Candidate Index Number

Anglo - Chinese School (Independent)

FINAL EXAMINATION 2023
YEAR 3 INTEGRATED PROGRAMME
CORE MATHEMATICS
PAPER 1

Friday **29th September 2023** **1 hour 30 minutes**

Candidates answer on the Question Paper.
No additional materials are required.

INSTRUCTIONS TO CANDIDATES

- Write your index number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Answer all questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- The maximum mark for this paper is 80.

For Examiner's Use

This paper consists of 16 printed pages.

[Turn over

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

*Answer **all** the questions in the spaces provided.*

1. [Maximum mark: 5]

(a) Evaluate $\dfrac{2\frac{3}{4}-\frac{1}{2}}{1-\frac{1}{4}}$. [2]

(b) Simplify $\dfrac{\left(2a^4b^2\right)^3}{14b^{-2}c^6} \div \dfrac{1}{7\sqrt[4]{b^{-16}}}$, leaving your answer in positive index. [3]

2. [Maximum mark: 4]

The graph $y = 2(x-3)^2 + m - 4$ has a minimum point at $(3, -8)$, where m is a constant.

(a) Find the value of m. [1]

(b) Hence, sketch the graph of $y = 2(x-3)^2 + m - 4$. Label the coordinates of the axes-intercepts and turning point clearly. [3]

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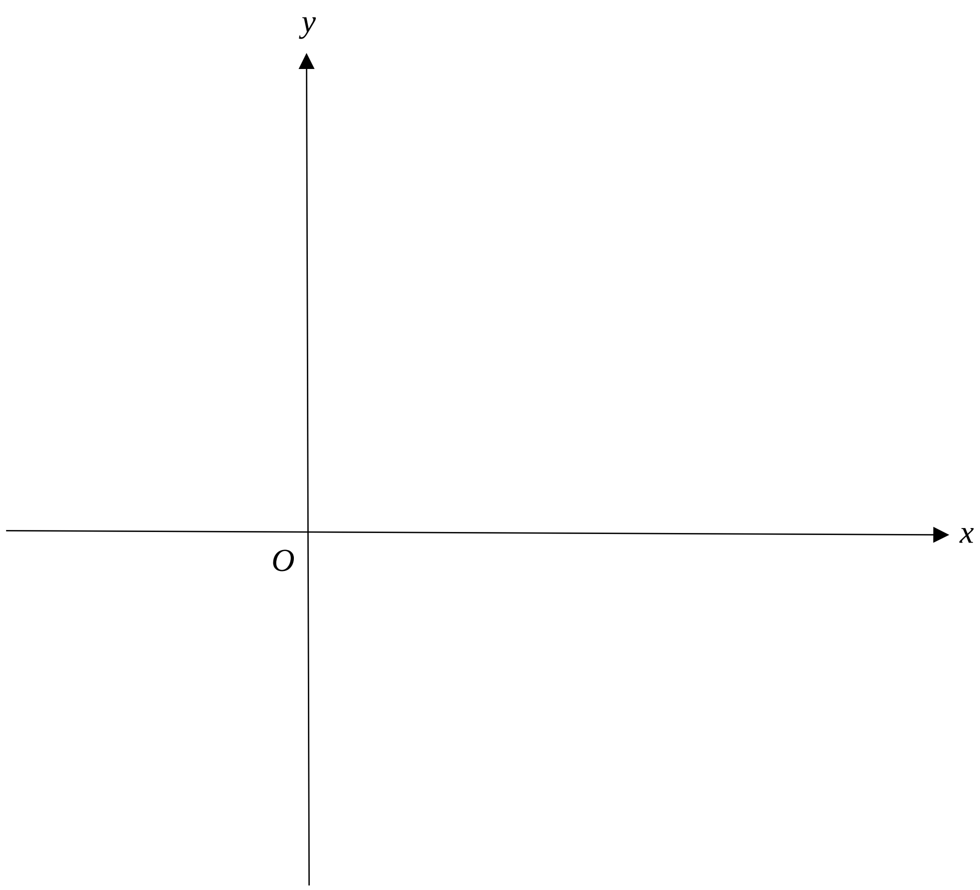

3. [Maximum mark: 7]

(a) Simplify $\frac{\sqrt{108}}{3}+18\sqrt{3}-\frac{4\sqrt{27}}{3}$. [2]

(b) (i) Given that the points $P\left(\sqrt{5},\ 1\right)$, $Q\left(x,\ \sqrt{5}\right)$ and $R\left(3\sqrt{5},\ 3\right)$ lie on the same straight line, find the value of x. [3]

(ii) Find the length of PR, in the form $a\sqrt{b}$ units, where a and b are integers. [2]

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[Continuation of working space for Question 3] ....

4. [Maximum mark: 5]

Solve the following simultaneous equations.

$$\left(\sqrt[3]{5}\right)^{3x} = 125^{y}$$

$$\log_3 x^2 + \log_3 y = 5$$

[5]

5. [Maximum mark: 8]

Given the three points $A(-4,\ -2)$, $B(3,\ 2)$ and $C(0,\ 5)$, find

(a) the equation of the line L_1, which passes through C and is perpendicular to BC, [2]

(b) the coordinates of D, the point where L_1 cuts the line $5y-10=2x$, [3]

(c) the area of the quadrilateral $ABCD$. [3]

6. [Maximum mark: 8]

(a) (i) Express $\dfrac{x}{x^2-1}+\dfrac{3}{x+1}$ as a single fraction in its simplest form. [2]

(ii) Hence, solve $\dfrac{x}{x^2-1}+\dfrac{3}{x+1}=0$. [1]

(b) Solve $2x-6<\dfrac{3(3-x)}{4}\le\dfrac{-x+4}{2}$ and state all the integers that satisfy the inequality. [5]

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[Continuation of working space for Question 6] ....

7. [Maximum mark: 7]

(a) Factorise $25-4a^2+20ab-25b^2$ completely. [3]

(b) Solve $\dfrac{32^{x^2}}{16}=2^{8x}$. [4]

8. [Maximum mark: 10]

The roots of the quadratic equation $2x^2 - 3x - 4 = 0$ are α and β.

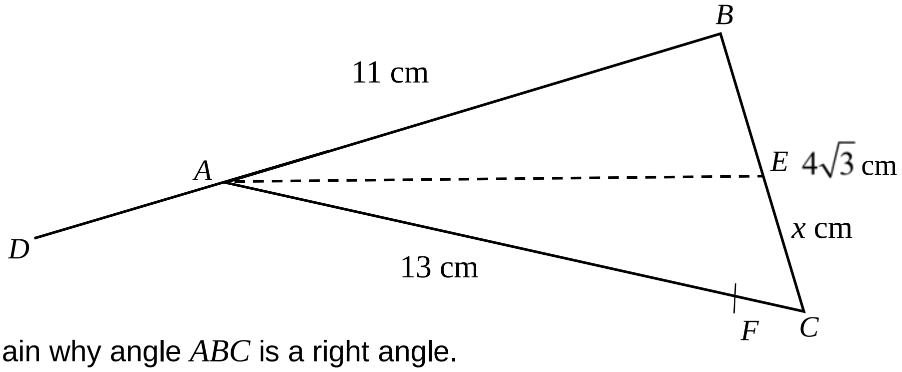
(a) State the value of $\alpha + \beta$ and of $\alpha\beta$. [2]

(b) Find the value of $\alpha^2 + \beta^2$. [3]

(c) Hence or otherwise, find the quadratic equation with roots $\alpha + \frac{\beta}{2}$ and $\frac{\alpha}{2} + \beta$. [5]

9. [Maximum mark: 9]

In triangle ABC, $AB = 11\,\text{cm}$, $BC = 4\sqrt{3}\,\text{cm}$ and $AC = 13\,\text{cm}$. BA is produced to D.



(a) Explain why angle ABC is a right angle. [1]

(b) Expressing your answer as a fraction in its simplest form, find

(i) $\cos\angle DAC$, [1]

(ii) $\dfrac{\tan\angle ACB}{\cos\angle BAC}$. [3]

(c) Given that E is a point on BC such that $EC = x\,\text{cm}$ and F is a point on AC such that AF is the reflection of AB in the line AE, calculate the value of x, leaving your answer in its simplest form. [4]

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[Continuation of working space for Question 9]

10. [Maximum mark: 7]

(a) The curve $y = (p-3)x^2 - 4x + p$ has a minimum point and it cuts the x-axis at two points. Find the range of values of p. [4]

(b) (i) Express $x^2 - 4x + 5$ in the form of $(x-h)^2 + k$. [1]

(ii) Hence, find the range of values of x if $\dfrac{x^2 - 4x + 5}{2x^2 + 2x - 40} > 0$. [2]

11. [Maximum mark: 10]

(a) Given that $\log_5 x = m$ and $\log_5 y = n$, express the following in terms of m and n.

(i) $\log_5 xy^2$, [2]

(ii) $\log_{\frac{x^2}{y^3}} 125$. [3]

(b) Given that $y = (\log_3 p)x - \frac{1}{4}\log_{\sqrt{3}} p$, where p is a constant and $p > 0$, is a tangent to the curve $y = x^2$, find the value(s) of p. [5]

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[Continuation of working space for Question 11]

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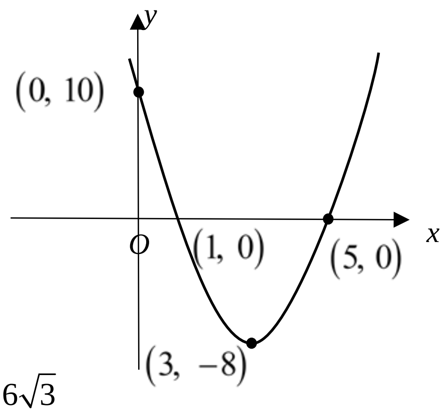
Answer keys

1(a) 3

(b) $\frac{4a^{12}b^4}{c^6}$

2(a) $m=-4$

(b)



3(a) $16\sqrt{3}$

(b)(i) $x=5$

(ii) $2\sqrt{6}$ units

4 $y=3,\ x=9$

5(a) $y=x+5$

(b) The Coordinates of *D* are (-5, 0).

(c) 24 units^2

6(a)(i) $\frac{4x-3}{x^2-1}$

(ii) $x=\frac{3}{4}$

(b) $\therefore 1\le x<3$, Integers that satisfy the inequality are 1 and 2

7(a) $(5-2a+5b)(5+2a-5b)$

(b) $x=-\frac{2}{5}$ or $x=2$

8(a) $\alpha+\beta=\frac{3}{2},\ \alpha\beta=-2$

(b) $\alpha^2+\beta^2=\frac{25}{4}$

(c) $x^2-\frac{9}{4}x+\frac{5}{8}=0$

9(a) By using Pythagoras' Theorem,
AB^2+BC^2
$=11^2+\left(4\sqrt{3}\right)^2$
$=121+16(3)$
$=13^2$
$=AC^2$
Hence, angle *ABC* is a right angle.

(b)(i) $\cos\angle DAC=-\frac{11}{13}$

(ii) $\frac{\tan\angle ACB}{\cos\angle BAC}=\frac{13\sqrt{3}}{12}$

(c) $x=\frac{13\sqrt{3}}{6}$

10(a) $3<p<4$

(b)(i) $(x-2)^2+1$

(ii) $x<-5$ or $x>4$

11(a)(i) $m+2n$

(ii) $\frac{3}{2m-3n}$

(b) $p=1$ or $p=9$