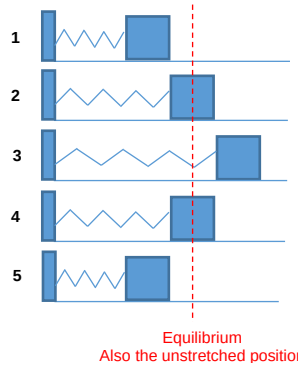
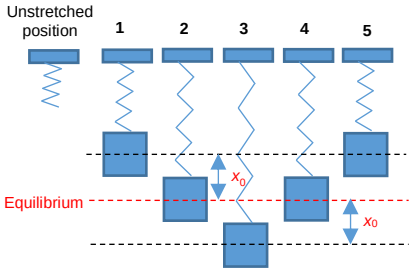
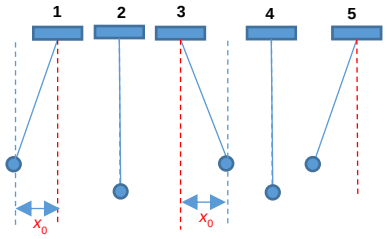
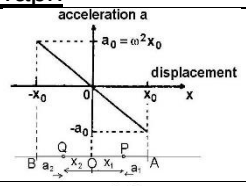
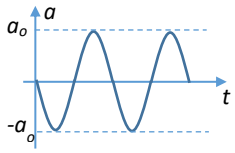
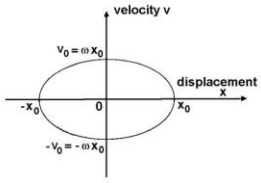
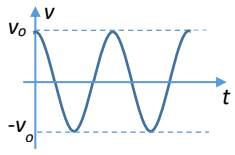
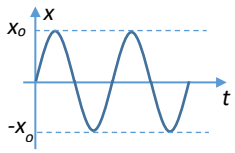
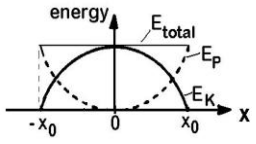
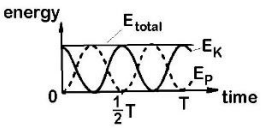
Oscillations

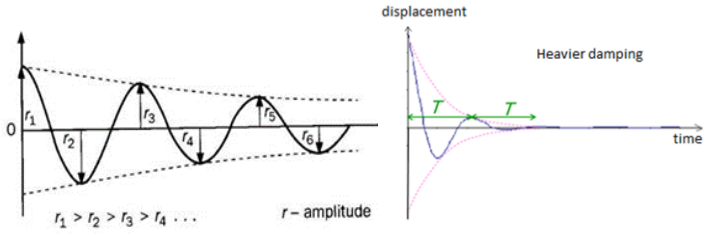
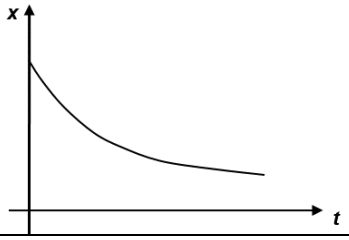
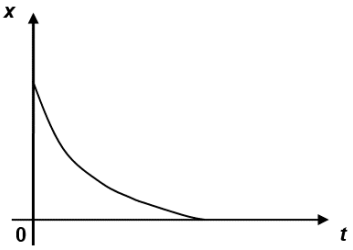
Simple Harmonic Motion (Free Oscillations) – Displacement of the object is proportional to its acceleration, with the acceleration pointing towards the equilibrium position.

Horizontal Spring Mass	Vertical Spring Mass	Pendulum
	 Note: Equilibrium is NOT at unstretched position	 Assume very small displacement. Arc length ≈ horizontal x

SHM Quantities and their variations with respect to displacement and time.

Quantity	**Variation w.r.t Displacement**		**Variation w.r.t Time**	
	Equation	Graph	Equation	Graph
Acceleration Max. acceleration	$a = -\omega^2 x$ $\lvert a_0 \rvert = \lvert \omega^2 x_0 \rvert$		$a = \mp a_0 \sin(\omega t)$ $a = \pm a_0 \cos(\omega t)$	
Velocity Max. velocity	$v = \pm\omega\sqrt{x_0^2 - x^2}$ $v_0 = \omega x_0$		$v = \pm v_0 \cos(\omega t)$ $v = \mp x_0 \sin(\omega t)$	
Displacement	N.A	N.A.	$x = \pm x_0 \sin(\omega t)$ $x = \pm x_0 \cos(\omega t)$	
Kinetic Energy	$E_k = \frac{1}{2} m\omega^2 (x_0^2 - x^2)$		$E_k = \frac{1}{2} m v_0^2 \cos^2(\omega t)$ $E_k = \frac{1}{2} m v_0^2 \sin^2(\omega t)$	
Potential Energy	$E_p = \frac{1}{2} m\omega^2 x^2$		$E_p = \frac{1}{2} m v_0^2 \sin^2(\omega t)$ $E_p = \frac{1}{2} m v_0^2 \cos^2(\omega t)$	
Total Energy	$TE = E_{k,max} = E_{p,max}$ $= \frac{1}{2} m\omega^2 x_0^2$		Constant value throughout time	
Angular Frequency	$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} = 2\pi f$	Period, T Frequency, f	$f = \frac{1}{T}$	

Damped Oscillations – Loss of total energy gradually with time → Amplitude decreases

Types of Damped Oscillations	Graphs
Light Damping or Underdamped • Still oscillating (negative and positive *x*) • Period is more than undamped case, but remains constant throughout oscillation	
Heavy Damping or Overdamped • Takes a long time (or never) reach equilibrium	
Critical Damping • Returns to equilibrium in the fastest possible time	 Use for 1. Car Suspensions 2. Analog meters 3. Door closers

Resonance – When driving frequency matches the natural frequency, maximum energy is transferred to the system, hence system oscillate at the maximum amplitude.

Frequency Response Curve	
More Damping • All amplitudes of oscillations decrease • Resonance frequency shift left Less Damping • All amplitudes of oscillations increase • Resonance frequency shift right Ideal case with no damping • All amplitudes of oscillations increase • Reach theoretical infinite amplitude at resonance frequency