RIVER VALLEY HIGH SCHOOL
2024 JC1 Promotional Examination
Higher 2

NAME	

CLASS							INDEX NUMBER		

MATHEMATICS **9758/01**

Paper 1 **26 September 2024**

3 hours

Additional Materials:
List of Formulae (MF27)
Cover Page
Answer Papers

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.
Write your name, class and index number in the space at the top of this page.

Write your name and class on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.
Up to **2 marks may be deducted** for poor presentation in your answers.

At the end of the examination, place the cover page on top of your answer paper and fasten all your work securely together.

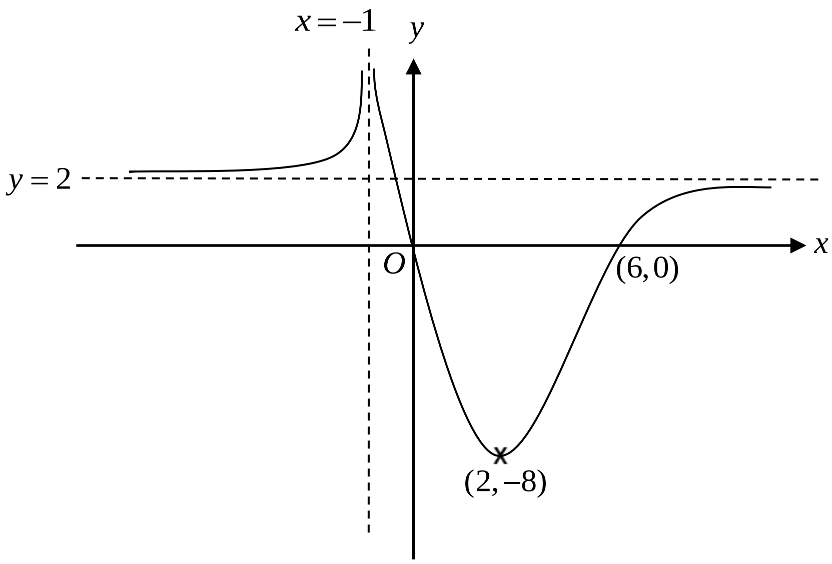
The number of marks is given in brackets [ ] at the end of each question or part question.

1 A curve C has equation $y=\frac{a}{x^2}+\frac{b}{x}+c$, where a, b and c are constants. It is given that C passes through the point with coordinates $\left(-2,-\frac{11}{2}\right)$ and has a turning point at $x=1$.

The gradient of C is $-\frac{3}{2}$ at the point where $x=2$.

Find the values of a, b and c and state the equation of C. [5]

2 The diagram below shows the graph of $y=\mathrm{f}(x)$. The curve crosses the x-axis at the origin and at the point $(6,0)$. The curve also has a minimum point at $(2,-8)$. The equations of the asymptotes of the curve are $x=-1$ and $y=2$.



On separate diagrams, sketch the following graphs, indicating clearly the coordinates of any points of intersections with both axes and any turning point(s), and the equations of any asymptotes where possible.

(i) $y=\mathrm{f}(3x+2)$ [2]

(ii) $y=\frac{1}{\mathrm{f}(x)}$ [3]

3 (i) Solve the inequality $\frac{9}{x^2-5}>-1$ exactly. [3]

(ii) Hence solve $\frac{9}{\mathrm{e}^{2x}-5}>-1$ exactly. [2]

4 It is given that $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$.

(a) Find $\sum_{r=n}^{2n}(r+1)^3$ where $n>1$. Express your answer in the form

$\frac{1}{a}(n+1)^2\left[a(2n+1)^2-n^2\right]$ where a is a constant to be determined. [4]

(b) Find $1^3+3^3+5^3+...+(2k-3)^3+(2k-1)^3$ where k is a positive integer. [2]

5 The curve C has equation $y=\frac{2x^2+ax+20}{x+b}$, where a and b are constants. The equations of the asymptotes of C are $x=4$ and $y=2x-3$.

(i) Find the values of a and b. [3]

(ii) Sketch C, stating the equations of any asymptotes and the coordinates of any axial intercepts and turning points. [3]

(iii) Hence deduce the number of real roots of the equation $2x^2+ax+20=-(x+b)^3$, where $x \neq -b$. [2]

6 (a) Find $\int \frac{\mathrm{P}(x)}{1+x^4}\,\mathrm{d}x$ where

(i) $\mathrm{P}(x)=x^3$, [2]

(ii) $\mathrm{P}(x)=x$. [2]

(b) Find $\int x\tan^{-1}x^2\;\mathrm{d}x$. [4]

7 (a) The function f is defined by $\mathrm{f}(x)=6x-x^2-2,\ x\le -1$.

Define f^{-1} in a similar form, stating its domain. [4]

(b) The function g is defined by $\mathrm{g}(x)=\frac{ax}{x-a},\ x<a,$ where a is a non-zero real number.

(i) Find $\mathrm{g}^2(x)$, giving your answer in simplified form. [2]

(ii) Without finding an expression for $\mathrm{g}^{-1}(x)$, show that $\mathrm{g}(x)=\mathrm{g}^{-1}(x)$. [1]

(iii) Find the range of values of a such that fg^{-1} exists. [2]

8 The curve C has cartesian equation $y = \frac{1}{x}\sqrt{x^2 - 1}$.

(i) Sketch the graph of C for $x \geq 1$. Label the coordinates of the axial intercept and equation of the asymptote. [2]

(ii) Find the equation of the normal to C at the point $P\left(\sqrt{2}, \frac{1}{\sqrt{2}}\right)$, leaving your answer in exact form. [3]

(iii) Using the substitution $x = \sec\theta$, find the exact value of $\int_1^{\sqrt{2}} \frac{1}{x}\sqrt{x^2 - 1}\,\mathrm{d}x$. [3]

(iv) Hence, find the exact value of the area of the region bounded by C, the normal to C at P, and the x-axis. [2]

9 A town developed a waste management system in the year 2014 to treat waste generated. The waste generated can either be sent to the landfill, or incinerated at the incineration plant.

(a) In the landfill, 1250 tonnes of waste is deposited in the year 2014. Due to the increase in activities in the town, experts predict that the amount of waste deposited in the landfill will increase by 12 tonnes in each subsequent year.

The landfill is projected to run out of space at the end of the year 2090. Determine the theoretical maximum amount of waste that the landfill can contain. [2]

(b) In the incineration plant, 1500 tonnes of waste is incinerated in the year 2014. As the town aims to reduce the amount of carbon emissions from incinerating waste, the amount of waste incinerated will decrease by 2% in each subsequent year.

(i) Determine the least number of years from the start of the year 2014 in which the total amount of waste incinerated exceeds 50 000 tonnes. [3]

(ii) In its lifetime, the incineration plant can handle 80 000 tonnes of waste. Explain, with working, whether the incineration plant can cope with the total amount of waste to be incinerated in the future. [2]

(c) Determine the number of years from the start of the year 2014 in which the amount of waste deposited in the landfill first exceeds the amount of waste incinerated in the incineration plant in that particular year. [3]

10 (a) It is given that $y = \ln(2+\sin 2x)$.

(i) Show that $\mathrm{e}^y\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right)+4\sin 2x = 0$. [2]

(ii) By further differentiation of the result in part (a)(i), find the Maclaurin series for $y = \ln(2+\sin 2x)$, up to and including the term in x^3. [3]

(iii) Hence find the Maclaurin series for $\frac{2\cos 2x}{2+\sin 2x}$, up to and including the term in x^2. [2]

(b) Given further that x is small such that x^3 and higher powers of x can be neglected, use appropriate standard series from the List of Formulae (MF27) to verify the correctness of the series of $\frac{2\cos 2x}{2+\sin 2x}$ obtained in part (a)(iii). [3]

11 The curve C has parametric equations

$$x = \frac{3}{\theta} - \theta, \quad y = \frac{2}{\theta} + 2, \quad \text{where } \theta > 0.$$

(a) Sketch the graph of C. [1]

(b) A particle is moving along the curve such that its x-coordinate is increasing at the constant rate of 10 units s^{-1}. Determine the rate of change of θ at the instant when the coordinates of the particle is $(2, 4)$. [3]

(c) (i) Find the equation of the tangent to C at the point P where $\theta = 1$. [2]

(ii) Show that this tangent will not meet the curve C again. [2]

(iii) Find the acute angle between the tangent to C at the point P where $\theta = 1$ and the normal to C at the point M where $\theta = 2$. [2]

(d) The curve C is transformed by a stretch with scale factor 2 in the x-direction and a reflection in the y-axis, to form the curve D. Find the equation of D in parametric form. [2]

12 (a) Engineers model the motion of an object. In a simple model, the engineer assumes that the rate of change of velocity, v ms^{-1}, with respect to time t seconds is a positive constant c.

(i) Write down a differential equation relating v, t and c. [1]

(ii) Using this model, predict the time taken for the velocity of the object to reach 30 ms^{-1}, given that its initial velocity is 0 ms^{-1} and $c = 10$ ms^{-2}. [4]

(b) Due to friction, the decrease in the rate of change of the velocity of the same object can be modelled as proportional to the square root of the velocity at that instant while the object is in motion. The rate of change of velocity of the object can then be more accurately modelled by the equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 10 - k\sqrt{v}.$$

(i) Find, in terms of k, the value of v for which $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$. State what this value of v represents. [2]

(ii) By considering the substitution $u = \sqrt{v}$, show that the general solution to this differential equation can be expressed as

$$2k\sqrt{v} + 20\ln\left|10 - k\sqrt{v}\right| = A - k^2 t,$$

where A is an arbitrary constant. [5]

(You may use the result $\frac{ku}{10 - ku} = \frac{10}{10 - ku} - 1$ where k is a constant.)

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