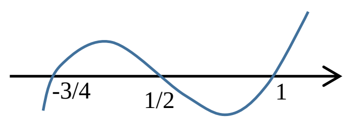
Chapter 1: Equations & Inequalities

Additional Practice Questions Solutions

1
$$\frac{x+2}{2x-1} < 2x+1$$
$$\frac{x+2-(2x+1)(2x-1)}{2x-1} < 0$$
$$\frac{-4x^2+x+3}{2x-1} < 0$$
$$(2x-1)\left(4x^2-x-3\right) > 0$$
$$(2x-1)(4x+3)(x-1) > 0$$
$$-\frac{3}{4} < x < \frac{1}{2} \quad \text{or} \quad x > 1 \quad \text{(ans)}$$



$$\frac{2x^2+1}{2-x^2} < \frac{2+x^2}{x^2}$$

$$\frac{2+\frac{1}{x^2}}{\frac{2}{x^2}-1} < \frac{2}{x^2}+1$$

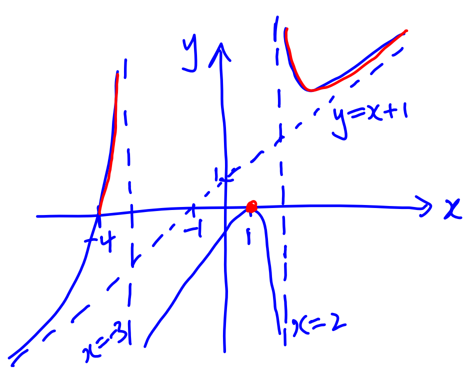
$$\Rightarrow -\frac{3}{4} < \frac{1}{x^2} < \frac{1}{2} \quad \text{or} \quad \frac{1}{x^2} > 1$$

$$\Rightarrow 0 < \frac{1}{x^2} < \frac{1}{2} \quad \text{or} \quad \frac{1}{x^2} > 1$$

$$\Rightarrow x^2 > 2 \quad \text{or} \quad x^2 < 1$$

$$\Rightarrow x > \sqrt{2} \;\; \textbf{or} \;\; x < -\sqrt{2} \quad \text{or} \quad -1 < x < 1,\; x \neq 0$$

2. Sketch $y = \dfrac{x(x+3)(x-3)+16}{(x+3)(x-2)} + 2 \;=\; x+1-\dfrac{2x-10}{(x+3)(x-2)}$

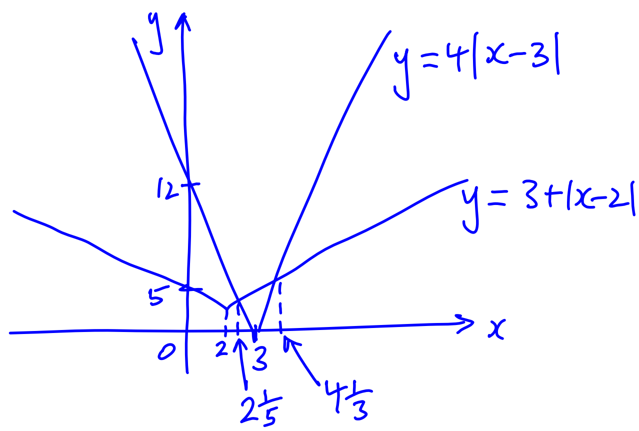


Find $y \geq 0$,
From GC, $-4 \leq x < -3$ or $x > 2$ or $x = 1$

Hence,

Let $y = 2^x$,
$-4 \leq 2^x < -3$(n.a.)
$2^x > 2 \Rightarrow x > 1$
$2^x = 1 \Rightarrow x = 0$
$\therefore x > 1$ or $x = 0$

3. (a)



$4|x-3| - |x-2| > 3$,
$4|x-3| > 3 + |x-2|$
From GC, $x < 2\frac{1}{5}$ or $x > 4\frac{1}{3}$

(b)

$\frac{x^2+3}{x+1} > 2$,
$(x+1)\left[x^2+3-2(x+1)\right] > 0$
$(x+1)(x^2-2x+1) > 0$
$(x+1)(x-1)^2 > 0$
$x+1 > 0$, since $(x-1)^2 \geq 0$
$x > -1$, $x \neq 1$

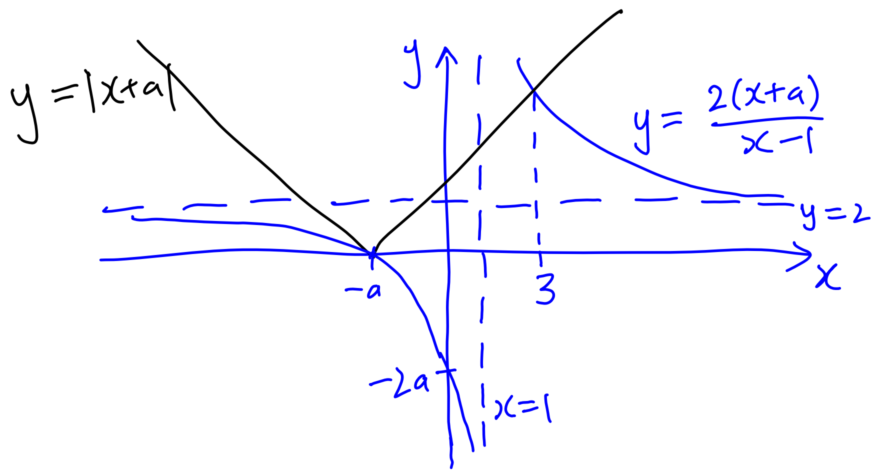
For $\dfrac{x^2+3}{1-x} > 2$, let $y=-x$

$-x > -1$

$x < 1,\ x \neq -1$

4. $y = \dfrac{2\left(x+a\right)}{x-1} = 2 + \dfrac{2-2a}{x-1}$

Horizontal asymptote: $y = 2$; vertical asymptote: $x = 1$



Therefore, solution set: $\left\{x : x \in \mathbb{R},\ x < 1 \text{ or } x \geq 3\right\}$.

5. (i) $y = ax^3 + bx^2 + cx + d$

When x = 0, $y = -8$, we get $d = -8$

(ii) So, $y = ax^3 + bx^2 + cx - 8$

When x = 4, y = 0, we get

$64a + 16b + 4c = 8 \Rightarrow 16a + 4b + c = 2$ ---- (1)

When x = 3, $y = -8 - 2(3) = -14$, we get

$27a+9b+3c-8=-14 \Rightarrow 9a+3b+c=-2$ ---- (2)

$$\frac{\mathrm{d}y}{\mathrm{d}x}=3ax^2+2bx+c$$

When x = 2, $\frac{\mathrm{d}y}{\mathrm{d}x}=-2$, we get

$12a+4b+c=-2$ ---- (3)

Putting equations (1), (2) and (3) together and using the GC to solve,
We get a = 1, b = −3 and c = −2.
Therefore, equation of curve is $y=x^3-3x^2-2x-8$.

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6. Let x, y and z be the cost of a ticket for "under 16 years", "between 16 and 65 years", and "over 65 years" categories respectively.

$9x + 6y + 4z = 162.03$
$7x + 5y + 3z = 128.36$
$10x + 4y + 5z = 158.50$

For "under 16", ticket costs $7.65.
For "between 16 and 65 years", ticket costs $9.85.
For "over 65 years", ticket costs $8.52.

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7. (a) Let x = 1, 2, 3, 4 (eg. x = 1 for year 1920), y represents the population in millions.
Let the cubic polynomial be $y=ax^3+bx^2+cx+d$

Sub. (1, 106), (2, 123), (3, 132), (4, 151) into the expression,

$a+b+c+d=106$

$8a+4b+2c+d=123$

$27a+9b+3c+d=132$

$64a+16b+4c+d=151$

By using GC, $a=3, b=-22, c=62, d=63$

$y=3x^3-22x^2+62x+63$

When $x=5$ (year 1925), $y=198$ millions

(b) My estimate is higher than the actual value. This value of x (for year 1960) is outside the data range for us to make a reliable prediction.

8.

Let x, y and z be the prices per gram of swordfish, salmon and tuna respectively.

$$4.6x + 2.4y + Tz = 64.60$$
$$8.6x + 3.6y + Tz = 102.20$$
$$4.8x + 2.8y + Tz = 68.00$$

By using GC: $x = 8.05882$, $y = 4.47059$, $Tz = 16.8$

Given $y < z < x$, so

$$4.47059 < z < 8.05882$$
$$\frac{1}{4.47059} > \frac{T}{16.8} > \frac{1}{8.05882}$$
$$\frac{16.8}{4.47059} > T > \frac{16.8}{8.05882}$$
$$3.75789 > T > 2.08467$$

Since T is an integer, so $T = 3$ and $z = 5.6$

Ans: $x = 8.06$, $y = 4.47$, $z = 5.6$, $T = 3$

9. 3. Since a, b, c are sides of a triangle, they satisfy the triangle inequality. Assume that $a + b > c$.

(i) show $\frac{a}{1+a} + \frac{b}{1+b} > \frac{c}{1+c}$.

$$\frac{a}{1+a} + \frac{b}{1+b} - \frac{c}{1+c} = \frac{a(1+b)(1+c) + b(1+a)(1+c) - c(1+a)(1+b)}{(1+a)(1+b)(1+c)}$$

$$= \frac{a(1+b+c+bc) + b(1+a+c+ac) - c(1+a+b+ab)}{(1+a)(1+b)(1+c)}$$

$$= \frac{a+b-c+2ab+abc}{(1+a)(1+b)(1+c)} > 0 \quad \text{since } a+b>c \text{ and } a, b, c > 0.$$

$\therefore \frac{a}{1+a}, \frac{b}{1+b}, \frac{c}{1+c}$ form sides of a $\triangle$.

(ii) show $\sqrt{a} + \sqrt{b} > \sqrt{c}$

consider $(\sqrt{a} + \sqrt{b})^2 = a + b + 2\sqrt{ab}$

$$> c + 2\sqrt{ab}$$
$$> c \quad \text{since } \sqrt{ab} > 0$$

$\Rightarrow \sqrt{a}+\sqrt{b} > \sqrt{c}$ since $\sqrt{a}, \sqrt{b}, \sqrt{c} > 0$ and the 3 terms form the 3 sides of a $\triangle$

(iii)
$\left(\sqrt{a(b+c-a)}+\sqrt{b(c+a-b)}\right)^2-\left(\sqrt{c(a+b-c)}\right)^2$

$= a(b+c-a)+b(c+a-b)+2\sqrt{ab(b+c-a)(c+a-b)} - c(a+b-c)$

$= ab+ac-a^2+bc+ab-b^2+2\sqrt{ab(b+c-a)(c+a-b)} - ac-bc+c^2$

$= -a^2-b^2+2ab+c^2+2\sqrt{ab(b+c-a)(c+a-b)}$

$= c^2-(a-b)^2+2\sqrt{ab(b+c-a)(c+a-b)}$

W/o loss of generality, assume $a>b$.
Then we required $c>a-b$, or else if $c<a-b$, then $c+b<a$ which is a contradiction since a, b, c are sides of a $\triangle$.

$b+c-a>0$ & $c+a-b>0$ for a, b, c being sides of a $\triangle$.

$\therefore c^2-(a-b)^2+2\sqrt{ab(b+c-a)(c+a-b)} > 0$

$\Rightarrow \sqrt{a(b+c-a)}+\sqrt{b(c+a-b)} > \sqrt{c(a+b-c)}$ and the 3 terms form the 3 sides of a $\triangle$.

10. **[2020 ASRJC**

Hence solve the inequality $\dfrac{2e^{2x}+3e^{x}}{e^{2x}+e^{x}-6} \geq 1$, leaving your answers in exact form. [2]

$\dfrac{2x^2+3x}{x^2+x-6} \geq 1$ $\dfrac{2x^2+3x}{x^2+x-6}-1 \geq 0$ $\dfrac{2x^2+3x-(x^2+x-6)}{x^2+x-6} \geq 0$ $\dfrac{x^2+2x+6}{x^2+x-6} \geq 0$ $\dfrac{(x+1)^2+5}{(x+3)(x-2)} \geq 0$ Since $(x+1)^2+5>0$ $(x+3)(x-2)>0$ $x<-3$ or $x>2$
For $\dfrac{2e^{2x}+3e^{x}}{e^{2x}+e^{x}-6} \geq 1$, replace x by e^x. $e^x<-3$ or $e^x>2$

For $e^x < -3$, there is no real solution.
For $e^x > 2 \Rightarrow x > \ln 2$

11. [**2019 EJC Promo Q1**]

At the start of the year, Mr Toh invested in three types of savings bonds, namely "Ucare", "Ushare" and "Ugain". The amount invested in "Ugain" is equal to the sum of the amounts invested in the other 2 bonds. In addition, the sum of the amount invested in "Ushare" and twice the amount invested in "Ugain" is 8 times the amount invested in "Ucare".

At the end of the year, "Ucare", "Ushare" and "Ugain" paid out interest at a rate of 2.5%, 1.75% and 3% respectively. Mr Toh received a total of $657.60 in interest.

Express this information as 3 linear equations and hence find the amount invested in savings bond "Ushare". [4]

Let c, s and g be the amount of investments in saving bonds 'Ucare', 'Ushare' and 'Ugain' respectively.

Amount invested in bond 'Ugain' equals the sum of the amounts invested in the other 2 bonds:

$c+s=g$

$c+s-g=0$ ----- (1)

The sum of the amount invested in "Ushare" and twice the amount invested in "Ugain" is 8 times the amount invested in "Ucare".

$s+2g=8c$

$8c-s-2g=0$ -----(2)

Total interest of $657.60:

$0.025c+0.0175s+0.03g=657.60$ ----(3)

Solving simultaneously,

$c=\$4384$, $s=\$8768$ and $g=\$13152$

Total amount of investments for Ushare is $8768

12. [**2020 TJC J1 MYE Q3**]

(i) Solve algebraically the inequality $\frac{2a}{x-a} \le 1$, where a is a positive constant. [3]

(ii) Hence solve the inequality $\frac{2}{(\ln x)-\frac{1}{3}} \le 3$, leaving your answers in exact form. [4]

$$\frac{2a}{x-a} \le 1 \quad \Rightarrow \quad \frac{2a}{x-a} - 1 \le 0$$

$$\Rightarrow \quad \frac{2a-(x-a)}{x-a} \le 0$$

$$\Rightarrow \quad \frac{3a-x}{x-a} \le 0$$

$$\Rightarrow \quad (3a-x)(x-a) \le 0 \quad \text{and} \quad x \ne a$$

$$\Rightarrow \quad x < a \quad \text{or} \quad x \ge 3a$$

$$\frac{2}{(\ln x)-\frac{1}{3}} \le 3 \quad \Rightarrow \quad \frac{2\left(\frac{1}{3}\right)}{(\ln x)-\frac{1}{3}} \le 1$$

Replacing x by $\ln x$ and let $a = \frac{1}{3}$, we have

$$\ln x < \frac{1}{3} \quad \text{or} \quad \ln x \ge 1$$

$$\Rightarrow \quad 0 < x < \mathrm{e}^{\frac{1}{3}} \quad \text{or} \quad x \ge \mathrm{e}$$