BENDEMEER SECONDARY SCHOOL
2024 PRELIMINARY EXAMINATION
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

CANDIDATE NAME

CLASS

INDEX NUMBER

MATHEMATICS
Paper 2

4052/02
21 August 2024
2 hours 15 minutes

Candidates answer on the Question Paper.
No additional materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO NOT WRITE ON ANY BARCODES.

MARKING SCHEME

Answer **all** the questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total number of marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

FOR EXAMINER'S USE
90

MATHEMATICAL FORMULAE

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 **(a)** Simplify $\left(\frac{3a^{-\frac{3}{2}}b^2}{4^{\frac{1}{2}}b^3}\right)^{-2}$ and leave your answer in positive index notation.

$= \left(\frac{4^{\frac{1}{2}}b^3}{3a^{-\frac{3}{2}}b^2}\right)^2$ M1

$= \frac{4b^6}{9a^{-3}b^4}$ M1

$= \frac{4a^3b^2}{9}$ A1

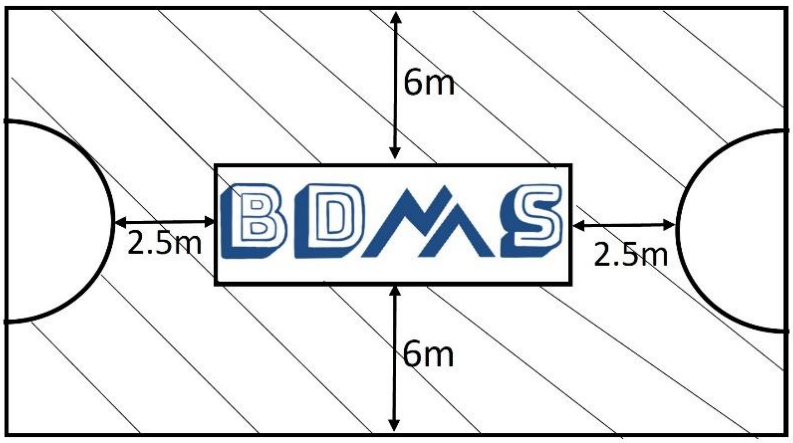
Answer (a) _____ [3]

(b) Simplify $\frac{(a-5b)^2}{a^2-25b^2} \div \frac{a-5b}{5b}$.

$= \frac{(a-5b)^2}{(a-5b)(a+5b)} \times \frac{5b}{a-5b}$ ..... M1 for $(a-5b)(a+5b)$

$= \frac{5b}{a+5b}$ A1

Answer (b) _____ [2]

2	The diagram below shows a school Tchoukball court of dimension 27m by 16m. On each end is a semicircle of radius 3m. The shaded part is to be painted yellow.  		
	(a)	Find the area of the court that is painted yellow. Area that is painted yellow $= (27 \times 16) - (\pi \times 3^2) - (16 \times 4)$ ……. M1 = 339.726 = 340 m^2 (3sf)…….. A1 *Answer* (a) _____m^2	[2]

(b) 3 workers can paint the court in 5 days. After painting the first day, the workers had to stop on the second and third day because it was raining.
Teachers, Mr Chin and Mr Lee, decided to help with the painting on the fourth and fifth day.
Assuming the 3 workers and 2 teachers paint at the same rate, can they finish painting the shaded area on the fifth day?
Show your working clearly. [3]

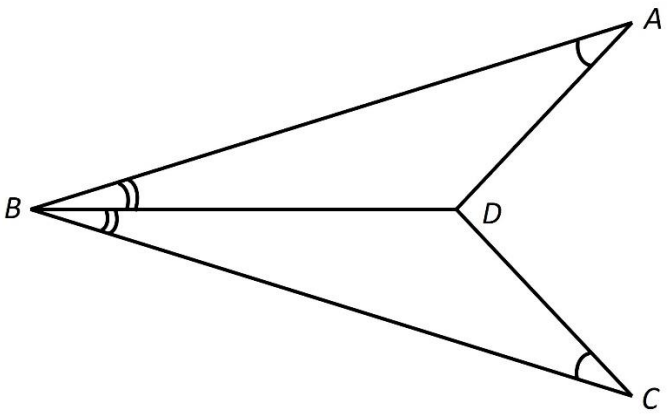
Man-days needed to paint the court
= 5 x 3
= 15 ……. M1

Man-days needed after the first day
= 15 – (3 x 1)
= 12

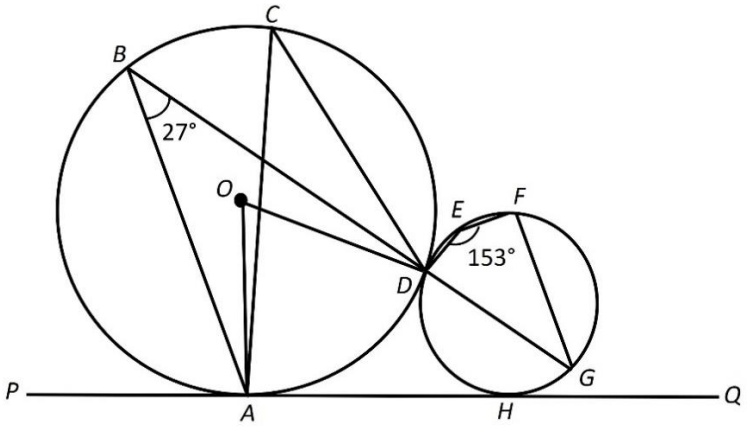
Number of people needed for the fourth and fifth days
= 12 ÷ 2
= 6 ……. M1

Number of workers and teachers painting on the fourth and fifth days
= 5

Therefore, they cannot finish painting on the fifth day. } A1

3	**(a)**	In the diagram, $\angle ABD = \angle CBD$ and $\angle BAD = \angle BCD$. Prove that ΔABD and ΔCBD are congruent.  In ΔABD and ΔCBD , BD is common $\angle ABD = \angle CBD$ (given) $\angle BAD = \angle BCD$ (given) B2 for 3 correct statements B1 for 2 correct statements Therefore, ΔABD and ΔCBD are congruent (AAS) ……. B1	[3]

(b) In the diagram below, *O* is the centre of the circle and *BDG* is a straight line. *PQ* is a tangent to the circles at *A* and *H* respectively. $\angle ABD = 27^0$ and $\angle DEF = 153^0$.



Stating the angle properties of circles clearly, find

(i) $\angle ACD$

= 27^0 (Angle in the same segment) …… B1

Answer (i) _____^{0} [1]

(ii) $\angle DAH$

$\angle OAH$ = 90^0 (radius perpendicular to tangent)
$\angle AOD$ = 54^0 (Angle at centre = 2x angle at circumference) } M1 for either property

$$\angle OAD = \frac{180 - 54}{2}$$
$$= 63^0$$

$$\angle DAH = 90 - 63$$
$$= 27^0 \text{ …… A1}$$

[2]

Answer (ii) _____^{0}

(iii) Is *BA* parallel to *FG*? Justify your answer clearly with working. [3]

$\angle DGF = 180 - 153$ (Angles in opposite segments)
= 27^0 …….. M1

Since $\angle ABD = \angle DGF = 27^0$, …….. M1
By the converse of alternate angles, *BA* is parallel to *FG* …….. A1

Note: Minus 1 mark from Q3(b) when angle properties of circles are not given / written wrongly.

4 Starkids Centre is organising a talent competition for children. Children can choose to compete in 3 categories – singing, dancing and acting.

Matrix **E** shows the number of boys and girls in each category for the age group 5 to 8 years old.

$$\mathbf{E} = \begin{matrix} \\ \text{Singing} \\ \text{Dancing} \\ \text{Acting} \end{matrix} \begin{matrix} \text{Boys} & \text{Girls} \\ \end{matrix} \begin{pmatrix} 5 & 8 \\ 7 & 4 \\ 6 & 7 \end{pmatrix}$$

Matrix **F** shows the number of boys and girls in each category for the age group 9 to 12 years old.

$$\mathbf{F} = \begin{matrix} \\ \text{Singing} \\ \text{Dancing} \\ \text{Acting} \end{matrix} \begin{matrix} \text{Boys} & \text{Girls} \\ \end{matrix} \begin{pmatrix} 12 & 9 \\ 8 & 10 \\ 15 & 11 \end{pmatrix}$$

(a) Evaluate the matrix **T = E** + **F**.

$$\mathbf{T} = \begin{pmatrix} 5 & 8 \\ 7 & 4 \\ 6 & 7 \end{pmatrix} + \begin{pmatrix} 12 & 9 \\ 8 & 10 \\ 15 & 11 \end{pmatrix}$$

$$= \begin{pmatrix} 17 & 17 \\ 15 & 14 \\ 21 & 18 \end{pmatrix}$$ ……. B1

Answer (a) **T** =_____ [1]

(b) Each child is charged a registration fee for the competition. The registration is \$30 for singing, \$25 for dancing and \$22 for acting.
Represent the fees in a 1 x 3 matrix **C**.

$\mathbf{C} = (30 \quad 25 \quad 22)$ …….. B1

Answer (b) **C** =_____ [1]

(c) Evaluate the matrix **M = CT** and state what the elements in matrix **M** represent.

$$\mathbf{M} = \begin{pmatrix}30 & 25 & 22\end{pmatrix}\begin{pmatrix}17 & 17\\ 15 & 14\\ 21 & 18\end{pmatrix}$$

$= (1347 \quad 1256)$ …….. B1, B1

M represents the registration fees collected from boys and girls respectively …….. B1

Answer (c) **M** =_____ [2]

_____

_____ [1]

(d) Using matrix multiplication, calculate the total amount of registration fees collected for this competition.

$(1347 \quad 1256) \times \begin{pmatrix}1\\ 1\end{pmatrix}$ …… M1

$= (2603)$ …… A1

Answer (d) $_____ [2]

(e) Starkids Centre decides to organise a similar competition for teenagers aged 13 to 18 years old. As compared to the competition for children, the registration fees for teenagers is increased by 10% for singing, 20% for dancing and 15% for acting.

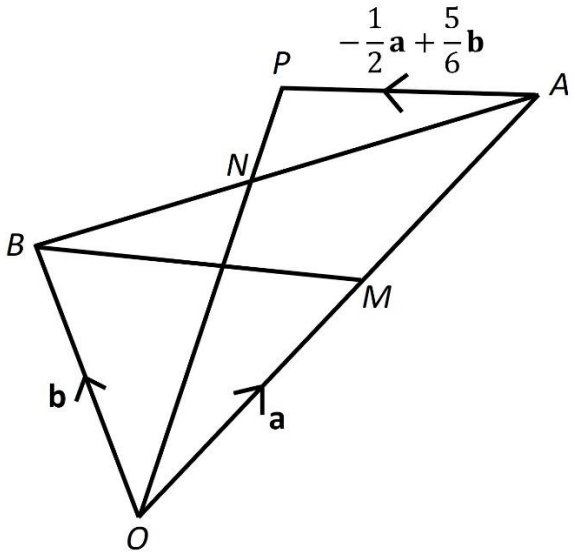
Using matrix multiplication, calculate the registration fees for teenagers for each category. Give your answer in matrix form.

$$(30 \quad 25 \quad 22) \times \begin{pmatrix}1.1 & 0 & 0\\ 0 & 1.2 & 0\\ 0 & 0 & 1.15\end{pmatrix}$$ ……. M1

$= (33 \quad 30 \quad 25.3)$ …….. A1

Answer (e) _____ [2]

5 In the diagram, M is the midpoint of OA.
$\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{AP} = -\frac{1}{2}\mathbf{a} + \frac{5}{6}\mathbf{b}$ and $BN : NA = 3 : 5$.



(a) Express, as simply as possible, in terms of **a** and/or **b**,

(i) $\overrightarrow{BN}$

(ii) $\overrightarrow{BM}$

(iii) $\overrightarrow{NM}$

$\overrightarrow{BN}$
$= \frac{3}{8}\overrightarrow{BA}$
$= \frac{3}{8}\mathbf{a} - \frac{3}{8}\mathbf{b}$ …… B1

$\overrightarrow{BM}$
$= \frac{1}{2}\mathbf{a} - \mathbf{b}$ ……. B1

$\overrightarrow{NM}$
$= \overrightarrow{NB} + \overrightarrow{BM}$
$= -\frac{3}{8}\mathbf{a} + \frac{3}{8}\mathbf{b} + \frac{1}{2}\mathbf{a} - \mathbf{b}$ …….. M1
$= \frac{1}{8}\mathbf{a} - \frac{5}{8}\mathbf{b}$ ……... A1

Answer (a) (i) _____ [1]

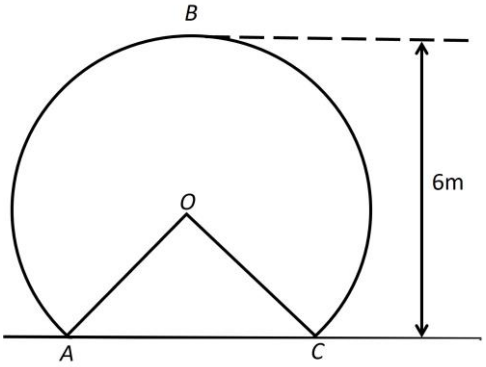
(ii) _____ [1]

(iii) _____ [2]

	(b)	Find $\frac{\text{Area of } \Delta OBM}{\text{Area of } \Delta OBA}$ $= \frac{1}{2}$ …… B1 *Answer* (b) _____	[1]
	(c)	If *ONAC* is a parallelogram, find $\overrightarrow{CA}$ $\overrightarrow{CA}$ $= \overrightarrow{ON}$ $= \overrightarrow{OB} + \overrightarrow{BN}$ $= \mathbf{b} + \frac{3}{8}\mathbf{a} - \frac{3}{8}\mathbf{b}$ …… M1 $= \frac{3}{8}\mathbf{a} + \frac{5}{8}\mathbf{b}$ …… A1 *Answer* (c) _____	[2]
	(d)	Given that $\frac{OP}{ON} = k$, find the value of *k*. $\overrightarrow{OP}$ $= \overrightarrow{OA} + \overrightarrow{AP}$ $= \mathbf{a} - \frac{1}{2}\mathbf{a} + \frac{5}{6}\mathbf{b}$ $= \frac{1}{2}\mathbf{a} + \frac{5}{6}\mathbf{b}$ **…… M1** $= \frac{1}{6}(\mathbf{3a} + 5\mathbf{b})$ $\overrightarrow{ON} = \frac{3}{8}\mathbf{a} + \frac{5}{8}\mathbf{b}$ $\overrightarrow{ON} = \frac{1}{8}(3\mathbf{a} + 5\mathbf{b})$ Therefore, $\frac{OP}{ON} = \frac{\frac{1}{6}(\mathbf{3a}+5\mathbf{b})}{\frac{1}{8}(3\mathbf{a}+5\mathbf{b})}$ …… M1 $\frac{OP}{ON} = \frac{4}{3}$ $k = \frac{4}{3}$ …… A1 *Answer* (d) *k* = _____	[3]

6 The diagram represents the cross-section of a tunnel where *AC* is on level ground and *B* is the highest point of the tunnel. The height of the tunnel is 6m and *AC* = 5m.

(a) Show that radius of the tunnel, r = 3.5m, corrected to 2 significant numbers. [2]



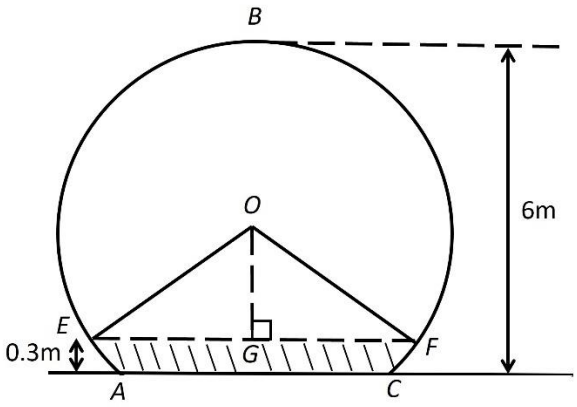
$(6 - r)^2 + 2.5^2 = r^2$ ……. M1

$36 - 12r + r^2 + 6.25 = r^2$

$12r = 42.25$

$r = 3.5$ cm (shown) …….. A1

(b) During a flood, water in the tunnel rose up to *EF*, 0.3m above ground level.



(i) Find $\angle EOF$ in radian.

$\frac{1}{2}\angle EOF = \cos^{-1}\left(\frac{6-3.5-0.3}{3.5}\right)$ …… M1

$\frac{1}{2}\angle EOF = \cos^{-1}\left(\frac{2.2}{3.5}\right)$

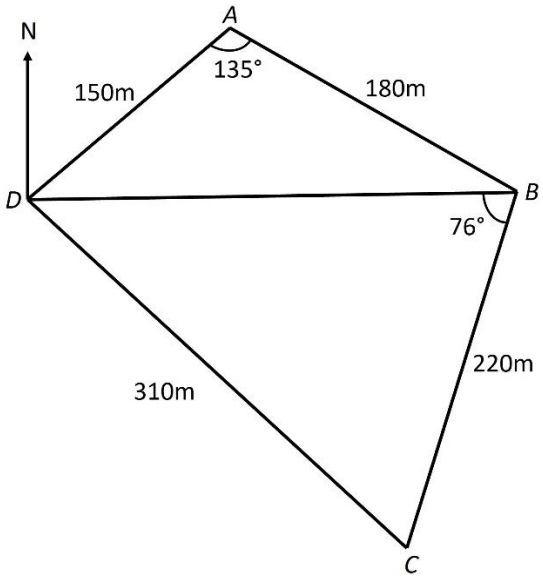
$\angle EOF = 1.78216$ rad

$\angle EOF = 1.78$ rad (3sf) …… A1

Answer (b) (i) _____ radian [2]

		(ii) Find the length of the major arc *EBF*. $3.5(2\pi - 1.78216)$ …… M1 = 15.8 m …….. A1 *Answer* (ii) _____m	[2]
		(iii) Find the cross-section area of the tunnel which is dry. $\frac{1}{2}(3.5^2)(2\pi - 1.78216) + \frac{1}{2}(3.5^2)\sin 1.78216$ …….. M1, M1 = 33.55747 = 33.6 m^2 (3sf) …….. A1 *Answer* (iii) _____m^2	[3]
		(iv) Given that the length of the tunnel is 1.4km, find the volume of the tunnel which is dry. 33.55747 x 1400 ……. M1 = 47 000 m^3 (3sf) …… A1 *Answer* (iv) _____m^3	[2]

7 The diagram shows a park *ABCD* on horizontal ground. *DB* represents a pebbled path.
AB = 180m, *BC* = 220m, *CD* = 310m and *DA* = 150m.
$\angle DAB = 135^0$ and $\angle DBC = 76^0$.



(a) Find the distance of the pebbled path *DB*.

$DB^2 = 150^2 + 180^2 - 2(150)(180)\cos 135^0$ ……. M1

DB = 305.0963

DB = 305 m (3sf) …… A1

Answer (a) _____m [2]

(b) Find $\angle BDC$.

$\frac{\sin \angle BDC}{220} = \frac{\sin 76^0}{310}$ …….. M1

$\angle BDC = 43.5191^0$

$\angle BDC = 43.5^0$ (1dp) …….. A1

Answer (b) _____^{0} [2]

	(c)	Find the shortest distance from *A* to *DB*. $\frac{1}{2} \times 150 \times 180 \times \sin 135^0 = \frac{1}{2} \times 305.0963 \times d$ …… M1, M1 d = 62.5766 d = 62.6 m (3sf) ……. A1 (ft) *Answer* (c) _____m	[3]
	(d)	A lamp post, 12m, is erected vertically at *A*. Find the greatest angle of elevation of the top of the lamp post from *DB*. $\tan^{-1}\left(\frac{12}{62.5766}\right)$ ……. M1 = 10.9^0 (1dp) …… A1 (ft) *Answer* (d) _____^{0}	[2]
	(e)	Given that *B* is due east of *D*, find the bearing of *D* from *C*. $360^0 - (180^0 - 90^0 - 43.5191^0)$ ……. M1 = 313.5^0 (1dp) …….. A1 (ft) *Answer* (e) _____	[2]

8 In July, the price of durians was \$$x$ per kg.
In August, the price of durians dropped by \$4.50 per kg.
Don could buy 2.8 kg more durians in August than in July with \$200.

(a) Write down an expression, in terms of x, to show the mass of durians that can be bought for \$200 in July.

$\frac{200}{x}$ …….. B1

Answer (a) _____ kg [1]

(b) Write down an expression, in terms of x, to show the mass of durians that can be bought for \$200 in August.

$\frac{200}{x-4.5}$ ……. B1

Answer (b) _____ kg [1]

(c) Form an equation in terms of x and show that it reduces to $14x^2 - 63x - 4500 = 0$. [3]

$\frac{200}{x-4.5} - \frac{200}{x} = 2.8$ ……. M1

$200(x) - 200(x - 4.5) = 2.8x(x - 4.5)$

$900 = 2.8x^2 - 12.6x$ ……. M1

$28x^2 - 126x - 9000 = 0$

$14x^2 - 63x - 4500 = 0$ (shown) …….. A1

	(d)	Solve the equation $14x^2 - 63x - 4500 = 0$. Give your answers corrected to 2 decimal places. $x = \frac{63 \pm \sqrt{(-63)^2 - 4(14)(-4500)}}{2(14)}$ ……. M1 $x = \frac{63 \pm \sqrt{255969}}{2(14)}$ $x = -15.82$ or 20.32 ……. A1, A1 *Answer* (d) x = _____or_____	[3]
	(e)	Find the price of the durians per kg in August. 20.3191 – 4.50 = \$15.82 (2dp) ……. B1 (ft) *Answer* (e) \$_____	[1]
	(f)	Find the percentage drop in price in August compared to July. $\frac{4.50}{20.3191} \times 100\%$ = 22.1% (3sf) …….. B1 (ft) *Answer* (f) _____%	[1]

9 The variables x and y are connected by the equation $y = \frac{1}{2}x^3 + \frac{1}{5x}$.

Some corresponding values of x and y, correct to 1 decimal place, are given in the table below.

x	-3	-2	-1	-0.5	-0.1	0.1	0.5	1	2	3
y	-13.6	p	-0.7	-0.5	-2.0	2.0	0.5	0.7	4.1	13.6

(a) Calculate the value of p.

$p = -4.1$ ……. B1

Answer (a) p = _____ [1]

(b) On the axes given on Page 19, draw the graph of $y = \frac{1}{2}x^3 + \frac{1}{5x}$. [2]

Correct points …… B1 Smooth curve …….. B1

(c) Using your graph, find the y when $x = -2.4$.

$y = -7 \pm 0.25$ …… B1

Answer (c) y = _____ [1]

(d) By drawing a tangent, find the gradient of the curve at $x = 2$.

Tangent …… M1

Gradient = 6 ± 2 ……. A1

Answer (d) _____ [2]

(e) By drawing a suitable straight line, find the solution of $\frac{1}{2}x^3 + \frac{1}{5x} + 2.5x + 5 = 0$.

Line: $y = -2.5x - 5$…… M1

$x = -1.4 \pm 0.1$ ……. A1

Answer (e) x = _____ [2]

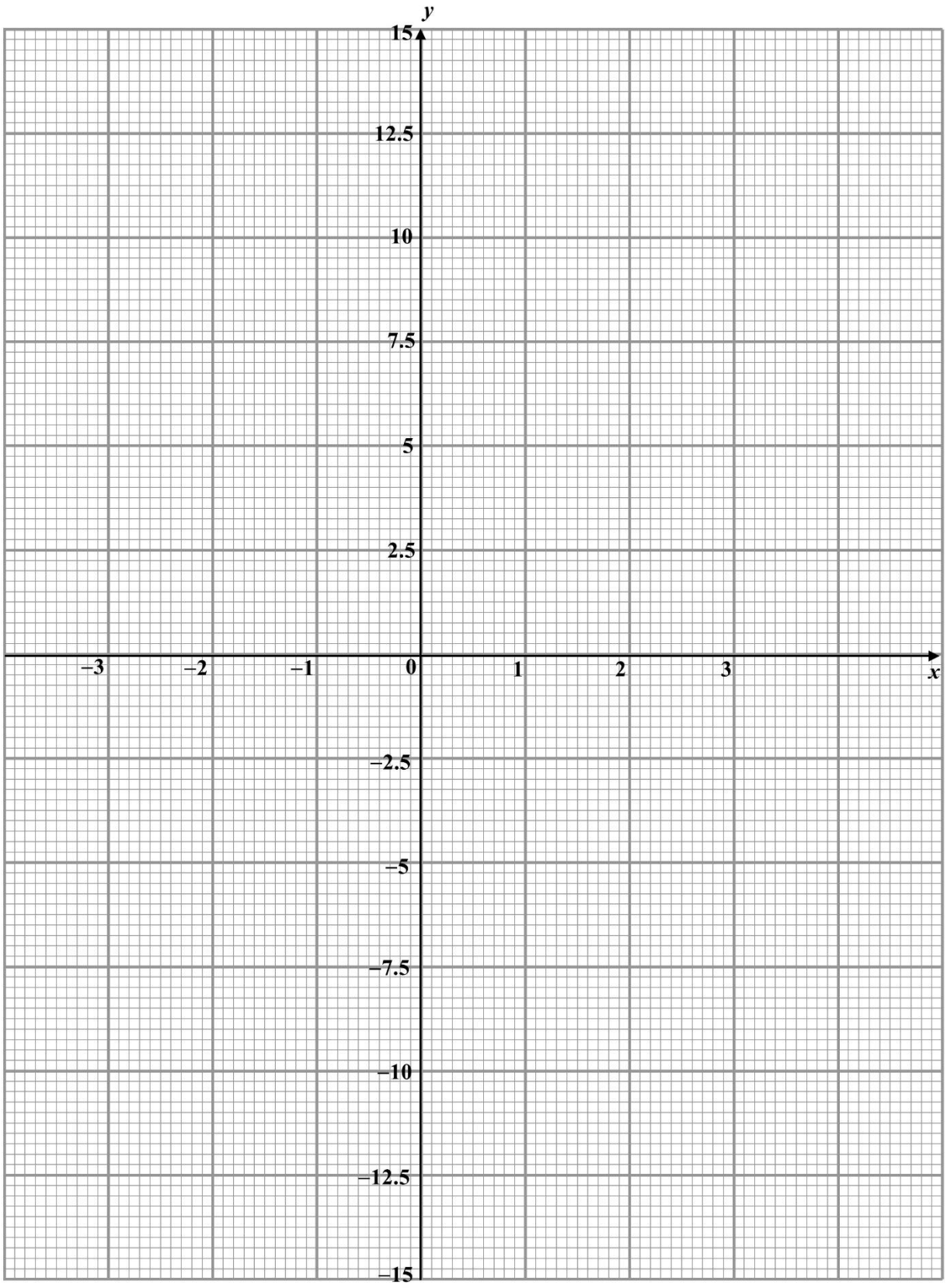
(f) Explain why the curve does not touch the y-axis.

At y-axis, $x = 0$, the curve is undefined…… B1

Answer (f) _____

_____ [1]

Answer (9b)



10 The tables below show the start-up kits for two food delivery companies, Get Food and Food Pack.

Get Food	Includes	Cost
Motorcycle/Car/Bicycle Basic Starter Pack	1 x Backpack 1 x Halal Bag 2 x Long Sleeve T-Shirt	$72
Motorcycle/Car/Bicycle Upsized Pack	1 x Basic Starter Pack 1 x Sling Bag	$105
Walker Starter Pack	1 x Sling Bag 1 x Halal Bag 2 x Long Sleeve T-Shirt	$60

Food Pack	Includes	Cost
Bicycles Starter Pack	Reflective jacket Long sleeve base Phone mount (optional) Helmet (optional) Insulated backpack	$50
Scooters / Motorcycles Starter Pack	Reflective jacket Long sleeve base Phone mount (optional) Small thermal bag Insulated backpack	$50
Walkers Starter Pack	Reflective jacket Long sleeve base Walker's lanyard Insulated backpack	$50

The table below shows the payments and benefits of the two food delivery companies.

	Get Food	Food Pack
Non-peak hours	$5.50 per delivery	$6 per delivery
Peak hours	$6.50 per delivery	$7 per delivery
Late fees	No late fees, but rider will not be given priority for the job after 5 late deliveries	Deduct $1.50 per delivery
Other benefits	Bonus of $10 will be given after every 13 deliveries. Bicycle rental at special rates: 7-Day Pass : $6.90 30-Day Pass : $19.90	Free bicycle rental

	(a)	**(i)**	Ben wants to work as a food delivery rider (Bicycle) during the one-month school holiday. He is choosing between two companies – Get Food and Food Pack. Ben wants to work 5 days a week, during non-peak hours. He can make an average of 6 deliveries a day during non-peak hours. Ben does not own a bicycle. Assuming that he makes three late deliveries in one month, how much does he earn in that month if he works with Get Food and Food Pack? <u>Get Food</u> Revenue = (5 x 6 x 4 x $5.50) + (9 x $10) = $750 …….. M1 Earnings = $750 - $72 - $19.90 ……...M1 = $658.10 …… A1 <u>Food Pack</u> Revenue = (5 x 6 x 4 x $6) = $720 …….. M1 Earnings = $720 - $50 – (3 x $1.50) ……...M1 = $665.50 …… A1	[6]

(ii) Besides making more deliveries, what is one other way Ben can increase his earnings?

work peak hours, don't be late for deliveries, other appropriate answers …… B1

Answer (a) (ii) _____ [1]

(b) **(i)** Karl wants to work as a food delivery walker with Get Food during peak hours. He aims to earn at least $200.

Fill in the table below to show how much Get Food pays a food delivery walker during peak hours.

	Minimum payment	**Maximum payment**
1 to 12 deliveries	$6.50	$78
13 to 25 deliveries	$94.50	$ 172.50 … B1
26 to 38 deliveries	$ 189 … B1	$267

[2]

(ii) What is the minimum number of deliveries Karl must make in order to earn at least $200?

$\frac{200+60-20}{6.50}$ ……. M1

= 36.9

Therefore, he must make a minimum of 37 deliveries….. A1

OR

$n \times 6.50 - 60 + 20 \geq 200$ …… M1
$n \geq 36.9$

Therefore, he must make a minimum of 37 deliveries….. A1

Answer (b) (ii) _____ [2]

End of Paper