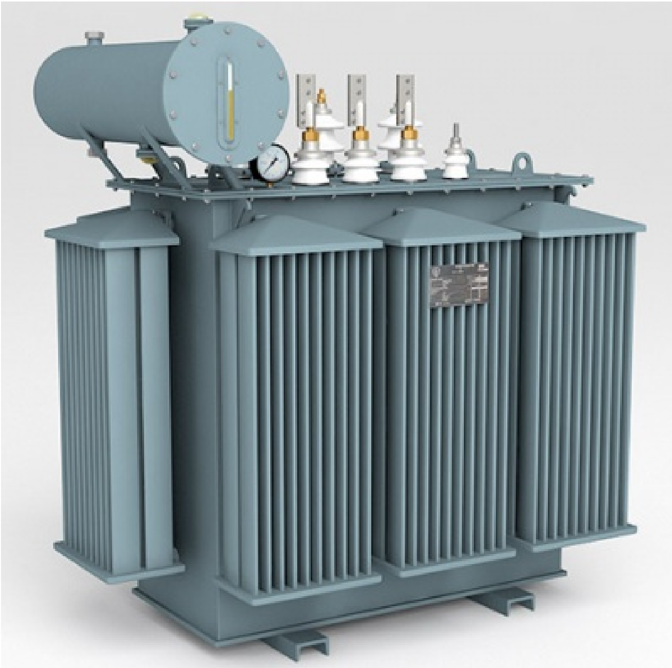
H2 Topic 17

Alternating Currents



A typical electrical transformer found at power sub-stations

A typical power adaptor for an electrical device

Learning Objectives

Content

- Characteristics of alternating currents
- The transformer
- Rectification with a diode

Learning Outcomes

Candidates should be able to:

(a) show an understanding and use the terms period, frequency, peak value and root-mean-square value as applied to an alternating current or voltage.

(b) deduce that the mean power in a resistive load is half the maximum power for a sinusoidal alternating current.

(c) represent an alternating current or an alternating voltage by an equation of the form $x = x_0 \sin \omega t$.

(d) distinguish between r.m.s. and peak values and recall and solve problems using the relationship $I_{rms} = I_0 / 2$ for the sinusoidal case.

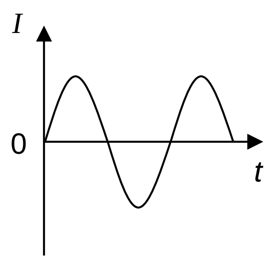
(e) show an understanding of the principle of operation of a simple iron-cored transformer and recall and solve problems using $N_s / N_p = V_s / V_p = I_p / I_s$ for an ideal transformer.

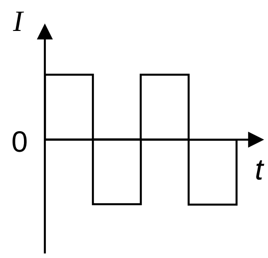
(f) explain the use of a single diode for the half-wave rectification of an alternating current.

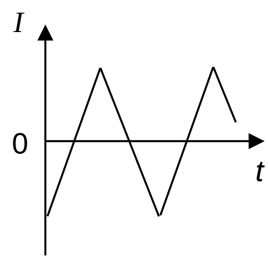
17.0 Introduction

It is important to understand alternating currents (a.c.) because they are so much a part of our everyday life. Every time we turn on a television set, computer or any of a multitude of other electric appliances, we are using a.c. to provide the power to operate them. At the microscopic level, a.c. can be considered as having the charge carriers oscillate about a fixed point.

An alternating current (a.c.) is a current that varies *periodically* with time in magnitude and direction (Polarity of the voltage source constantly changes). Below are some examples of a.c.





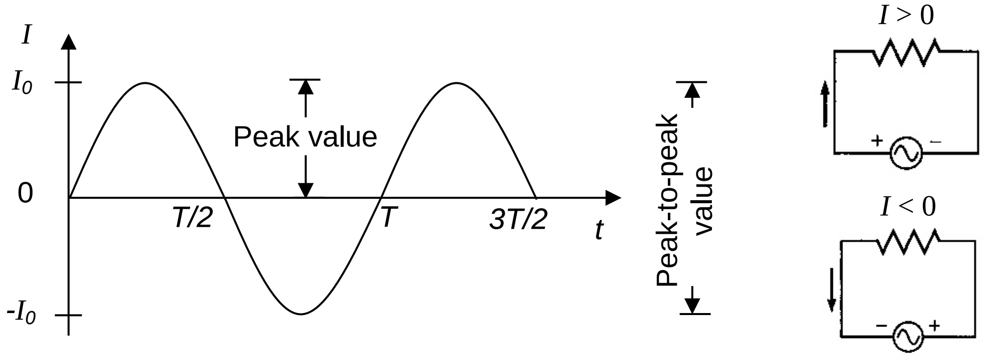


The principles of direct current in resistors learned in previous topics can be applied to resistors in a.c. circuits. However, major differences in circuit analysis arise when inductors and capacitors are to be considered.

17.1 Quantities Characterising an Alternating Current

Quantity	***Symbol***	***Description***
Period	T	Time taken by alternating current to make one complete alternation.
Frequency	f	The number of times the current passes through its zero value in the same direction in unit time.
Angular Frequency	ω	A way of expressing the frequency of the alternating current in terms of radians per second instead of cycles per second.
Peak Value (Amplitude)	I_o	Maximum value of the alternating current in either direction of zero value in a periodic cycle.
Peak to Peak Value		Difference between the positive peak value and the negative peak value of the a.c. within a cycle.
Mean Value	$<I>$	The average value of an a.c. over a given time interval.
Root Mean Square Value	I_{rms}	The value of alternating current that is equal to the steady direct current which would dissipate heat at the same average rate in a given resistor.

The most commonly encountered form of a.c. is the sinusoidal form, that is, it varies with time according to a sine or cosine function.



The current across the resistor can be expressed by the equation $I = I_o \sin \omega t$

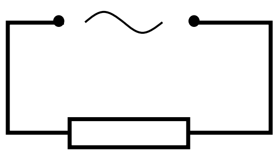
Similarly, the potential difference across the resistor is given by $V = V_o \sin \omega t$

17.1.1 Mean Value of a.c.

For the case of sinusoidal current, any positive value of current, there will be a corresponding negative value within a complete cycle, thus the mean value of current I is zero. However heat is dissipated when an a.c. flows in a resistor, implying that the mean value of an a.c. does not represent the effective value of the a.c.

17.1.2 Root-mean-square value of a.c.

Let us consider a simple circuit consisting of a resistor and an a.c. source as shown.



As with d.c., a.c. also causes heating effect in resistors. Recall that the electrical power P dissipated by a resistor is

$$P = I^2R$$

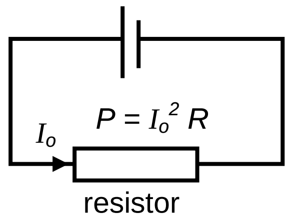
In an a.c., the current keeps changing magnitude from zero to I_o, hence the instantaneous power dissipated from a resistor is given by

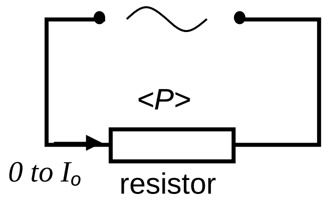
$$P_{instantaneous} = I^2R$$

Mean power dissipated from a resistor over a time interval

$$\begin{aligned} P_{mean} &= \text{mean value of } P_{instantaneous} \\ &= \text{mean value of } I^2R \\ &= (\text{mean value of } I^2) \times R \\ &= I_{rms}^2 R \end{aligned}$$

I_{rms} is the square root of the mean value of I^2 and hence is known as the root-mean-square (r.m.s.) current of the a.c.





To find what value of current in a d.c. will dissipate the same power as the **mean power dissipated by an a.c.** with maximum current I_o, we equate:

$$P_{dc} = \langle P_{ac} \rangle$$

$$I_{dc}^2 R = \langle I_{ac}^2 R \rangle$$

$$I_{dc}^2 R = \langle I_{ac}^2 \rangle R \qquad \text{since } R \text{ is constant}$$

$$I_{dc} = \sqrt{\langle I_{ac}^2 \rangle} = \boldsymbol{I_{rms}}$$

Hence, the r.m.s. value of an a.c. is the value of a steady d.c. which will dissipate energy at the same rate as the mean power dissipated by an a.c. in a given resistor. In other words, a d.c. of magnitude $\boldsymbol{I_{rms}}$ will produce the same heating effect as the a.c. Thus the r.m.s. value can be considered as the effective value of the a.c.

The r.m.s. value of an a.c. is the value of the steady direct current which would dissipate energy at the same rate as the a.c. in a given resistor.

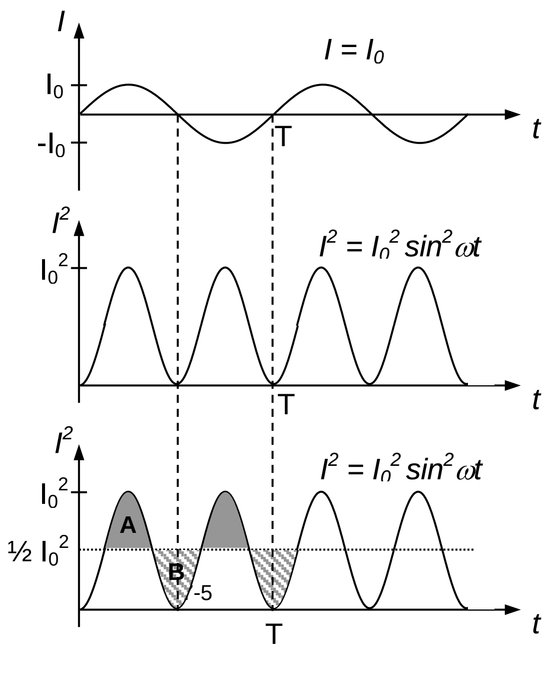
Graphically, it is the square root of the mean value of the square of the instantaneous current over one cycle.

$$I_{rms} = \sqrt{\langle I_{ac}^2 \rangle} = \sqrt{\frac{\int_0^T I^2 dt}{T}} = \sqrt{\frac{\text{area under } I^2 \text{ curve for one cycle}}{\text{period}}}$$

So to find the r.m.s. current, first we square the current, next find its average value and then take the square root of this average value.

For a <u>sinusoidal</u> a.c., $I = I_o \sin \omega t$

To find the r.m.s. value:



Squaring

Mean = $\frac{1}{2} I_0^2$

Square **R**oot = $\frac{I_0}{\sqrt{2}}$

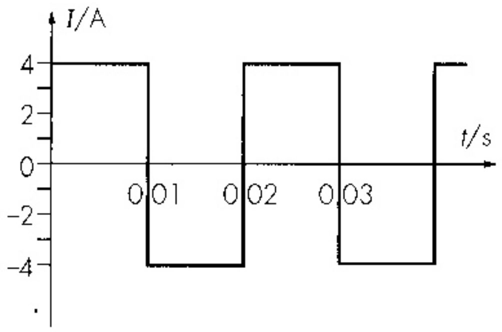
For a <u>sinusoidal</u> current, $I_{\text{rms}} = \dfrac{I_o}{\sqrt{2}}$ refer to Annex A

Similarly, $V_{rms} = \dfrac{V_o}{\sqrt{2}}$

Example 1

Calculate the mean value and r.m.s. value for each of the a.c. shown

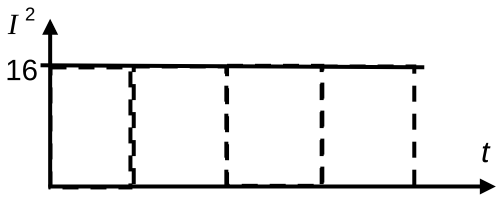
(a)



Mean value:

$$\langle I \rangle = \frac{\int_0^T I dt}{T} = \frac{4(0.01) - 4(0.01)}{0.02} = 0 \text{ A}$$

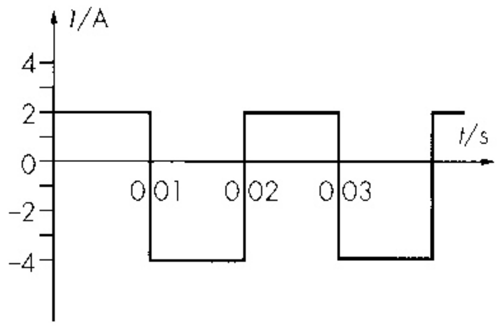
r.m.s. value:



$$\langle I^2 \rangle = \frac{\int_0^T I^2 dt}{T} = \frac{16(0.02)}{0.02} = 16 \text{ A}^2$$

$$\therefore I_{rms} = \sqrt{\langle I^2 \rangle} = 4 \text{ A}$$

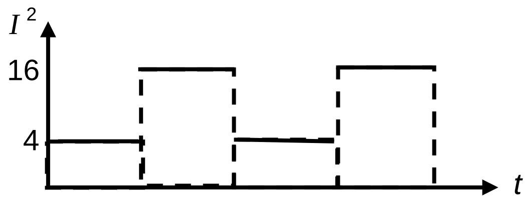
(b)



Mean value:

$$\langle I \rangle = \frac{\int_0^T I dt}{T} = \frac{2(0.01) - 4(0.01)}{0.02} = -1 \text{ A}$$

r.m.s. value:



$$\langle I^2 \rangle = \frac{\int_0^T I^2 dt}{T} = \frac{4(0.01)+16(0.01)}{0.02} = 10 \text{ A}^2$$

$$\therefore I_{rms} = \sqrt{\langle I^2 \rangle} = 3.16 \text{ A}$$

17.1.3 Mean power of of a.c.

For a sinusoidal alternating current,

$$I = I_o \sin \omega t$$

The instantaneous power dissipated in the resistor

$$P_{\text{instantaneous}} = I^2 R = I_o^2 R \sin^2 \omega t$$

Mean power,

$$P_{\text{mean}} = (\text{mean value of } I^2)\, R = I_o^2 R \langle \sin^2 \omega t \rangle = \frac{1}{2} P_{\max}$$

So the mean power *P* dissipated by a resistive load is **half** the maximum power available.

The r.m.s. values of an a.c. will obey the many of the same equations as a d.c., for example,

$$V_{rms} = I_{rms} R$$

Power dissipation in a resistor,

$$P_{mean} = \langle I^2 \rangle R = I_{rms}^2 R = I_{rms} V_{rms} = \frac{V_{rms}^2}{R}$$

These equations also apply if the maximum values of the current and voltage are used.

Example 2

A tourist from U.S.A. brings an electric water kettle designed to operate with 110 V to Singapore and plugs it into a 240 V outlet. The heater breaks down due to overheating.

(a) If the heater normally consumes 500 W, what is the resistance of the heating coil?

$$\langle P \rangle = \frac{V_{rms}^2}{R}$$

$$500 = \frac{110^2}{R} \quad R = 24.2\ \Omega$$

(b) How much power does it consume using our 240 V electrical supply?

Assuming that *R* remains constant

$$\langle P \rangle = \frac{V_{rms}^2}{R}$$

$$= \frac{240^2}{24.2}$$

$$= 2380\ \text{W}$$

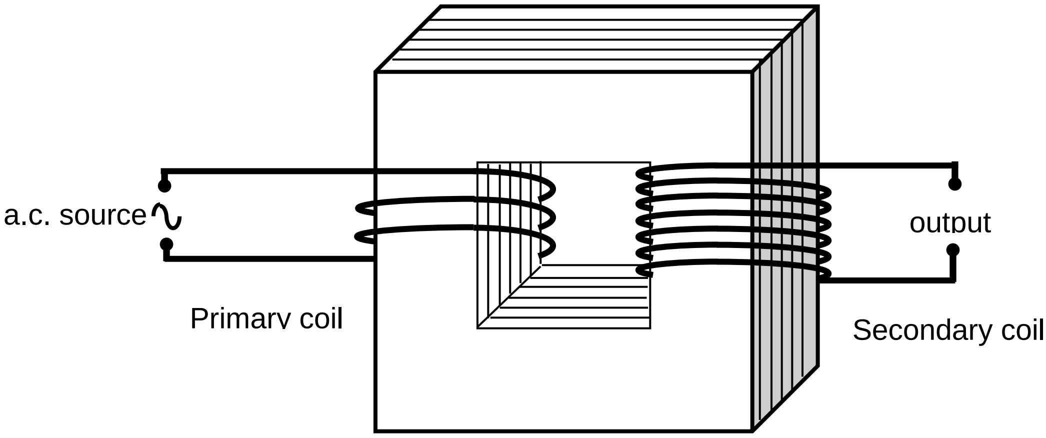
(c) What is the r.m.s. current that flows when the potential difference is 240 V?

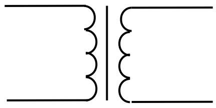
$$I_{rms} = \frac{V_{rms}^2}{R}$$

$$= \frac{240}{24.2} = 9.9\ \text{A}$$

17.2 The Transformer

The transformer is a device for increasing or decreasing an a.c. voltage using electromagnetic induction and it plays an important role in the transmission of electricity. Power plants are often situated some distance from the city and electricity must be transmitted over long distances to reach the consumers. Inevitably, some power is lost in the transmission lines. This power loss can be minimized if the power is transmitted at high voltages (low currents).





circuit symbol

Two insulated copper coils, called the primary and secondary coils, not electrically connected to one another, are wound on separate limbs of a soft iron core (which is laminated to reduce eddy current losses). The soft iron core ensures that most of the flux associated with one coil also passes through the other. When an alternating voltage is applied to the primary coil, the resulting current produces a large alternating magnetic flux which links the secondary coil and induces an e.m.f. in it.

17.2.1 The Ideal Transformer

An ideal transformer is one in which there is no energy loss when stepping up or down the primary e.m.f. In addition, the same flux passes through each turn of both the primary and secondary coils, that is, there is no flux leakage.

Using the Laws of Electromagnetic Induction, it can be shown (in Annex B) that for an ideal transformer,

$$\frac{N_s}{N_p} = \frac{V_s}{V_p}$$

where, N_p = no. of turns in the primary coil,
N_s = no. of turns in the secondary coil,
V_p = e.m.f. in the primary coil,
V_s = e.m.f. in the secondary coil

**It should be noted that the output waveform in the secondary coil is π rad out of phase with the input waveform to the primary coil.*

Also for an ideal transformer, the power input at the primary coil is equal to the power output at the secondary circuit. Hence:

$$I_p V_p = I_s V_s$$

$$\frac{V_s}{V_p} = \frac{I_p}{I_s}$$

Thus for an <u>ideal</u> transformer:

$$\boxed{\frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{I_p}{I_s}}$$

<u>Concepts related to transformers:</u>

(a) If $N_s > N_p$, the transformer is called a step-up transformer since $V_s > V_p$. A step-down transformer has $N_s < N_p$.

(b) The above equation tells us that an ideal transformer that steps up the voltage simultaneously steps down the current and a transformer that steps down the voltage steps up the current. However, the power is neither stepped up nor stepped down as energy is conserved.

17.2.2 The Practical Transformer

Practical transformers lose power through

(i) *Resistance of the windings (coils):*
The wires used for the windings of the coils have resistance and so heating occurs, resulting in power loss (I^2R). This form of power loss is also known as Joule heating. Thicker wires made of material with low resistivity (i.e. high purity copper) are used to reduce this power loss.

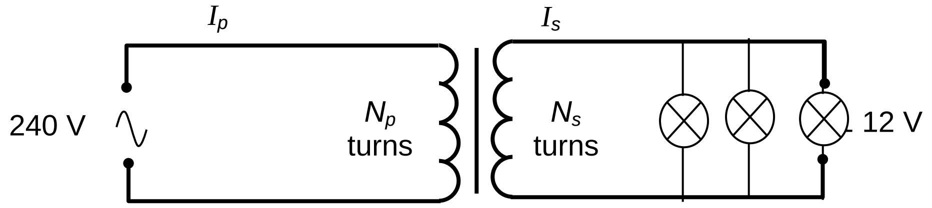
(ii) *Eddy Currents:*
The alternating magnetic flux induces eddy currents in the iron core and cause heating. This effect is reduced by laminating the iron core. Laminations reduce the area of circuits in the core, and thus reduce the e.m.f. induced and current flowing within the core, which leads to a reduction in the energy lost.

(iii) *Hysteresis:*
There are hysteresis losses in the core. The magnetization of the core is repeatedly reversed by the alternating magnetic field. The energy required to magnetise the core (while the current is increasing) is not entirely recovered during demagnetisation. The difference in energy is lost as heat in the core. The energy loss is kept to a minimum by using a magnetic material with low hysteresis loss.

(iv) Flux Leakage:
The flux due to the primary may not all link to the secondary coil if the coil is badly designed or has air gaps in it. When flux is "leaked "to the surrounding, power is loss and thus not all the power from the primary coil can be transferred to the secondary coil.

Example 3

10 lamps rated 24 W, 12 V are connected in parallel with the mains supply of 240 V.



(a) Calculate the turns ratio of the transformer needed.

Turns ratio: $\frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{12}{240} = \frac{1}{20}$

(b) What would be the current drawn from the mains supply if the transformer is 100% efficient?

Current in each lamp, $I_{rms} = \frac{\langle P \rangle}{V_{rms}} = \frac{24}{12} = 2.0\text{ A}$

Since the lamps are connected in parallel, total current drawn in the secondary coil = 20 A.

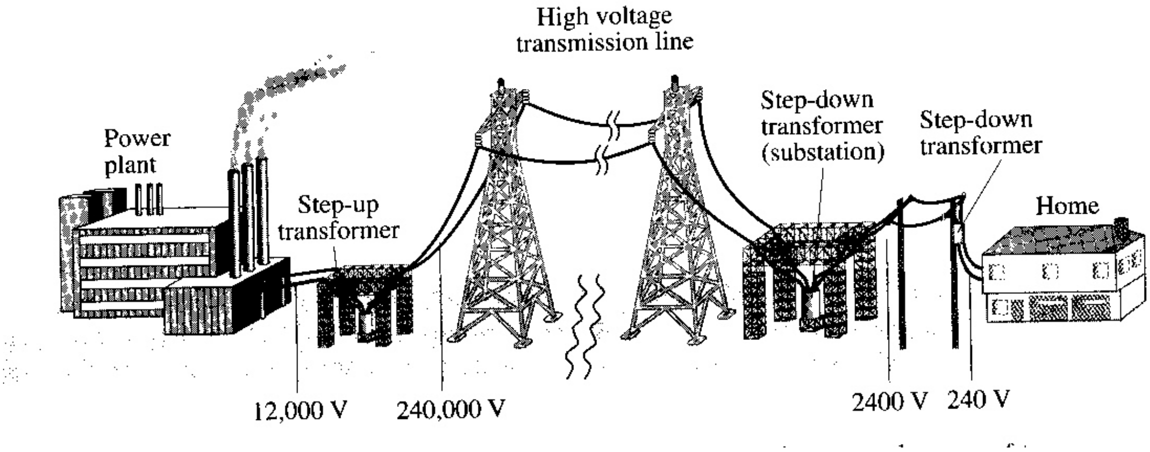
Since transformer is assumed to be 100 % efficient,

$$I_P V_P = I_s V_s$$
$$240 I_P = 12(20)$$
$$I_P = 1.0\text{ A}$$

(c) What would be the current drawn from the mains supply if the transformer is 91% efficient?

$$0.91 I_P V_P = I_s V_s$$
$$0.91(240) I_P = 12(20)$$
$$I_P = 1.1\text{ A}$$

17.2.3 Transmission of electricity



Step-up transformers at the power plants increase the potential to 240 kV or more for long distance transmission. Step-down transformers operate in stages and are used to supply electricity to local users.

There are two reasons why a.c. is used in the transmission of electrical power.

(a) ***Minimise power loss***

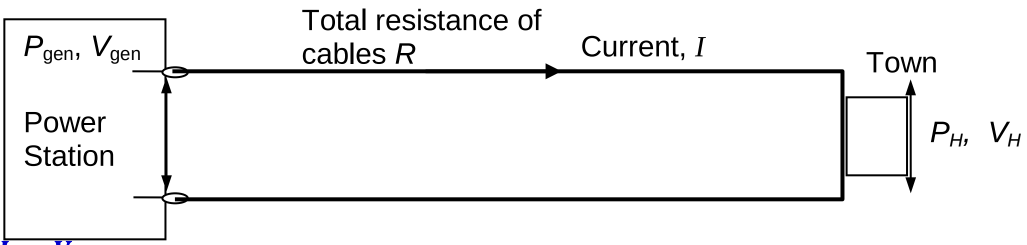
Transformers play an important role in the transmission of electricity. Power plants are often situated some distance from the city and electricity must be transmitted over long distances to reach the consumers. Inevitably, some power is lost in the transmission lines. This power loss can be minimised if the power is transmitted at **high voltages (low currents)**.

(b) ***Transformers only work with a.c.***

However, the high voltages are extremely dangerous. Normally, household appliances do not need high voltages. There is a need to convert to lower voltage for household use, that is, to step down the voltage. Transformers meet this requirement too but they only work with a.c.

Example 4

An average of 120 kW of electric power is sent to a small town from a power plant 10 km away. The transmission lines have a total resistance of 0.40 Ω. Calculate the power loss if the power is transmitted at (a) 240 V (b) 24 000 V.



$\langle P \rangle = I_{rms} V_{rms}$

(a) At 240 V, $I_{rms} = \frac{\langle P \rangle}{V_{rms}} = \frac{120 \times 10^3}{240} = 500 \text{ A}$

Power loss in the lines $\langle P \rangle = I_{rms}^2 R_{cable} = 500^2(0.4) = 1.0 \times 10^5 \text{ W}$

(b) At 24 000 V, $I_{rms} = \frac{\langle P \rangle}{V_{rms}} = \frac{120 \times 10^3}{24000} = 5 \text{ A}$

Power loss in the lines $\langle P \rangle = I_{rms}^2 R_{cable} = 5^2(0.4) = 10 \text{ W}$

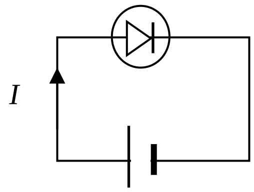
17.3 Rectification

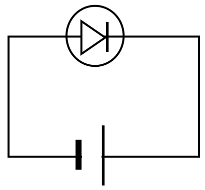
The e.m.f. obtained from the mains supply is alternating in nature, but we often require constant voltage to operate the electronic circuitries of many household devices. A means of deriving d.c. from a.c. is desired.

The process of converting a.c. to d.c. is known as rectification. This is achieved through the use of rectifiers which conduct current in one direction only. An example of a rectifier is the p-n diode which will be covered in the last topic on semiconductors and lasers.

17.3.1 Half Wave Rectification

A diode is forward-biased when it is connected to a supply in such a way that current flows through it. It is reverse-biased if connected such that little or no current flows through it.

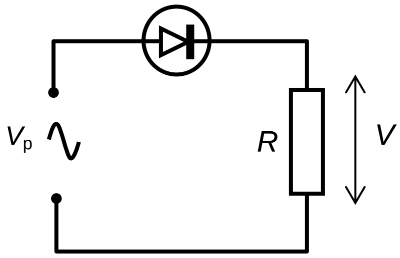




Forward-biased – current flows

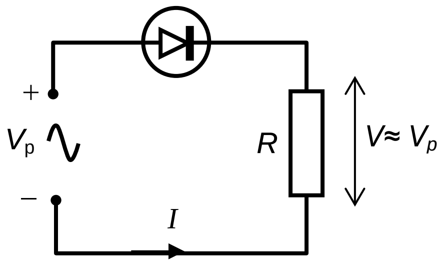
Reverse-biased – negligible current flow

The a.c. to be rectified is connected in series with the diode and the electronic equipment that requires d.c. output (represented by resistor *R*) as shown below.

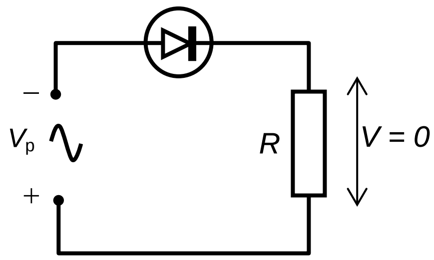


Let us consider the case of a sinusoidal a.c. If the first half cycle acts in the forward-biased direction of the diode (i), current flows in the circuit and a p.d., V is obtained across *R* (which will have almost the same value as the applied p.d., V_p, due to the low resistance of the diode when it is forward-biased). During the second half cycle, the diode is reverse-biased (ii). No current flows in the circuit and the p.d. across *R* is zero. This is repeated for each cycle of the a.c.

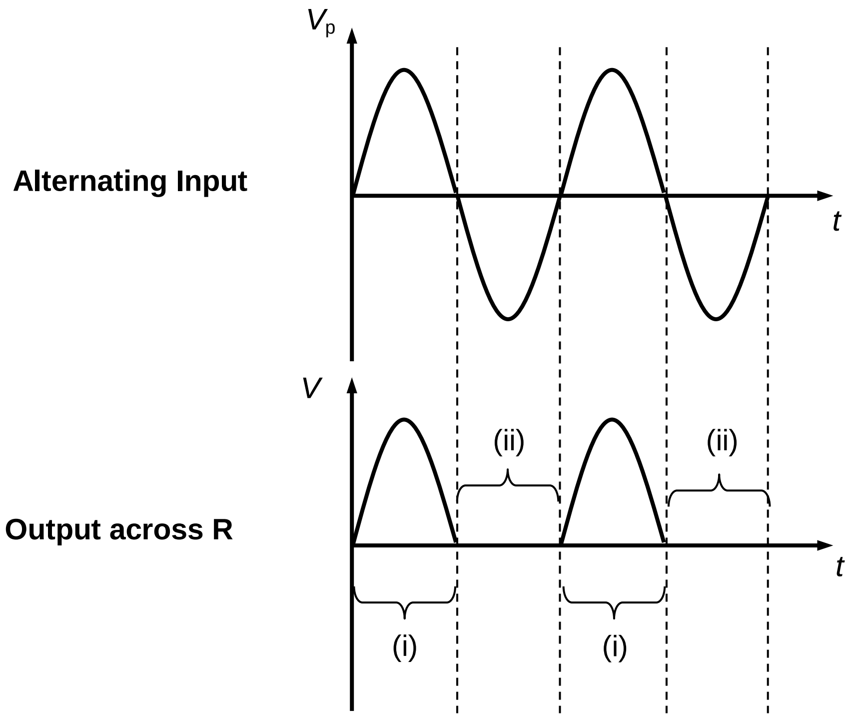
(i) forward-biased



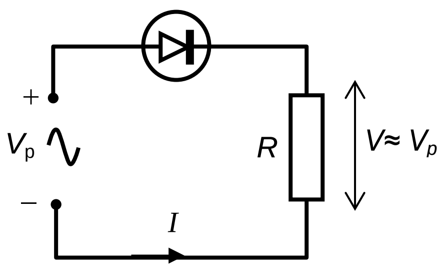
(ii) reverse-biased



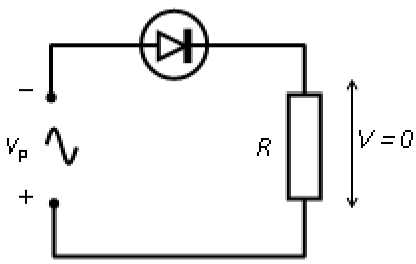
Hence, the output p.d. across *R* is a pulsating d.c.



(i) forward-biased



(ii) reverse-biased

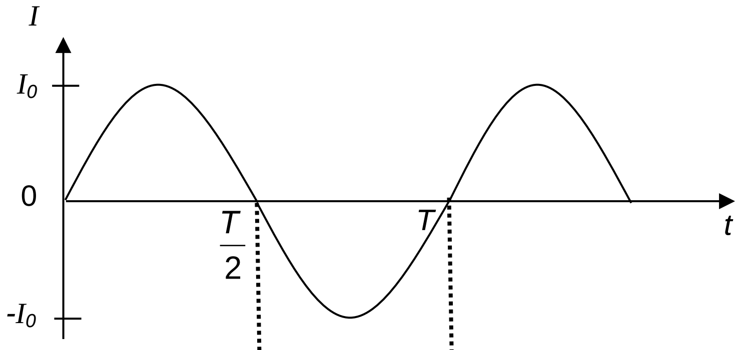


17.3.2 Full Wave Rectification

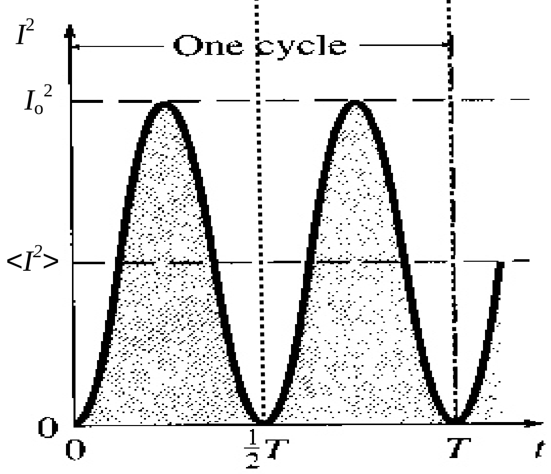
Full-wave rectification is achieved with a four diode bridge connection as shown in Annex C.

Annex A *To obtain the root-mean-square value of sinusoidal a.c.*

For a sinusoidal a.c. $I = I_o \sin \omega t$



Mean current $\langle I \rangle = \dfrac{\int_0^T I\,dt}{T} = 0$



Mean-square current $\langle I^2 \rangle = \langle I_o^2 \sin^2 \omega t \rangle = I_o^2 \langle \sin^2 \omega t \rangle$

but $\langle \sin^2 \omega t \rangle = \dfrac{\int_0^T \sin^2 \omega t\,dt}{T} = \dfrac{\int_0^T (1 - \cos 2\omega t)\,dt}{2T} = \dfrac{1}{2T}\left[t - \dfrac{1}{2\omega}\sin 2\omega t\right]_0^T = \dfrac{1}{2}$

so $\langle I^2 \rangle = \dfrac{I_o^2}{2}$

or $I_{rms} = \sqrt{\langle I^2 \rangle} = \dfrac{I_o}{\sqrt{2}}$

Annex B *The Ideal Transformer Equation*

Consider the case of an ideal transformer. Suppose that the secondary circuit is not connected to any load. We apply an alternating e.m.f. to the primary coil, resulting in an alternating current flowing. The following happens:

(a) The a.c. sets up a changing magnetic field associated with the primary coil. The changing magnetic flux linkage through the primary coil induces an e.m.f. in it given by

$$V_p = -N_p \frac{d\Phi}{dt} \qquad \text{... (1)}$$

where N_p : number of turns in the primary coil

$\frac{d\Phi}{dt}$: rate at which magnetic flux through the primary coil changes

(b) The iron core links this magnetic flux to the secondary coil. This changing magnetic flux linkage thus induces an e.m.f. in the secondary coil of N_s turns, given by

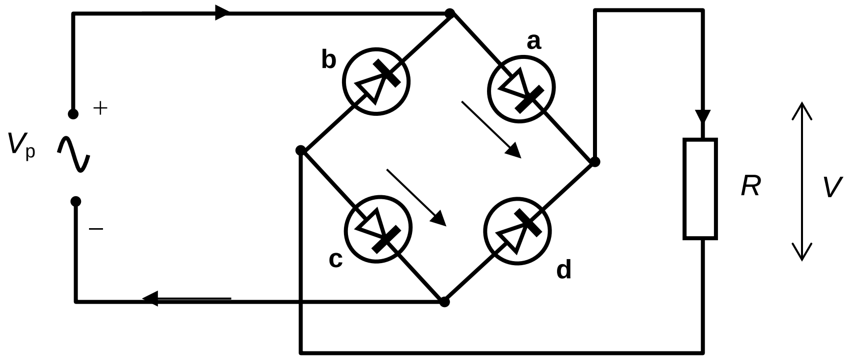
$$V_s = -N_s \frac{d\Phi}{dt} \qquad \text{... (2)}$$

Dividing Equation (2) by (1),

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

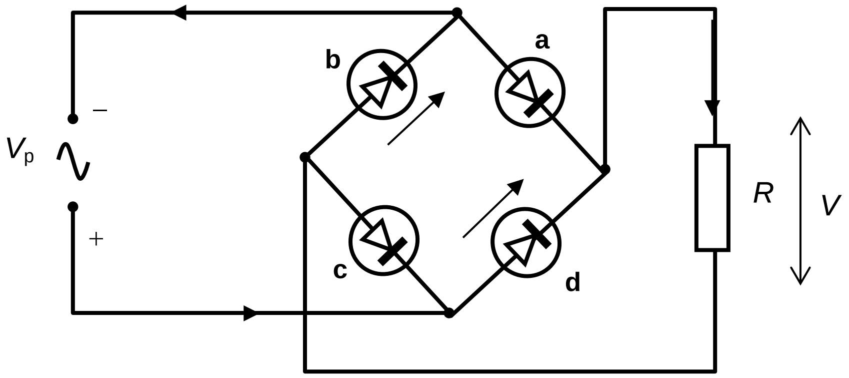
Annex C *Full-wave rectification*

Full-wave rectification is achieved with a four diode bridge connection as shown.

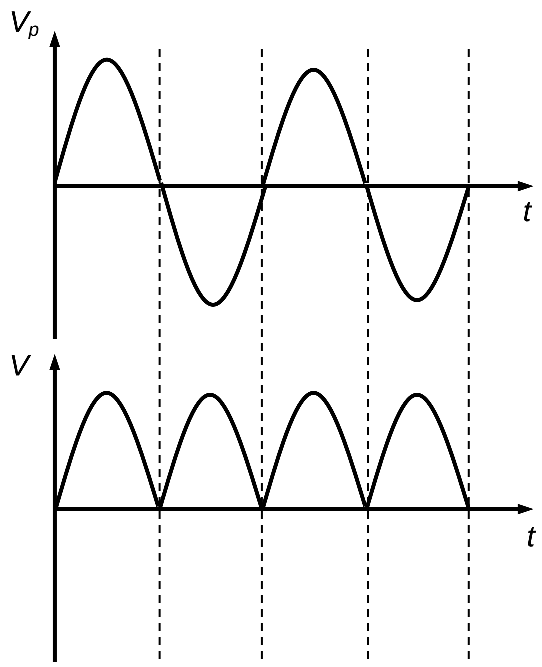


For the first half cycle, only two of the four diodes (a and c) are forward-biased and the current flows through *R* as shown.

For the second half cycle (when the polarity of the a.c. changes), the other two diodes (b and d) are forward-biased and the current flows through *R* as shown.



Notice that the direction of the current through *R* remains the same during both half cycles of the a.c. The output p.d. across *R* is a pulsating d.c.



德明政府中学
Dunman High School

Topic 17: Alternating Currents Tutorial

H2 PHYSICS (2013)

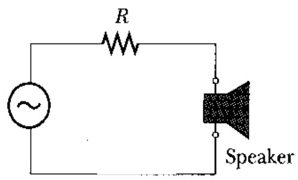
Self Attempt

1. A sinusoidal current flowing in a 10 Ω resistor varies with time according to the equation $I = 1.2 \sin(100\pi t)$. Calculate the instantaneous current and the instantaneous power dissipated at
   (i) t = 0.005 s
   (ii) t = 0.010 s
   (iii) t = 0.018 s

   (1.2 A, 14.4 W; 0 A, 0 W; –0.705 A, 4.98 W)

2. A steady current I dissipates a certain power in a variable resistor. The resistance has to be halved to obtain the same power when a sinusoidal alternating current is used. What is the r.m.s. value of the alternating current? ($I\sqrt{2}$)

3. An audio amplifier, represented by an a.c. source and the resistor R delivers alternating voltages at audio frequencies to the speaker. If the source puts out an alternating voltage of 15V (r.m.s.) resistance R is 8.20 Ω and the speaker is equivalent to a resistance of 10.4 Ω, calculate the average power delivered to the speaker. (6.76 W)



4. A power station generates 100 kW of power at a voltage of 10 kV. Given that the connecting cables have a total resistance of 20 Ω, find the turn ratio required for an ideal step-down transformer to bring electrical energy to the home at 240 V.

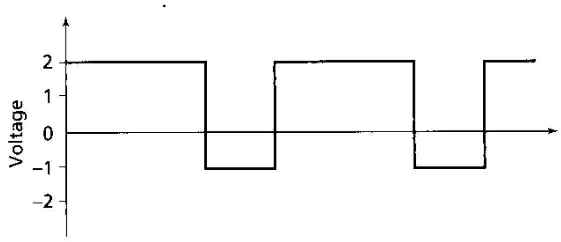
   (245:6)

Tutorial Discussion

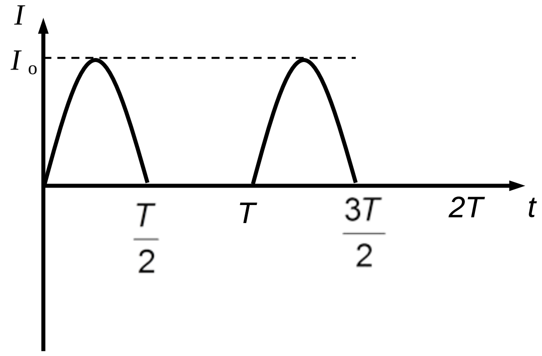
5. Determine the r.m.s. current in each case.

   10 minutes

   (a) A sinusoidal current of peak value 2.0 A.
   (b) A full-wave rectified sinusoidal current of peak value 3.0 A.
   (c) A square-wave current with a frequency of 1 Hz which is 0.1 A for one half cycle and –0.1 A for the next half cycle.
   (d) An uneven square wave current as shown below.



6. Calculate the r.m.s. current I_{rms}, in terms of its peak current I_o, for a half-wave rectified sinusoidal a.c. as shown below.

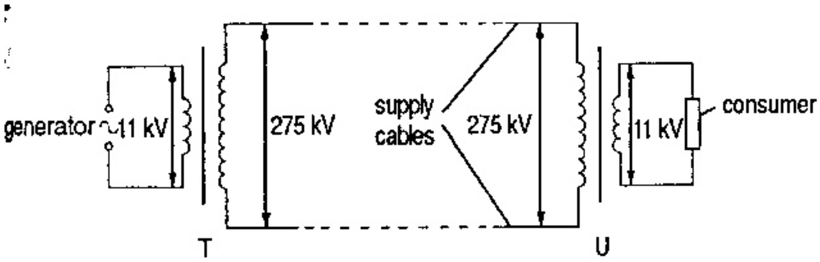


3 minutes

7. A sinusoidal current described by the equation I = 9.0 sin *wt* flows through a 12. 0 Ω resistor. Calculate
   (a) the r.m.s. value of the current and voltage.
   (b) the maximum instantaneous power dissipated in the resistor.
   (c) the average power dissipated in the resistor.

6 minutes

8. Jun98/II/5
   Electrical power of 4400 kW is supplied to an industrial consumer at a considerable distance from a generating station. This is represented below.



12 minutes

In order to do this, the electricity supply company makes use of a circuit containing two transformers T and U. The transformers can be considered to be ideal and the supply cables to have negligible resistance.

(a) The power is generated at 11 kV r.m.s. and is supplied to the consumer at 11 kV r.m.s. Calculate the r.m.s. current supplied to the consumer.

(b) There is a potential difference of 275 kV r.m.s. between the supply cables. Calculate

   1. the ratio $\frac{N_s}{N_p}$ for each transformer

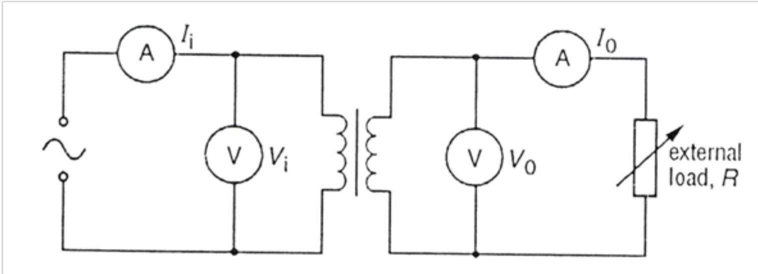
   2. the current in the supply cables

(c) Explain why, when the resistance of the supply cables cannot be neglected, this arrangement is preferable to a system which generates and transmits the power at the same voltage of 11 kV r.m.s.

(d) Find the peak value of the current in (a).

9. N91/II/8
This question is about the current through, and the potential difference across, the coils of a transformer. The figure below shows the circuit used.

22 minutes



The resistance of each coil is as follows:
primary coil, 470 Ω
secondary coil, 2950 Ω

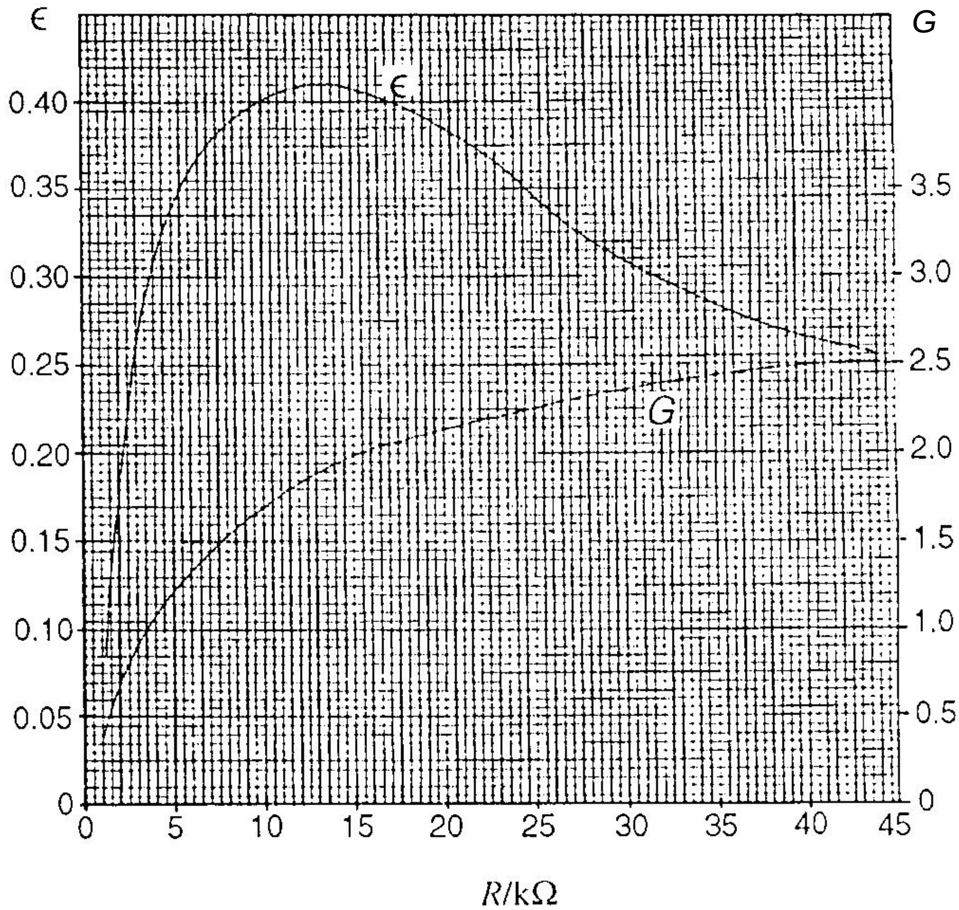
V_i, the r.m.s. potential difference across the primary coil is maintained at 6.00 V.

The efficiency, ε of the transformer is defined as

$$\varepsilon = \frac{output\ power}{input\ power}$$

The ratio $\frac{V_o}{V_i}$ is represented by *G*.

The graph below shows the variation of ε and *G* with *R*, the resistance of the external load.



(a) Read off from the graph, a value for the maximum efficiency of the transformer.

(b) For the case where the transformer is operating at this maximum efficiency, determine
- **(i)** the value of *G*
- **(ii)** the value of *R*
- **(iii)** the current in *R*
- **(iv)** the input current

(c) Use your answers to **(b)** and any other data, to deduce the following for the transformer when operating at maximum efficiency:
- **(i)** the total power loss in the transformer
- **(ii)** the power loss in the resistance of the primary coil
- **(iii)** the power loss in the resistance of the secondary coil

(d) Use your answers in **(c)** to show that there must be some other power loss in the transformer besides power loss in the resistance of the coils.

(e) What would be the effect on the efficiency of using
- **(i)** a very large external load resistance,
- **(ii)** a very small external load resistance

A student who is well-prepared for 'A' levels will be able to complete all the tutorial questions in less than 1 hour. Suggested target time for each tutorial question is given based on 'A' level standards.