Answer **all** the questions.

1 Differentiate each function with respect to $x$.

**(a)** $y = 2 - x^2 - \frac{16}{x^2}$, leaving your answer in positive index. [1]

$\frac{dy}{dx} = -2x - 16(-2)(x)^{-3}$

$= -2x + \frac{32}{x^3}$ . —(B1)

**(b)** $y = \frac{5x^3 + 4x^2 - 7x + 3}{x^2}$, leaving your answer in positive index. [2]

$= 5x + 4 - \frac{7}{x} + \frac{3}{x^2}$ —(M1)

$\frac{dy}{dx} = 5 - 7(-1)(x)^{-2} + 3(-2)(x)^{-3}$

$= 5 + \frac{7}{x^2} - \frac{6}{x^3}$ —(A1) / $\frac{5x^3 + 7x - 6}{x^3}$

**(c)** $y = 4(5 - 3x^2)^4$ [2]

$\frac{dy}{dx} = 4(4)(5 - 3x^2)^3(-6x)$ —(M1)

$= -96x(5 - 3x^2)^3$ —(A1)

2 (i) Differentiate $\frac{5x^2+3}{10x^2-6}$ with respect to $x$. [2]

$$\frac{d}{dx}\left(\frac{5x^2+3}{10x^2-6}\right)$$

$u = 5x^2+3$ | $v = 10x^2-6$

$\frac{du}{dx} = 10x$ | $\frac{dv}{dx} = 20x$

$$\frac{d}{dx}\left(\frac{5x^2+3}{10x^2-6}\right) = \frac{10x(10x^2-6) - 20x(5x^2+3)}{(10x^2-6)^2} \quad \text{— (M1)}$$

$$= \frac{100x^3 - 60x - 100x^3 - 60x}{(10x^2-6)^2}$$

$$= \frac{-120x}{(10x^2-6)^2} \quad \text{— (A1)}$$

Also accept

$$= \frac{-30x}{(5x^2-3)^2}$$

(ii) Differentiate $\left(\frac{5x^2+3}{10x^2-6}\right)^4$ with respect to $x$, leaving your answer in the form $\frac{kx(5x^2+3)^m}{(5x^2-3)^n}$ where $k$, $m$ and $n$ are constants. [2]

$$\frac{d}{dx}\left(\frac{5x^2+3}{10x^2-6}\right)^4$$

$$= 4\left(\frac{5x^2+3}{10x^2-6}\right)^3\left(-\frac{120x}{(10x^2-6)^2}\right) \quad \text{— (M1)}$$

$$= \frac{-480x(5x^2+3)^3}{(10x^2-6)^5}$$

$$= \frac{-480x(5x^2+3)^3}{32(5x^2-3)^5}$$

$$= \frac{-15x(5x^2+3)^3}{(5x^2-3)^5} \quad \text{— (A1)}$$

3 The equation of a curve is $y = 3x\sqrt{5+x^2}$. Find the gradient(s) of the curve at the point(s) where it crosses the line $y = 6x$. [5]

$3x\sqrt{5+x^2} = 6x$

$3x\sqrt{5+x^2} - 6x = 0$

$3x\left(\sqrt{5+x^2} - 2\right) = 0$ — (m1)

$3x = 0$ or $\sqrt{5+x^2} - 2 = 0$

$x = 0$ $\quad\sqrt{5+x^2} = 2$

m1 for either $\quad 5+x^2 = 4$

$x^2 = -1$ (rej).

$\frac{dy}{dx} = \underline{3\sqrt{5+x^2}} + \underline{3x\left(\frac{1}{2}\right)(5+x^2)^{-\frac{1}{2}}(2x)}$ — (m1, m1)

$= 3\sqrt{5+x^2} + \frac{3x^2}{\sqrt{5+x^2}}$

at $x = 0$, $\frac{dy}{dx} = 3\sqrt{5}$ — (A1)

$= 6.71$

4 (i) By considering the general term, explain why the binomial expansion of $\left(\frac{3}{x^2}+x\right)^7$ does not have a term independent of $x$. [3]

$T_{r+1} = \binom{7}{r}\left(\frac{3}{x^2}\right)^{7-r}(x)^r$ — (M1)

$= \binom{7}{r} 3^{7-r} (x^{2r-14})(x)^r$

$= \binom{7}{r} 3^{7-r} (x^{3r-14})$

$\underbrace{3r-14=0}_{(M1)} \Rightarrow r = \frac{14}{3}.$

Since $r=\frac{14}{3}$, which is not an integer,
there is no term independent of $x$. } (A1)

(ii) Find the term independent of $x$ in the expansion of $\left(\frac{3}{x^2}+x\right)^7(5-2x^2)$. [3]

$\left(\frac{3}{x^2}+x\right)^7(5-2x^2)$

$\frac{1}{x^2}$ term from $\left(\frac{3}{x^2}+x\right)^7$ : $3r-14=-2$ — (M1)
$3r = 12$
$r = 4.$

$\therefore$ term indep. of $x$ : $\binom{7}{4}(3)^{7-4}\left(\frac{1}{x^2}\right)\cdot(-2x^2)$ — (M1)

$= -1890$ — (A1)

5 A tangent to a circle at the point $(-6,6)$ passes through the origin. The centre, $C$, of the circle lies on the line $4y-3x=40$.

(i) Show that the centre of the circle is $(-8,4)$. [4]

$m_{tangent}$ : $\frac{6-0}{-6-0} = -1$

$m_{radius}$ : $(-1) \div (-1) = 1$ — M1

$\therefore$ Eqn of radius : $y = x + c$

Sub $(-6, 6)$ into $y = x + c$

(M1) — $[6 = -6 + c] \Rightarrow c = 12$

$\therefore y = x + 12$ — ①

$4y - 3x = 40$ — ②

Sub ① into ② : $4(x+12) - 3x = 40$ — (M1)

$4x + 48 - 3x = 40$

$x = -8$

$\therefore y = -8 + 12$

$= 4$

$\therefore$ Centre : $(-8, 4)$ — (A1)

(ii) Find the equation of the circle. [2]

Radius : $\sqrt{\underbrace{(-6+8)^2 + (6-4)^2}_{(M1)}} = \sqrt{8}$

$\therefore$ Eqn : $(x+8)^2 + (y-4)^2 = 8$ — (A1)

$\therefore x^2 + 16x + y^2 - 8y + 72 = 0$

(iii) Find the coordinates of the circle which is nearest to the $y$-axis. [2]

$(\underbrace{-8+\sqrt{8}}_{(B1)}, \underbrace{4}_{(B1)})$ also accept $(-8+2\sqrt{2}, 4)$ or $(-5.17, 4)$

(iv) Explain why the point $P(-10, 6.5)$ lies outside the circle. [2]

Dist. b/w P and centre: $\sqrt{(-10+8)^2+(6.5-4)^2}$ — (M1)

$= \sqrt{10.25}$

Since dist. b/w P & centre of circle is longer than length of radius, P lies outside the circle. } (A1)

END OF PAPER