SERANGOON GARDEN SECONDARY SCHOOL



Vision:Learners with Grit, Leaders with HeartMission:Nurturing and empowering individuals to lead purposeful lives

PRELIMINARY EXAMINATION 2021

CANDIDATE NAME	
CLASS	REGISTER NUMBER

MATHEMATICS Paper 2 Secondary 4 Express/ 5 Normal Academic

4048/02 24 Aug 2021 2 hours 30 minutes 0800 - 1030

Candidates answer on the Question Paper.

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READ THESE INSTRUCTIONS FIRST

Write your name, class and class register number on all the work you hand in. Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question, it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question. The total of the marks for this paper is 100.

		FOR MARKER'S USE
Name/Signature of Parent/Guardian	Date	100

This question paper consists of <u>19</u> printed pages and <u>1</u> blank page.

Setter: Mr Poh WR

Vetter: Ms Tay HY

Mathematical Formulae

Compound interest

Total amount =
$$P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Surface area of a sphere $= 4\pi r^2$ Volume of a cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of a cone = πrl

Volume of a sphere =
$$\frac{4}{3}\pi r^3$$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

Mean =
$$\frac{\sum fx}{\sum f}$$

Standard deviation = $\sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$

Answer **all** the questions.

1 (a) Simplify $\frac{4ab^2}{3} \div \frac{2a^{-2}}{b}$, leaving your answer in positive index.

(b) Solve the equation $9^{(2x+1)} = 3^{-x}$

(c) Given that $x = \frac{2}{a} + \frac{1}{b}$, express b in terms of a and x.

$$Answer \ b = \dots \qquad [2]$$

(d) Simplify
$$\frac{4x^2 - 16}{3x^2 - 5x - 2}$$
.

(e) Solve these simultaneous equations.

$$x - 2y = 10$$
$$2x - y = 8$$

- 2 A motorist cycled 55 km from Town *A* to Town *B* at an average speed of *x* km/h. He returned from Town *B* to Town *A* by the same route. His average speed for the return journey was 10 km/h less than the journey from Town *A* to Town *B*.
 - (a) Write down an expression, in terms of *x*, for the number of hours taken to travel from Town *A* to Town *B*.

Answer hours [1]

(b) Write down an expression, in terms of *x*, for the number of hours taken to travel from Town *B* to Town *A*.

Answer hours [1]

- (c) He took 1 hour 15 minutes more on the return journey than on the outward journey.
 - (i) Write down an equation in x and show that it reduces to $x^2 10x 440 = 0$.

Answer

[3]

(ii) Solve the equation $x^2 - 10x - 440 = 0$.

Answer $x = \dots$ or \dots [3]

(iii) Hence, find the total time he was cycling in hours and minutes, rounding your answer to the nearest minute.

3 (a) Two spinners with numbers on them are shown below. They are divided equally into two sectors and three sectors respectively.



(i) List all possible outcomes <u>using a tree diagram</u> when both spinners are spun. [2]

Tree Diagram

(ii) Find the probability that the sum of the two numbers where the pointers of the spinners land on is more than 5.

(b) Dylan picked 2 balls from a bag containing 3 red balls and x green balls without replacement. If the probability of picking 1 red and 1 green ball is $\frac{15}{28}$ find the

replacement. If the probability of picking 1 red and 1 green ball is 28, find the possible number of green balls in the bag initially.

The variables *x* and *y* are connected by the equation

Answer

$$y = \frac{1}{4}x(x^2 - 5)$$

Some corresponding values of *x* and *y* are given in the table below.

4

x	-3	-2	- I	0	1	2	3
У	-3	0.5	1	0	- I	-0.5	3

(a) On graph paper below, draw the graph of $y = \frac{1}{4}x(x^2 - 5)$ for $-3 \le x \le 3$.



[3]

[4]

(b)	$\frac{1}{4}x(x^2-5) = 2$ The equation $\frac{1}{4}x(x^2-5) = 2$ has only 1 solution. Explain how this can be seen from your graph.	
	Answer	
		[2]

(c) By drawing a tangent, find the gradient of the curve at the point (2, -0.5).

(d) (i) On the same grid, draw the line $y = \frac{1}{2}x + 1$ for $-3 \le x \le 3$. [2]

(ii) Write down the *x*-coordinates of the points where this line intersects the curve.

(iii) Show that the points of intersection of the line and the curve satisfy the equation $x^3 - 7x - 4 = 0$

- (e) Using your graph, find the coordinates of another point on the curve with the same gradient as the answer in part (c).
- Answer (....., ,) [1]
 5 A shop makes mini cakes and operates 7 days a week. The matrix, *M*, shows the number of cakes of different types that are made each day.

	Regular	Deluxe	Premiun	n
	_(60	40	30)	Caramel
M	= 40	50	20)	Strawberry

(a) Evaluate the matrix P = 7M

Answer $\boldsymbol{P} = \dots$ [1]

(b) Regular cakes cost \$1 each to make. Deluxe cakes cost \$1.50 each to make. Premium cakes cost \$2.50 each to make. By representing these amounts in a 3×1 column matrix *N*, evaluate the matrix T = PN.

Answer $T = \dots$ [2]

(c) State what each of the elements of *T* represents.

Answer	
	[1]

(d) The shop sells each cake for 40% more than the cost price. On a certain week, they $\underline{6}$

sold $\overline{7}$ of each type for both caramel and strawberry cakes. The unsold cakes were given away to staff.

(i) Show that the cakes sold for that week can be represented by the matrix [2]

L	_	1	

(360	240	180
240	300	120)

Answer

(ii) Hence, calculate the profit the bakery made that week.

7 , 13 , 19 , 25

(i) Write down the 8^{th} term of the sequence.

Answer [1]

(ii) Find an expression, in terms of *n*, for the *n*th term of the sequence

(iii) Explain whether 256 is part of the sequence. Show your working clearly.

Answer

(b) The 4th and 8th term of the sequence are the base area (in cm²) of two geometrically similar cones, A and B, respectively. If the volume of cone A is 437.5 cm³, find the volume of cone B.



The diagram shows a semicircle, centre at the origin $O^{(0,0)}$, with radius 13 cm.

13

Chord AB is parallel to the x-axis and M is the midpoint of AB. Angle AOB = 134.8° .

(a) Calculate the shaded area.

(b)

7

The equation of the line *OA* is y = kx, where *k* is a real number. Show that *k* is approximately 0.4 when rounded off to 1 decimal place.



E

In the diagram, A, B, C, D, E and F are points on a circle with centre O. Angle $BAD = 60^{\circ}$ and angle $OEF = 40^{\circ}$.

Find (i) angle *ADB*

Answerº [2]

[4]

(ii) angle *EOF*

8 (a)

Answer^o [2]

(iii) angle ECF

Answer^o [1]

(b) Given *OF* is 10 cm, find the length of *EF*.

9



E

15

The diagram shows *ABCDE* with side 20 E is vertically above base, *X*.

a solid square pyramid cm. the centre of the square

(a) Given that angle EAX is 60°, show that the vertical height EX is 24.5 cm.

(b) Find the total surface area of the pyramid.

Answer cm² [3]

(c) The pyramid *ABCDE* is fully filled with sand. The sand is then poured into the container below, made up of two cuboids both with length 40 cm. Find the height of the sand in this new container.



10 A spot light was placed on the roof of a building *BCDE* at *D* to focus on a guest who will be walking from *A* to *B* (the entrance of the building). The angle of the spot light can

change and is indicated by θ (see picture below).



(a) Find the angle of depression of the entrance of the building from the spotlight.

Answer^o [2]

(b) (i) Find the shortest distance from *C* to the line *AB*.

Answer m [3]

(ii) Given that the smallest possible θ is 45°, is it possible for the spotlight to be on the guest all the time as he walks from *A* to *B*? Justify your answer with clear working.

11 The figure shows a pedestrian walkway joining a multi-storey car park and a departmental store.



To estimate its length, the walkway is modelled by the arc ABC as shown in the figure below, where A is the entrance to the departmental store and C is the exit to the car park. The arc ABC is part of a sector with centre O.



19

Given that AC = 49.65 and angle $ABC = 120.6^{\circ}$, show that

(a) AO = 28.8 m,

(b) the length of arc ABC is 59.8 m.

[2](c) As a safety measure, John is conducting an emergency evacuation drill. Based on his calculations, the success of the drill heavily depends on whether shoppers can evacuate the departmental store via the walkway from A to C in less than a minute.

The table below shows the average walking speed of visitors along the walkway to the departmental store from the previous 5 emergency drills at various timings of the day.

	Average walking speed in km/h		
Time Frame	Weekends	Week days	
11 am to 2 pm	4.5	5.0	
2 pm to 5 pm	4.0	4.5	
5 pm to 7 pm	3.5	3.5	

John plans to conduct the emergency drill during one of the time frames shown in the table above. As a colleague, you want the evacuation drill to be a success and decides to advise John to conduct the drill only at certain time frames.

With clear mathematical working, advise John the time frame(s) where he should avoid conducting the evacuation drill.

Answer

END OF PAPER

[3]