\ x\in\mathbb{R}$		Yes **Any** horizontal line $y=k$ cuts the graph at most once. Hence the function is one-one.	$y=\mathrm{e}^x,\ x\in\mathbb{R}$ $x=\ln y$ $\mathrm{p}^{-1}(x)=\ln x,\ x\in\mathbb{R}^+$
(b) $\mathrm{f}(x)=(x-1)^2,$ $x>0$			

In the above exercise, which function is not one-one?

<u>RESTRICTED DOMAIN FOR EXISTENCE OF INVERSE FUNCTION</u>

Can we restrict the domain of f(x) = $(x-1)^2$, $x > 0$ so that the function is one-one?
What are the domains that you can find so that the function is one-one?

$D_f = (0,1)$, , etc

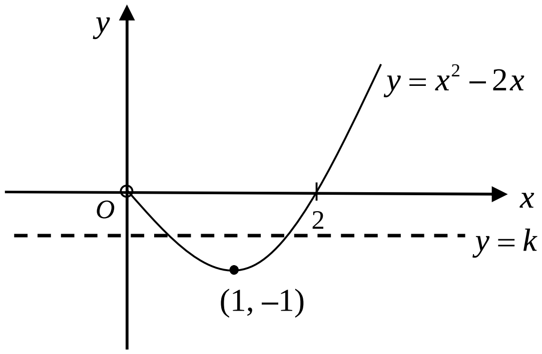
The largest possible domain among these is $[1,\infty)$.

Example 10

The function f is defined by $\mathrm{f} : x \mapsto x^2 - 2x, \ \ x > 0$. State a reason why f^{-1} does not exist.

Find the smallest value of *a* such that the inverse of f exists in [*a*, ∞). Hence find f^{-1} in a similar form.

Solution :



There exists a horizontal line $y = k$ that cuts the graph of $y = \mathrm{f}(x)$ more than once. Hence, f is not one-to-one. So, f^{-1} does not exist.

For f to be one-one, the largest possible domain is

$\therefore \ a = 1$.

Let $y = x^2 - 2x \, , x \in [1, \infty)$.

Alternatively,

Let $y = x^2 - 2x$

$x^2 - 2x - y = 0$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-y)}}{2}$$

$$x = \frac{2 \pm \sqrt{4(1+y)}}{2} = 1 \pm \sqrt{1+y}$$

since $x \geq 1, \ x = 1 + \sqrt{y+1}$

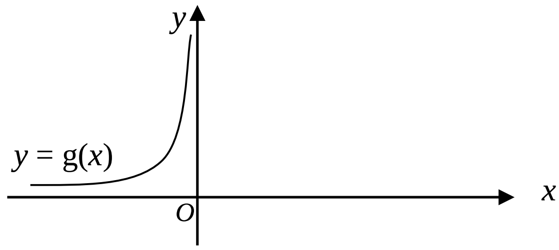
$\mathrm{D}_{\mathrm{f}^{-1}} = \mathrm{R}_{\mathrm{f}} = [-1, \infty)$

Thus, $\mathrm{f}^{-1} : x \mapsto 1 + \sqrt{x+1}, \ \ x \geq -1$

<u>Example 11</u>

Find the inverse of the functions (i) $\mathrm{g}(x) = \dfrac{1}{x^2}$, $x < 0$, (ii) $\mathrm{h}(x) = \dfrac{1}{x^2}$, $x > 0$.

Solution:

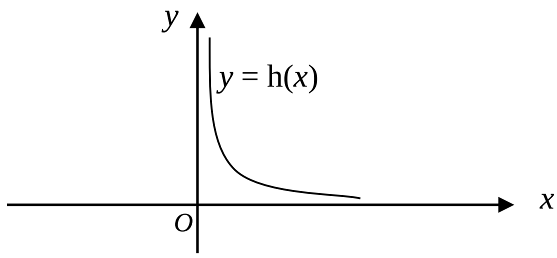


(ii) Let $y = \dfrac{1}{x^2}$, where $x > 0$.

$\therefore\ x^2 = \dfrac{1}{y} \Rightarrow x = \pm\dfrac{1}{\sqrt{y}}$

Since $x > 0$, $x = \dfrac{1}{\sqrt{y}}$

Hence, $\mathrm{h}^{-1}(x) = \dfrac{1}{\sqrt{x}}$, $x > 0$



<u>2.3 Relationship between graph of f and graph of f $^{-1}$</u>

<u>Example 12</u>

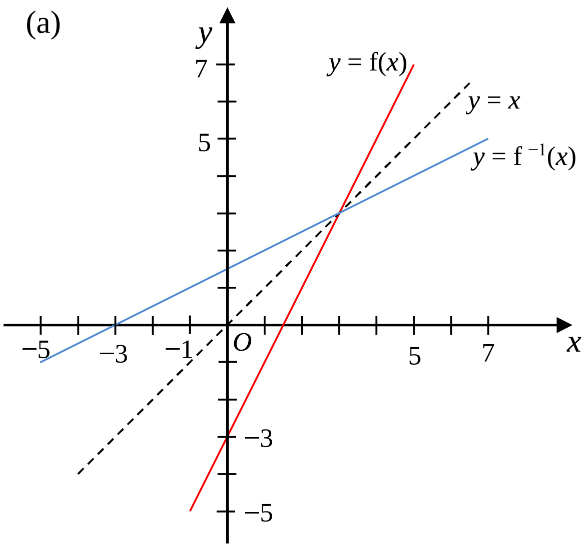
The diagram below shows the graphs of

(a) $\mathrm{f}(x) = 2x - 3,\ x \in [-1, 5]$ and $\mathrm{f}^{-1}(x) = \dfrac{x+3}{2},\ x \in [-5, 7]$

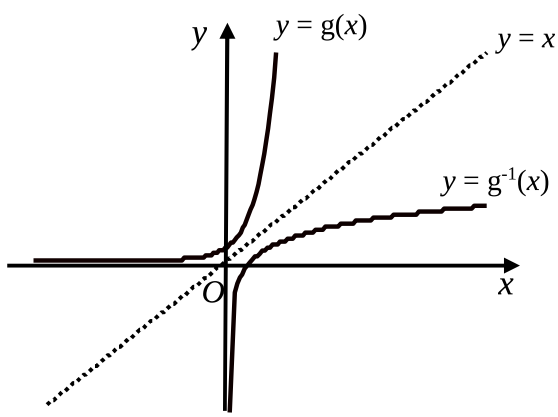
(b) $\mathrm{g}(x) = \mathrm{e}^x,\ x \in \mathbb{R}$ and $\mathrm{g}^{-1}(x) = \ln(x),\ \ x \in \mathbb{R}^+$.

How are the graphs related to each other?

(a)



(b)



Solution:
From the above example (a), we can observe that the graph of f and the graph of f^{-1} are reflections of each other in the line $y = x$.

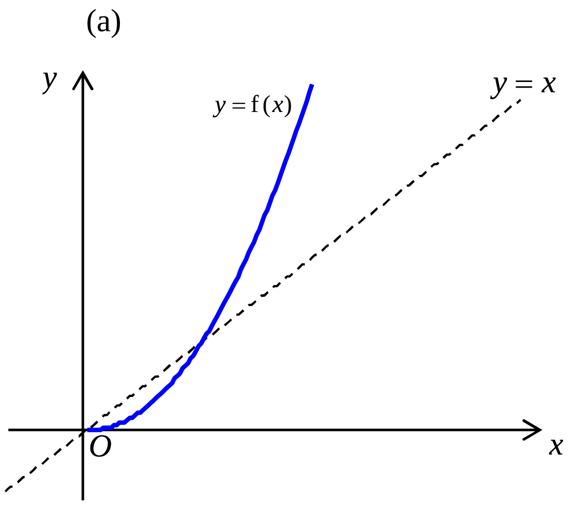
Similarly from example (b) the graph of g and the graph of g^{-1} are reflections of each other in the line $y = x$.

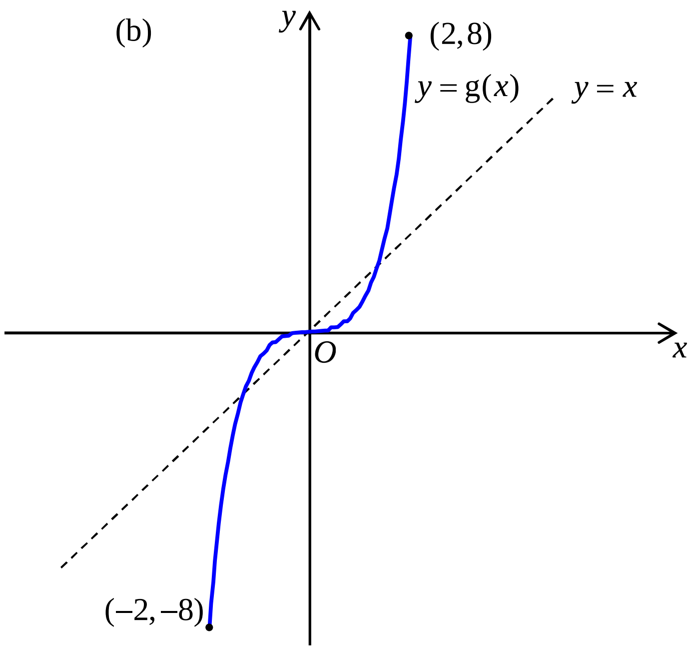
In general,

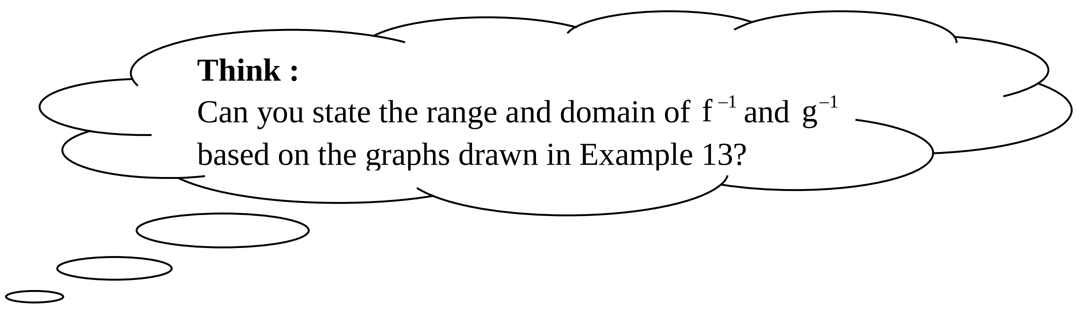
1) Graphs of the function and its inverse are reflections of each other in the line $y = x$.

2) Graphs of the function and its inverse intersect (if any) along $y = x$. i.e. graphs of $y = f(x)$, $y = f^{-1}(x)$ and $y = x$ have common point(s) of intersection, if any.

<u>Example 13</u>
On the same diagram, sketch the graph of the inverse function of
(a) $f(x) = x^2$, $x \geq 0$ (b) $g(x) = x^3$, $x \in [-2, 2]$.
Showing clearly the relationship of the function and its inverse.

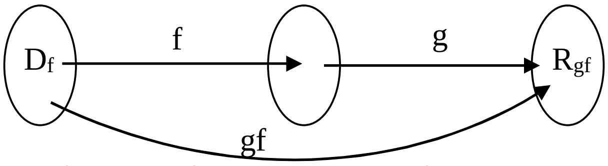






3. Composite Functions

The term "composite function" refers to the combining of functions in a manner where the output(range) from one function becomes the input(domain) for the next function.



If we have two functions f and g, we can define a composite function gf .

For example, if $\mathrm{f}(x)=x^3$ and $\mathrm{g}(x)=2x-1$,
we have $\mathrm{gf}(x)=\mathrm{g}(\mathrm{f}(x))$

$$=\mathrm{g}(x^3)$$
$$=2(x^3)-1$$

On the other hand, we can define the composite function fg .
In this case we have $\mathrm{fg}(x)=\mathrm{f}(\mathrm{g}(x))$

$$=\mathrm{f}(2x-1)$$
$$=(2x-1)^3$$

Notice that $\mathrm{gf}(x)$ and $\mathrm{fg}(x)$ are different.

Note:

- The order of the functions is important: gf and fg are not the same function
- gf(x) means "apply f to x first to get f(x), then apply g to f(x) to obtain gf(x)."
- $\mathrm{f}^2(x)$ does not mean $\left[\mathrm{f}(x)\right]^2$. $\mathrm{f}^2(x)$ refers to the composite function $\mathrm{ff}(x)$.
  Hence, in general, $\mathrm{f}^n(x)$ refers to the composite function $\mathrm{f}\ldots\mathrm{f}\left(x\right)$

Looking at the diagram above, can you deduce what is the **domain of gf** ?

Example 14

Functions f and g are defined by $\mathrm{f}: x \to x^2, x \in \mathbb{R}^+$,
$\mathrm{g}: x \to 2-3x, x \in \mathbb{R}$.

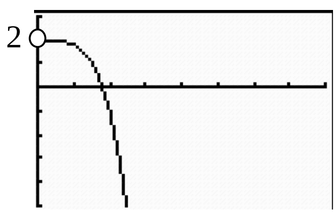
For the composite function gf, find its rule, domain and range.

Solution:

gf(x) = g(f(x)) = g(x^2) = 2 – 3x^2

Domain of gf = Domain of f = $(0,\infty)$

Range of gf is $(-\infty,2)$



Graph of y = gf (x)

3.1 Existence of composite function

In section 2, we saw that a function f has an inverse f^{-1} provided f is one-one. Similarly, certain conditions must be satisfied before a composite function fg can be formed (or "for fg to exist".)

The condition is: $\boxed{\textbf{Composite function fg exists if and only if } \mathbf{R_g \subseteq D_f}\,.}$

To understand this condition, recall: a function comprises a rule and domain, such that the rule can be applied to ***every*** element in the domain. Therefore, **why is this condition $\mathbf{R_g \subseteq D_f}$ necessary for fg to be formed?**

In composing the function $\mathrm{f}(\mathrm{g}(x))$, you have to be careful that the range of g is in the domain of f: that is, that g doesn't have any outputs for which f is undefined.

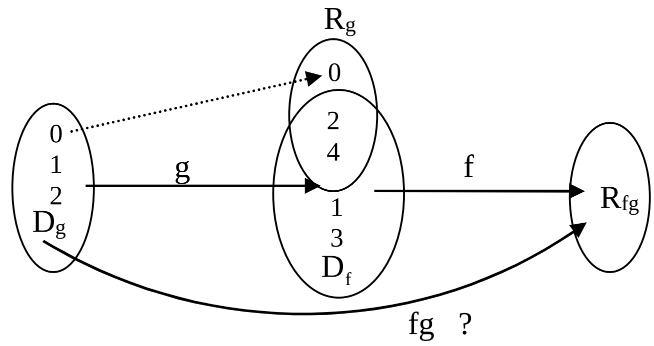
If $\mathbf{R_g} \not\subset \mathbf{D_f}$, then there are elements in $\mathbf{R_g}$ that cannot be mapped under f.

Consider the following functions: $\mathrm{f}: x \to \dfrac{1}{x},\quad x \in \{1, 2, 3, 4\}$,

$\mathrm{g}: x \to 2x,\quad x \in \{0, 1, 2\}$.

Then $\mathrm{fg}(0) = \mathrm{f}(0)$ which is NOT defined under mapping of f.

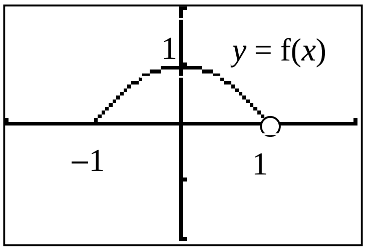
Therefore, the composite function fg does not exist.

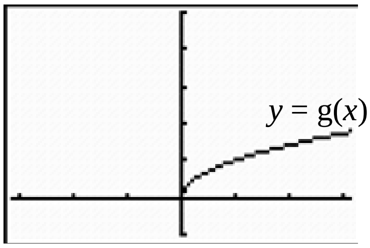


Example 15

Given that f(x) = $1-x^2$, $-1 \le x < 1$ and g(x) = $\sqrt{x}$, $x \ge 0$.
(i) Sketch separately the graphs of f and g.
(ii) State, giving your reasons, whether fg and gf exist. For the composite function that exists, find its rule, domain and range.

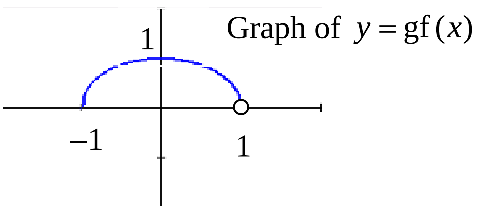
Solution:
(i)





(ii)

Thus, only function gf exists, and is defined by
$\text{gf}\,(x) = \text{g}\left(1-x^2\right) = \sqrt{1-x^2}$, $-1 \le x < 1$



Range of gf is [0,1]

Alternatively, when f(x) = $1-x^2$, $-1 \le x < 1$ and g(x) = $\sqrt{x}$, $x \ge 0$,

Range of gf can also be obtained by a mapping method shown below:

$\overset{\text{D}_\text{f}}{[-1,1)} \xrightarrow{\text{f}} [0,1] \xrightarrow{\text{g}} \overset{\text{R}_\text{gf}}{[0,1]}$

$\therefore \text{R}_{\text{gf}} = [0,1]$

Note :

Since a composite function is a function too, it is possible for us to talk about its inverse, if it exists. The following example gives a good illustration on this.

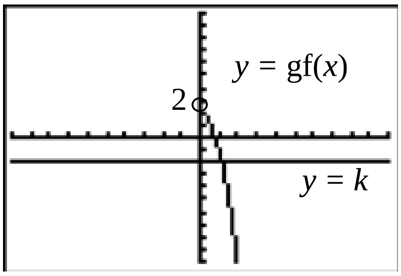
Example 16

The functions f and g are defined by f: $x \to x^2$, $x \in \mathbb{R}^+$ and g: $x \to 2-3x$, $x \in \mathbb{R}$. Use the answer in Example 14 to show that the function $(\mathrm{gf})^{-1}$ exists. State its rule, domain and range. Sketch the graph of $y = (\mathrm{gf})^{-1}(x)$.

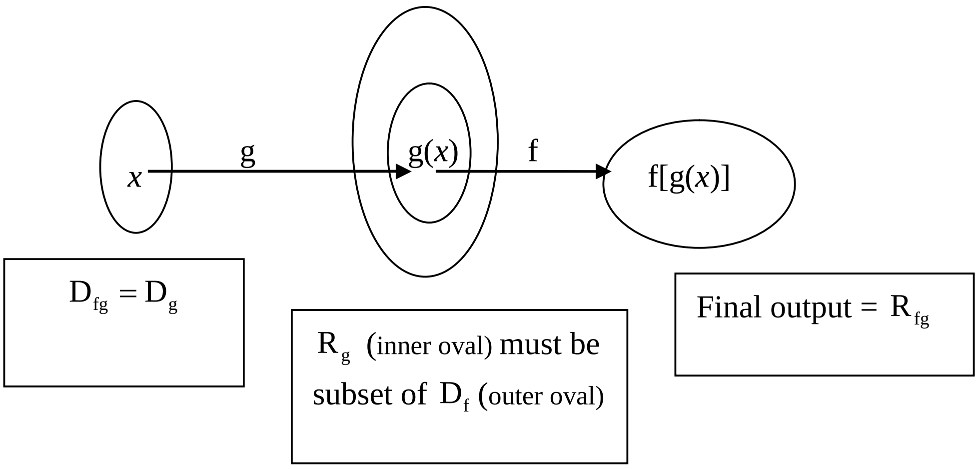
Solution:

From Example 14, we have the composite function $\mathrm{gf}(x) = 2-3x^2, \ \ x > 0$.

Any horizontal line $y = k$ cuts the graph at most once, gf is a one-one function, hence $(\mathrm{gf})^{-1}$ exists.



3.2 Pictorial Summary of Composite Function fg



4. Some special functions

(1) Periodic functions

A function f satisfies $f(x)=f(x+a)$ for all real x where a is a positive constant , is called a periodic function with period a. The graph of $f(x)$ will repeat itself every a units.

Example: $f(x)=\sin x$ is a periodic function with period 2π.

Name other periodic functions that you have learnt at O level.

(2) Even function

A function f satisfies $f(-x)=f(x)$ for all real x is called an even function. The graph of f is symmetrical about the y-axis.

Example: $f(x)=x^2$ is an even function.

Name other even functions that you have learnt at O level.

(3) Odd function

A function f satisfies $f(-x)=-f(x)$ for all real x is called an odd function. The graph of f is symmetrical about the origin.

Example: $f(x)=x^3$ is an odd function.

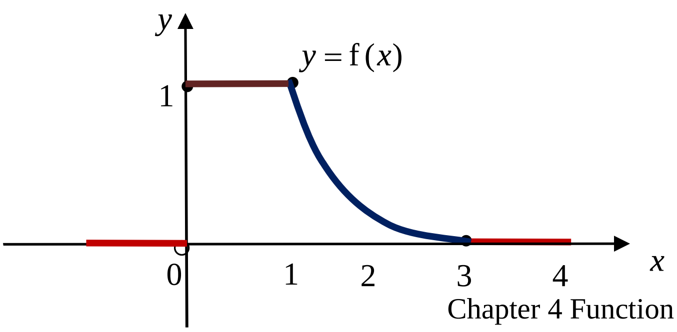
Name other odd functions that you have learnt at O level.

(4) Piecewise function

A **piecewise function** is a **function** which is defined by multiple sub-**functions**, each sub-**function** applying to a certain interval of the main **function's** domain (a sub-domain).

Example: A curve has equation $y=f(x)$, where

$$f(x)=\begin{cases} 1 & \text{for } 0\le x\le 1, \\ \frac{1}{4}(x-3)^2 & \text{for } 1<x\le 3, \\ 0 & \text{otherwise.} \end{cases}$$



<u>Example 17</u> [2009/I/Q4(i)(ii)]

It is given that

$$f(x) = \begin{cases} 7 - x^2 & \text{for } 0 < x \le 2, \\ 2x - 1 & \text{for } 2 < x \le 4, \end{cases}$$

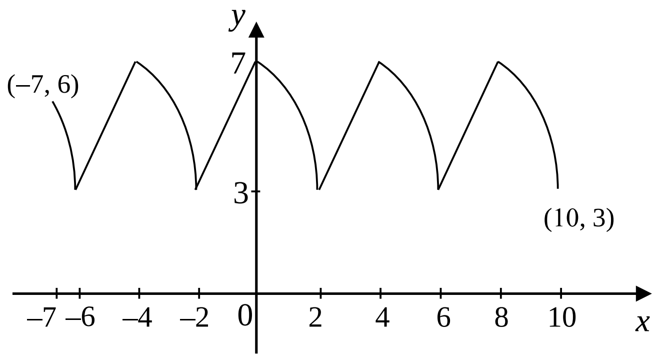
and that $f(x) = f(x+4)$ for all real values of x.

(i) Evaluate f(27) + f(45). [2]

(ii) Sketch the graph of $y = f(x)$ for $-7 \le x \le 10$. [3]

Solution:

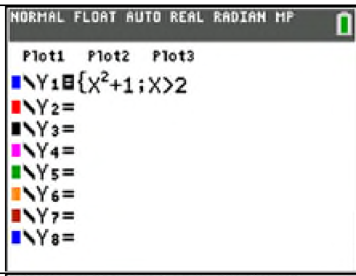
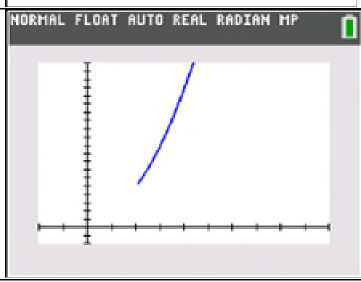
(ii)



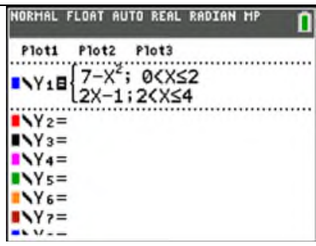
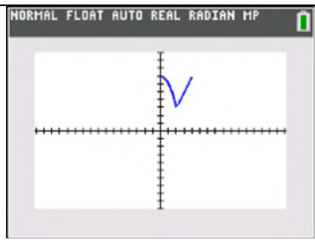
<u>5. Annex</u>

5.1 Using GC to sketch the graphs of f in D_f.

Example 1: Sketch the graph of $f(x) = x^2 + 1, \quad x > 2$.

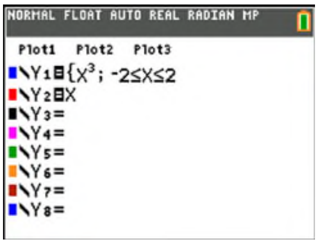
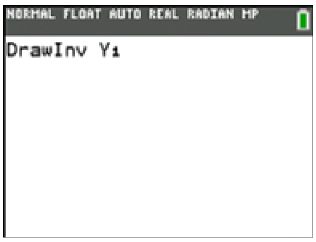
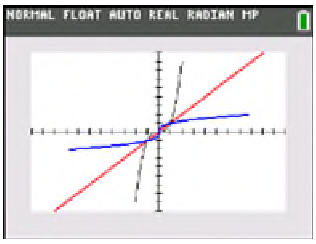
Steps / Explanation		Screen Display
Step 1	In the [Y=] window, press [MATH] then scroll up or down and select **B: piecewise(** . Use [◄] or [►] to select 1 for the number of pieces. Press [▼] [enter] to select OK Key in the function and the domain. **Note:** Use [2nd] [MATH] 3: to select >	
Step 2	Press **[GRAPH]**. (You may adjust the window setting.)	

Example 2: Sketch the graph of $\mathrm{f}(x)=\begin{cases}7-x^2 & \text{for} \quad 0<x\le 2\\ 2x-1 & \text{for} \quad 2<x\ge 4\end{cases}$

	Steps / Explanation	Screen Display
Step 1	In the **Y=** window, press **math** and select **B: piecewise(** . Enter the number of sub-functions and key in the respective sub-functions and sub-domains for $\mathbf{Y_1}$.	
Step 2	Press **GRAPH**.	

5.2 Using GC to sketch the graphs of f and f^{-1} .

E.g: Given $\mathrm{f}: x \mapsto x^3, \quad -2\le x\le 2$, sketch the graph of f and f^{-1} on the same diagram.

	Steps / Explanation	Screen Display
Step 1	To sketch the graph of $y=\mathrm{f}(x)$ and $y=x$: In the **Y=** window, enter $\mathbf{X^3}$ for $\mathbf{Y_1}$ and its domain and **X** for $\mathbf{Y_2}$. **Note**: We enter **X** for $\mathbf{Y_2}$ to illustrate the relationship between the graph of f and f^{-1} . [To prepare for step 2, use Zoom In and Zoom Square]	
Step 2	To sketch the graph of $y=\mathrm{f}^{-1}(x)$: Press **2nd** **prgm** (draw) and select **8:DrawInv**. This takes you to the Home Screen. Now press **alpha** **trace** (f4) and select $\mathbf{Y_1}$. Upon pressing **enter**, it returns you to the **GRAPH** window, where you will see the graphs of f and f^{-1} together. Press **zoom** and choose 5: Zsquare to use the same scale for both the *x*- and *y*-axis. (You need to do Step 2 again every time you change the Zoom settings.)	 

Note: (i) To clear the graph of the inverse function: 2nd prgm 1:ClrDraw

(ii)The GC will always show a graph using the "DrawInv" function even if f does not have an inverse.