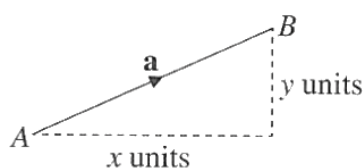


## Secondary 4 Mathematics: Vectors

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### 1. Vectors

- A vector is a quantity that has both **magnitude** and **direction**.
- A vector can be represented by a directed line segment as shown below.



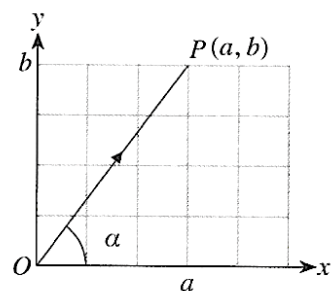
- Point  $A$  is known as the **starting point** and point  $B$  is the **ending point**.
- The vector above can be denoted by  $\overrightarrow{AB}$  or  $\begin{pmatrix} x \\ y \end{pmatrix}$  or  $a$ .

#### A. Magnitude of a Column Vector

- For  $\overrightarrow{AB} = \mathbf{a} = \begin{pmatrix} x \\ y \end{pmatrix}$ , magnitude of  $\mathbf{a} = |\mathbf{a}| = \sqrt{x^2 + y^2}$

#### B. Position Vector and Coordinates of a Point

- If a point  $P$  has coordinates  $(a, b)$ , the position vector of  $P$  is  $\overrightarrow{OP} = \begin{pmatrix} a \\ b \end{pmatrix}$ .



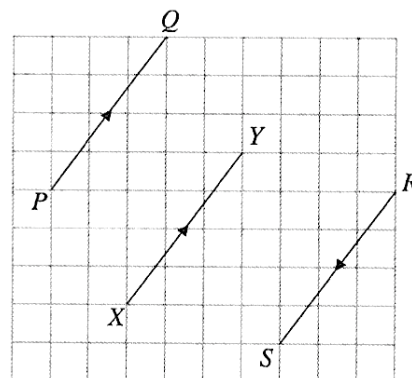
### 2. Equal and Negative Vectors

- Equal vectors** are vectors with the same magnitude and direction.

$$\overrightarrow{PQ} = \overrightarrow{XY}$$

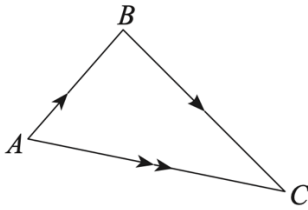
- Negative vectors** are vectors with the same magnitude but are in opposite directions.

$$\overrightarrow{PQ} = -\overrightarrow{RS}$$



### 3. Sum and Differences of Two Vectors

#### A. Triangular Law of Addition



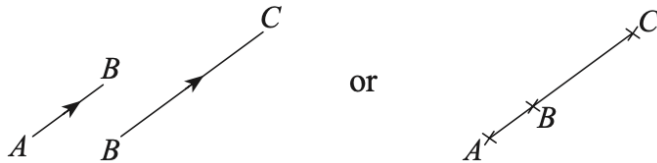
$$\vec{AB} + \vec{BC} = \vec{AC}$$

#### B. Subtraction of Vectors

$$\vec{BC} = \vec{AC} - \vec{AB}$$

### 4. Multiplying a Vector by a Scalar

For parallel vectors or points that are collinear,  $\vec{AB} = k\vec{BC}$ .



### 5. Ratio of Areas of Triangles

Method 1: If both triangles are similar

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$

Method 2: If both share a common height,

$$\frac{A_1}{A_2} = \frac{\frac{1}{2} \times b_1 \times h}{\frac{1}{2} \times b_2 \times h} = \frac{b_1}{b_2}$$

## Vectors (Worksheet 1)

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1. Find the magnitude of the following vectors:

(a)  $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

Answer: (a) \_\_\_\_\_ units

(b)  $\overrightarrow{BC} = \begin{pmatrix} -12 \\ -5 \end{pmatrix}$

Answer: (b) \_\_\_\_\_ units

(c)  $\overrightarrow{CD} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$

Answer: (c) \_\_\_\_\_ units

2. Given that  $A$  is the point  $(-4, 6)$ ,  $B$  is the point  $(-1, y - 2)$  and  $\overrightarrow{AC} = \begin{pmatrix} 4 \\ -10 \end{pmatrix}$ , find
- (a) the position vectors of points  $A$  and  $B$ ,

Answer: (a)  $\overrightarrow{OA} =$  \_\_\_\_\_,  $\overrightarrow{OB} =$  \_\_\_\_\_

- (b)  $\overrightarrow{OC}$  and thus the coordinates of  $C$ ,

Answer: (b)  $\overrightarrow{OC} =$  \_\_\_\_\_,  
 $C = ($  \_\_\_\_\_, \_\_\_\_\_)

- (c)  $\overrightarrow{AO}$ .

Answer: (c)  $\overrightarrow{OA} =$  \_\_\_\_\_

- (d) Hence, using your answer in (c), find the value of  $y$  if  $A$ ,  $B$  and  $C$  are collinear.

Answer: (d)  $y =$  \_\_\_\_\_

3. Given that  $P$  is the point  $(0, 3)$ ,  $\overrightarrow{PQ} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$  and  $R$  is a point on the  $x$ -axis, such that  $P$ ,  $Q$  and  $R$  lies on the same straight line, find

(a)  $|\overrightarrow{PQ}|$ ,

Answer: (a)  $|\overrightarrow{PQ}| = \underline{\hspace{2cm}}$  units

(b) the coordinates of  $Q$ ,

Answer: (b)  $Q = ( \underline{\hspace{1cm}}, \underline{\hspace{1cm}} )$

$R$  is a point on the  $x$ -axis such that point  $R$  is  $(x, 0)$ . Find

(c)  $\overrightarrow{PR}$  in terms of  $x$ ,

Answer: (c)  $\overrightarrow{PR} = \underline{\hspace{2cm}}$

Since  $P$ ,  $Q$  and  $R$  lies on the same straight line, it is given that  $\overrightarrow{PQ} = k\overrightarrow{PR}$ . Find  
(d) the value of  $k$ ,

Answer: (d)  $k =$  \_\_\_\_\_

(e) the position vector of  $R$ .

Answer: (e)  $\overrightarrow{OR} =$  \_\_\_\_\_

4. Given that  $A$  is the point  $(5, 2)$  and  $\overrightarrow{OB} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$ .

(a) Calculate  $|\overrightarrow{AB}|$ .

Answer: (a)  $|\overrightarrow{AB}| =$  \_\_\_\_\_ units

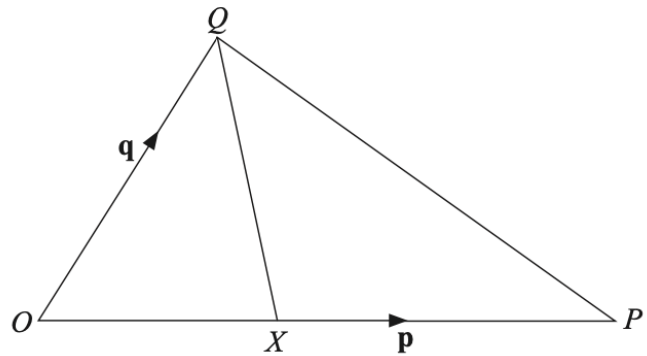
(b) Find the equation of the line  $AB$ .

Answer: (b)  $y =$  \_\_\_\_\_

(c) Given that  $C$  is the midpoint of  $AB$ , find the coordinates of  $C$ .

Answer: (c)  $C = ( \rule{1cm}{0.4pt}, \rule{1cm}{0.4pt} )$

5. In the figure below,  $\overrightarrow{OP} = \mathbf{p}$  and  $\overrightarrow{OQ} = \mathbf{q}$ . If  $\overrightarrow{OX} = \frac{3}{5}\overrightarrow{OP}$ , find  $\overrightarrow{QX}$  in terms of  $\mathbf{p}$  and  $\mathbf{q}$ .

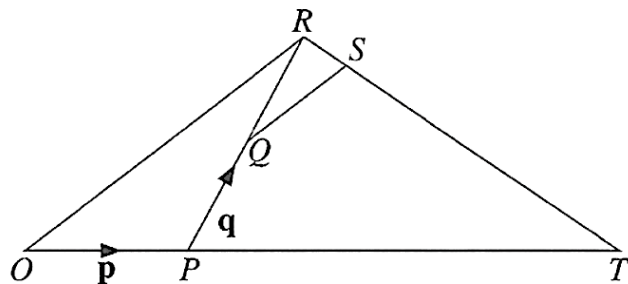


Answer:  $\overrightarrow{QX} = \rule{2cm}{0.4pt}$



6. In the diagram,  $OT = 3OP$ ,  $RT = 6RS$  and  $Q$  is the midpoint of  $PR$ .

It is given that  $\overrightarrow{OP} = \mathbf{p}$  and  $\overrightarrow{PQ} = \mathbf{q}$ .



- (a) Express, as simply as possible, in terms of  $\mathbf{p}$  and  $\mathbf{q}$ ,

- (i)  $\overrightarrow{OR}$ ,
- (ii)  $\overrightarrow{TR}$ ,
- (iii)  $\overrightarrow{QS}$ .

Answer: (a)(i)  $\overrightarrow{OR} =$  \_\_\_\_\_

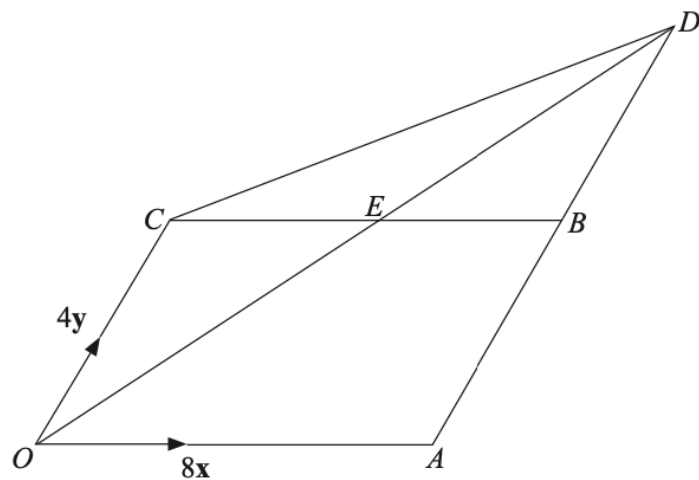
(a)(ii)  $\overrightarrow{TR} =$  \_\_\_\_\_

(a)(iii)  $\overrightarrow{QS} =$  \_\_\_\_\_

(b) Determine if the lines  $OR$  and  $QS$  are parallel, showing your reasons clearly.

7. In the figure below,  $OABC$  is a parallelogram.

The vectors  $\overrightarrow{OA} = 8\mathbf{x}$  and  $\overrightarrow{OC} = 4\mathbf{y}$ . The lines  $AB = 2BD$  and  $CE = EB$ .



(a) Express the following vectors in terms of  $\mathbf{x}$  and  $\mathbf{y}$ .

(i)  $\overrightarrow{AC}$ ,

Answer: (a)(i)  $\overrightarrow{AC} = \underline{\hspace{2cm}}$

(ii)  $\overrightarrow{OD}$ .

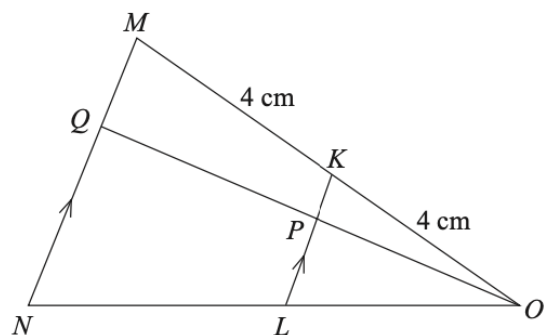
Answer: (a)(ii)  $\overrightarrow{OD} = \underline{\hspace{2cm}}$

(b) Find the value of the ratio of the area of triangle  $OCE$  to the area of triangle  $OAD$ .

Answer: (b)  $\frac{\text{Area of triangle } OCE}{\text{Area of triangle } OAD} = \underline{\hspace{2cm}}$

8. In the diagram below,  $OK = MK = 4$  cm,  $MN$  is parallel to  $KL$ ,  $OQ$  and  $KL$  intersect at  $P$  and  $\frac{KP}{PL} = \frac{1}{3}$ .

(a) Name a triangle which is similar to  $\triangle OMN$ .



Answer: (a)  $\triangle \underline{\hspace{2cm}}$

(b) Write down the numerical value of

(i)  $\frac{\text{Area of } \triangle OKP}{\text{Area of } \triangle OKL}$

Answer: (b)(i)  $\frac{\text{Area of } \triangle OKP}{\text{Area of } \triangle OKL} = \underline{\hspace{2cm}}$

(ii)  $\frac{\text{Area of } \triangle OKL}{\text{Area of } \triangle OMN}$

Answer: (b)(i)  $\frac{\text{Area of } \triangle OKL}{\text{Area of } \triangle OMN} = \underline{\hspace{2cm}}$

9. Given that  $P$  is the point  $(1, 1)$  and  $\overrightarrow{PQ} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$  and  $\overrightarrow{SR} = \begin{pmatrix} k \\ -12 \end{pmatrix}$ , find

(a)  $|\overrightarrow{PQ}|$ ,

Answer: (a)  $|\overrightarrow{PQ}| =$  \_\_\_\_\_ units

(b) the coordinates of  $Q$ ,

Answer: (b)  $Q = ( \text{_____, } \text{_____} )$

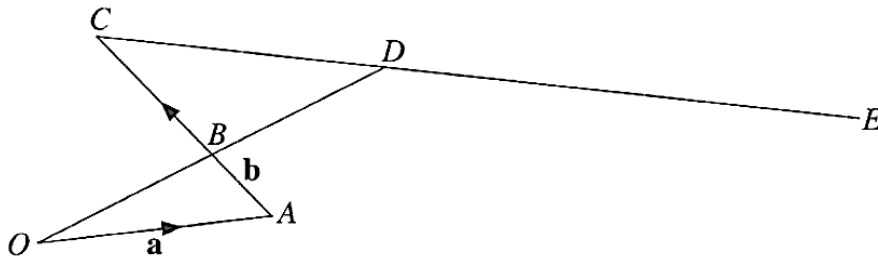
(c) the value of  $k$  if  $PQ$  is parallel to  $SR$ .

Answer: (c)  $k =$  \_\_\_\_\_

## Vectors (Worksheet 2)

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1. In the diagram below,  $\overrightarrow{OA} = \mathbf{a}$  and  $\overrightarrow{OB} = \mathbf{b}$ . It is given that  $\overrightarrow{OB} = \overrightarrow{BD}$ ,  $\overrightarrow{BC} = 2\overrightarrow{AB}$  and  $\overrightarrow{DE} = 2\overrightarrow{CD}$ .



(a) Express, as simply as possible, in terms of  $\mathbf{a}$  and/or  $\mathbf{b}$ ,

- (i)  $\overrightarrow{OB}$ ,
- (ii)  $\overrightarrow{CD}$ ,
- (iii)  $\overrightarrow{OD}$ ,
- (iv)  $\overrightarrow{OE}$ .

Answer: (a)(i)  $\overrightarrow{OB} =$  \_\_\_\_\_

(a)(ii)  $\overrightarrow{CD} =$  \_\_\_\_\_

(a)(iii)  $\overrightarrow{OD} =$  \_\_\_\_\_

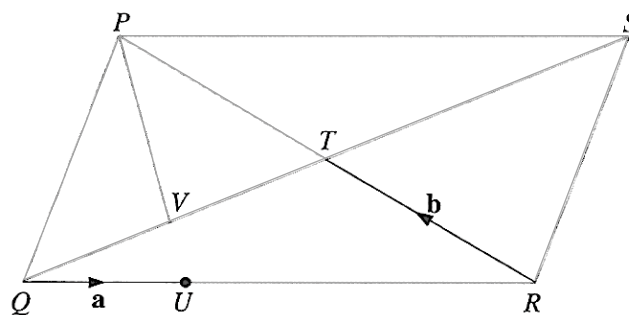
(a)(iv)  $\overrightarrow{OE} =$  \_\_\_\_\_

(b) Hence, write down two facts about  $O$ ,  $A$  and  $E$ .

(1) \_\_\_\_\_

(2) \_\_\_\_\_

2. In the diagram below,  $PQRS$  is a parallelogram. The diagonals  $PR$  and  $QS$  intersect at  $T$ .



$U$  is a point on  $QR$  such that  $QR = 3QU$ .  $V$  is the midpoint of  $QT$ .  $\vec{QU} = \mathbf{a}$  and  $\vec{RT} = \mathbf{b}$ .

(a) Express as simply as possible in terms of  $\mathbf{a}$  and/or  $\mathbf{b}$ .

(i)  $\vec{QP}$ ,

Answer: (a)(i)  $\vec{QP} =$  \_\_\_\_\_

(ii)  $\vec{QV}$ ,

Answer: (a)(ii)  $\vec{QV} =$  \_\_\_\_\_

(iii)  $\overrightarrow{PV}$

Answer: (a)(iii)  $\overrightarrow{PV} =$  \_\_\_\_\_

(b) Show that  $PV$  produced passes through  $U$ .

(c) Calculate the value of

(i)  $\frac{\text{Area of } \triangle PQS}{\text{Area of } \triangle PVS}$

(ii)  $\frac{\text{Area of } \triangle PVS}{\text{Area of } \triangle PQRS}$

Answer: (c)(i)  $\frac{\text{Area of } \triangle PQS}{\text{Area of } \triangle PVS} =$  \_\_\_\_\_

Answer: (c)(ii)  $\frac{\text{Area of } \triangle PVS}{\text{Area of } \triangle PQRS} =$  \_\_\_\_\_



3. In the diagram,  $\overrightarrow{OA} = \frac{1}{3}\overrightarrow{OP}$  and  $\overrightarrow{OB} = \frac{1}{4}\overrightarrow{OQ}$ .  $M$  is the midpoint of  $OQ$ , and  $MX = \frac{1}{5}MP$ .

(a) Given that  $\overrightarrow{OA} = \mathbf{a}$  and  $\overrightarrow{OB} = \mathbf{b}$ , express as simply as possible, in terms of  $\mathbf{a}$  and/or  $\mathbf{b}$ ,

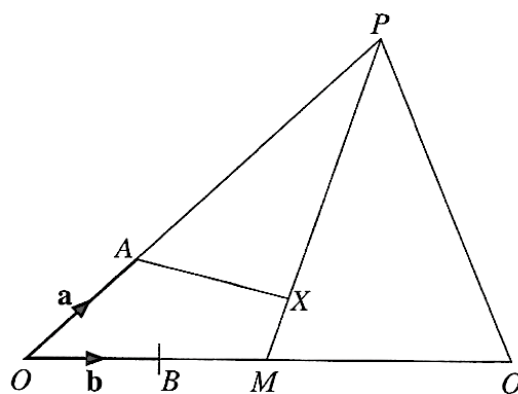
(i)  $\overrightarrow{OP}$ ,

(ii)  $\overrightarrow{OM}$ ,

(iii)  $\overrightarrow{AQ}$ ,

(iv)  $\overrightarrow{MP}$ ,

(v)  $\overrightarrow{MX}$ .



Answer: (a)(i)  $\overrightarrow{OP} =$  \_\_\_\_\_

(a)(iv)  $\overrightarrow{MP} =$  \_\_\_\_\_

(a)(ii)  $\overrightarrow{OM} =$  \_\_\_\_\_

(a)(v)  $\overrightarrow{MX} =$  \_\_\_\_\_

(a)(iii)  $\overrightarrow{AQ} =$  \_\_\_\_\_

(b) Prove that  $AX$  produced will pass through  $Q$ .

(c) Find the ratio of  $AX : XQ$ .

Answer: (b)  $AX : XQ =$  \_\_\_\_\_

(d) Given that the area of  $\triangle OPQ = 30 \text{ cm}^2$ , calculate the area of

(i)  $\triangle PMQ$ ,

Answer: (d)(i) Area of  $\triangle PMQ =$  \_\_\_\_\_  $\text{cm}^2$

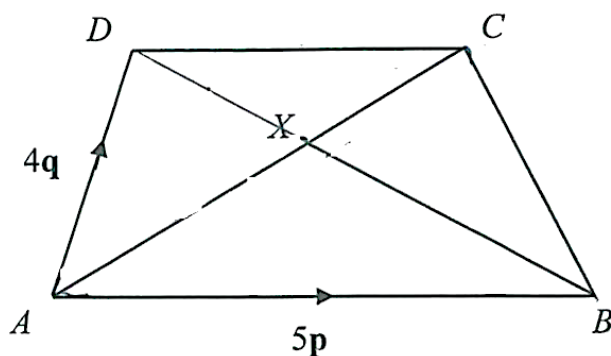
(ii)  $\triangle PQX$ ,

(d)(ii) Area of  $\triangle PQX$  = \_\_\_\_\_  $\text{cm}^2$

(iii)  $\triangle OAB$ .

(d)(iii) Area of  $\triangle OAB$  = \_\_\_\_\_  $\text{cm}^2$

4.



$ABCD$  is a quadrilateral.

$\overrightarrow{AB} = 5\mathbf{p}$ ,  $\overrightarrow{AD} = 4\mathbf{q}$ ,  $DC : AB = 3 : 5$ ,  $AX : AC = 5 : 8$ .

(a) Write down the following in terms of  $\mathbf{p}$  and  $\mathbf{q}$ .

(i)  $\overrightarrow{AC}$ ,

Answer: (a)(i)  $\overrightarrow{AC} =$  \_\_\_\_\_

(ii)  $\overrightarrow{BX}$ ,

Answer: (a)(ii)  $\overrightarrow{BX} =$  \_\_\_\_\_

(iii)  $\overrightarrow{XD}$ .

Answer: (a)(i)  $\overrightarrow{XD} =$  \_\_\_\_\_

(b) Explain why  $B$ ,  $X$  and  $D$  lie on a straight line.

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5. (a) The position vector of point  $A$  is  $\begin{pmatrix} -1 \\ 5 \end{pmatrix}$  and the position vector of point  $B$  is  $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ .

(i) Find the column vector of  $\overrightarrow{AB}$ .

Answer: (a)(i)  $\overrightarrow{AB} =$  \_\_\_\_\_

(ii) Find  $|\overrightarrow{AB}|$ .

Answer: (a)(ii)  $|\overrightarrow{AB}| =$  \_\_\_\_\_ units

(iii) Given that  $\overrightarrow{AC} = 3\overrightarrow{AB}$ , find the coordinates of  $C$ .

Answer: (a)(iii)  $C = ( \text{_____, } \text{_____} )$

(b) The point  $P$  has coordinates  $(4, -2)$  and  $\overrightarrow{PQ} = \begin{pmatrix} -8 \\ 12 \end{pmatrix}$ .

(i) Find the equation of the line  $PQ$ .

Answer: (b)(i)  $y =$  \_\_\_\_\_

(ii) The equation of another line  $3x + 2y = 11$ . Show how you can tell that this line does **not** intersect the line  $PQ$ .

6.  $A$  is the point  $(1, 1)$ ,  $\overrightarrow{AB} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$  and  $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ .

$D$  divides  $BC$  such that  $BD : DC = 1 : 1$ .

- (a) Find  $\overrightarrow{BC}$ .

Answer: (a)  $\overrightarrow{BC} =$  \_\_\_\_\_

- (b) Find  $|\overrightarrow{AD}|$ .

Answer: (b)  $|\overrightarrow{AD}| =$  \_\_\_\_\_ units

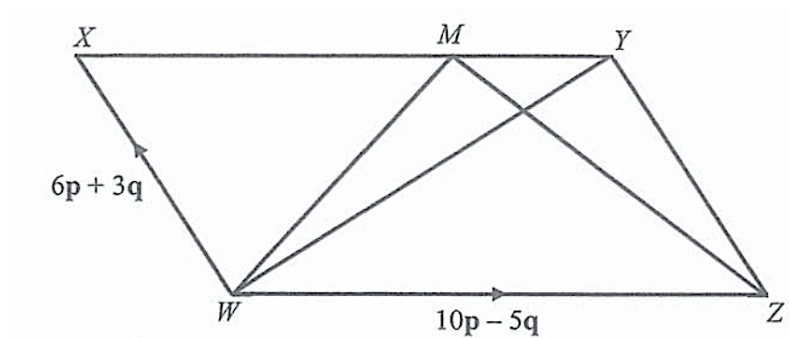
(c)  $P$  is the point  $(3, 9)$

Use the vectors to show whether or not  $ABPC$  is a parallelogram.



7. In the diagram,  $WXYZ$  is a parallelogram.

$M$  is a point on  $XY$  such that  $XM : MY = 3 : 2$ ,  $\overrightarrow{WX} = 6\mathbf{p} + 3\mathbf{q}$  and  $\overrightarrow{WZ} = 10\mathbf{p} - 5\mathbf{q}$ .



(a) Find, in terms of  $\mathbf{p}$  and/or  $\mathbf{q}$ ,

(i)  $\overrightarrow{WM}$ ,

Answer: (a)(i)  $\overrightarrow{WM} =$  \_\_\_\_\_

(ii)  $\overrightarrow{ZM}$

Answer: (a)(i)  $\overrightarrow{ZM} =$  \_\_\_\_\_

(b) Find

- (i) the area of triangle  $WMX$  : area of  $WXYZ$ ,

Answer: (b)(i) \_\_\_\_\_

- (ii) area of  $WXYZ$ , given that the area of triangle  $WMX$  is 8 units<sup>2</sup>.

Answer: (b)(ii) area of  $WXYZ$  = \_\_\_\_\_ units<sup>2</sup>

- (iii) Given that  $N$  is on  $WX$  produced such that  $ZMN$  is a straight line. Express  $\overrightarrow{WN}$  in terms of  $p$  and  $q$ .

Answer: (b)(iii)  $\overrightarrow{WN}$  = \_\_\_\_\_

8. Coordinates of  $A$  and  $B$  are  $(-3, 3)$  and  $(7, -13)$  respectively.

(a) Write  $\overrightarrow{AB}$  as a column vector.

Answer: (a)  $\overrightarrow{AB} =$  \_\_\_\_\_

(b) Find the acute angle formed by the line  $AB$  with the horizontal axis.

Answer: (b) *angle* = \_\_\_\_\_ °

(c) If the gradient of  $AB = -\frac{2m}{n}$ , express  $\overrightarrow{AB}$  in terms of  $m$  and  $n$ .

Answer: (C)  $\overrightarrow{AB} =$  \_\_\_\_\_

9.  $A$  is the point  $(-2, 5)$  and  $\overrightarrow{BA} = \begin{pmatrix} -4 \\ 8 \end{pmatrix}$ .

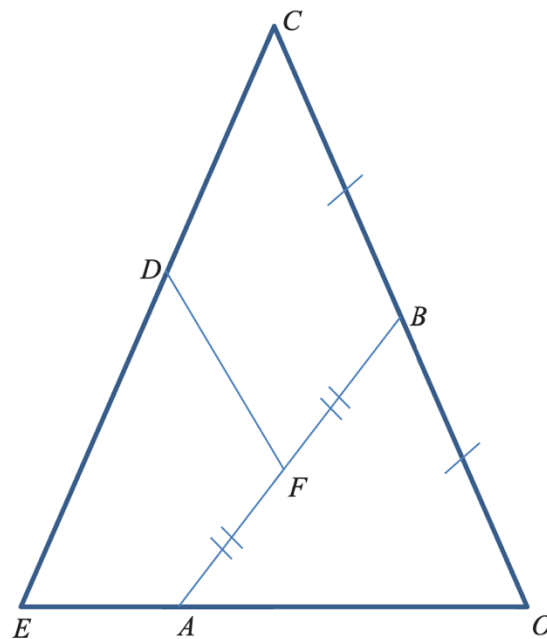
(a) Find the coordinates of point  $B$ .

Answer: (a)  $B = ( \rule{1cm}{0.4pt}, \rule{1cm}{0.4pt} )$

(b) Calculate  $|\overrightarrow{BA}|$ .

Answer: (b)  $|\overrightarrow{AB}| = \rule{2cm}{0.4pt}$  units

10. In the diagram below,  $OB = BC$  and  $AF = FB$ . It is given that  $OA : AE = 2 : 1$  and  $ED : DC = 4 : 3$ .  $\overrightarrow{OA} = 2\mathbf{a}$  and  $\overrightarrow{OB} = \mathbf{b}$ .



(a) Express, as simply as possible, in terms of  $\mathbf{a}$  and/or  $\mathbf{b}$ ,

(i)  $\overrightarrow{CE}$ ,

Answer: (a)(i)  $\overrightarrow{CE} =$  \_\_\_\_\_

(ii)  $\overrightarrow{CD}$ ,

Answer: (a)(ii)  $\overrightarrow{CD} =$  \_\_\_\_\_

(iii)  $\overrightarrow{BA}$ ,

Answer: (a)(iii)  $\overrightarrow{BA} =$  \_\_\_\_\_

(iv)  $\overrightarrow{OF}$ ,

Answer: (a)(iv)  $\overrightarrow{OF} =$  \_\_\_\_\_

(v)  $\overrightarrow{FD}$ .

Answer: (a)(ii)  $\overrightarrow{FD} =$  \_\_\_\_\_

(b) Find

(i)  $\frac{\text{Area of } \triangle OBA}{\text{Area of } \triangle OBE}$

Answer: (b)(i)  $\frac{\text{Area of } \triangle OBA}{\text{Area of } \triangle OBE} =$  \_\_\_\_\_

(ii)  $\frac{\text{Area of } \triangle OBA}{\text{Area of } \triangle OCE}$

Answer: (b)(ii)  $\frac{\text{Area of } \triangle OBA}{\text{Area of } \triangle OCE} =$  \_\_\_\_\_

### Vectors (Worksheet 3)

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1.  $OPQR$  is a parallelogram such that  $\overrightarrow{PQ} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$  and  $P$  is the point  $(3, 2)$ .

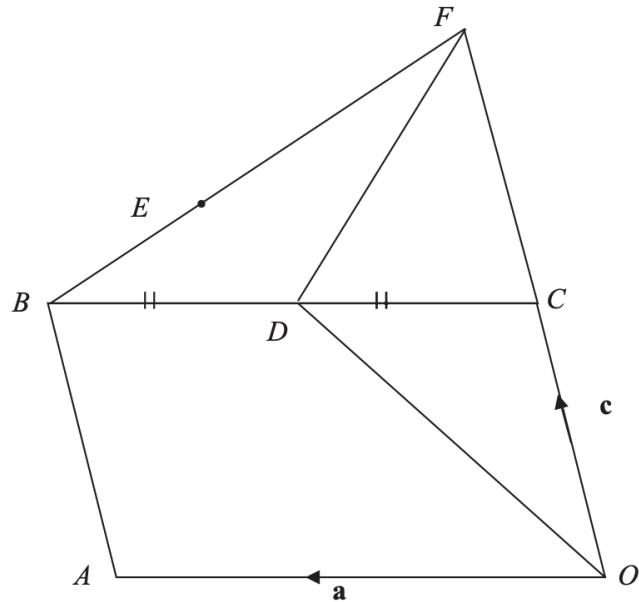
(a) Express  $\overrightarrow{RP}$  as a column vector.

Answer: (a)  $\overrightarrow{RP} =$  \_\_\_\_\_

(b) The point  $J$  lies on  $\overrightarrow{RP}$  produced such that  $\overrightarrow{PJ} = m\overrightarrow{RP}$ .

Show that  $\overrightarrow{OJ} = \begin{pmatrix} 3 + m \\ 2 - 2m \end{pmatrix}$ .

2. In the diagram,  $OABC$  is a parallelogram and  $D$  is the midpoint of  $BC$ .  $BE$  and  $OC$  produced intersect at the point  $F$ .  $BE : BF = 1 : 3$  and  $OC : OF = 1 : 2$ .  
Let  $\overrightarrow{OA} = \mathbf{a}$  and  $\overrightarrow{OC} = \mathbf{c}$ .



(a) Express and simplify the following vectors in terms of  $\mathbf{a}$  and  $\mathbf{c}$ .

(i)  $\overrightarrow{AC}$ ,

Answer: (a)(i)  $\overrightarrow{AC} =$  \_\_\_\_\_

(ii)  $\overrightarrow{BF}$ ,

Answer: (a)(ii)  $\overrightarrow{BF} =$  \_\_\_\_\_



(iii)  $\overrightarrow{OD}$ ,

Answer: (a)(iii)  $\overrightarrow{OD} =$  \_\_\_\_\_

(iv)  $\overrightarrow{OE}$ .

Answer: (a)(iv)  $\overrightarrow{OE} =$  \_\_\_\_\_

(b) State two facts about the vectors  $\overrightarrow{OD}$  and  $\overrightarrow{OE}$  from the results in (a).

(c) Find the ratio of the areas of

(i)  $\triangle ODF$  and  $\triangle OEF$ ,

Answer: (c)(i) \_\_\_\_\_

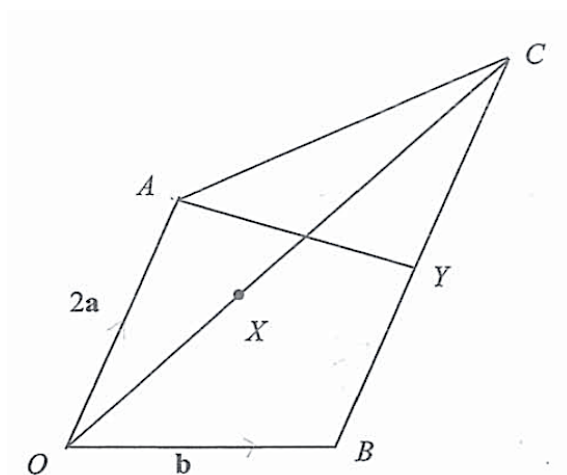
(ii)  $\triangle OCD$  and  $OABC$

Answer: (c)(ii) \_\_\_\_\_

(iii)  $\triangle OCD$  and  $OABF$

Answer: (c)(iii) \_\_\_\_\_

3.



In the diagram,  $\overrightarrow{OA} = 2\mathbf{a}$ ,  $\overrightarrow{OB} = \mathbf{b}$ .  $BC$  is parallel to  $OA$  and  $BC = \frac{3}{2}OA$ .  $X$  is a point on  $OC$  such that  $OX = \frac{2}{3}XC$ .  $Y$  is the midpoint of  $BC$ .

(a) Express in terms of  $\mathbf{a}$  and/or  $\mathbf{b}$ , as simply as possible,

(i)  $\overrightarrow{AB}$ ,

Answer: (a)(i)  $\overrightarrow{AB} =$  \_\_\_\_\_

(ii)  $\overrightarrow{OC}$ ,

Answer: (a)(ii)  $\overrightarrow{OC} =$  \_\_\_\_\_

(iii)  $\overrightarrow{OX}$ ,

Answer: (a)(iii)  $\overrightarrow{OX} =$  \_\_\_\_\_

(iv)  $\overrightarrow{AX}$ ,

Answer: (a)(iv)  $\overrightarrow{AX} =$  \_\_\_\_\_

(b) What can you deduce about  $A$ ,  $X$  and  $B$ ?

Justify your answer.

(c)  $AY$  produced meets  $OB$  at a point  $Z$ .

(i) Given that  $\overrightarrow{AZ} = h\overrightarrow{AY}$ , express  $\overrightarrow{AZ}$  in terms of  $\mathbf{a}$ ,  $\mathbf{b}$  and  $h$ .

Answer: (c)(i)  $\overrightarrow{AZ} =$  \_\_\_\_\_

(ii) Given also that  $\overrightarrow{OZ} = k\overrightarrow{OB}$ , express  $\overrightarrow{OZ}$  in terms of  $\mathbf{a}$ ,  $\mathbf{b}$  and  $k$ .

Answer: (c)(i)  $\overrightarrow{OZ} =$  \_\_\_\_\_

(iii) Hence, show that  $h = 4$  and  $k = 4$

(d) Find the value of,

(i)  $\frac{\text{Area of } \triangle OAX}{\text{Area of } \triangle OAC},$

Answer: (d)(i)  $\frac{\text{Area of } \triangle OAX}{\text{Area of } \triangle OAC} =$  \_\_\_\_\_

(ii)  $\frac{\text{Area of } \triangle OBX}{\text{Area of } \triangle ABC}$

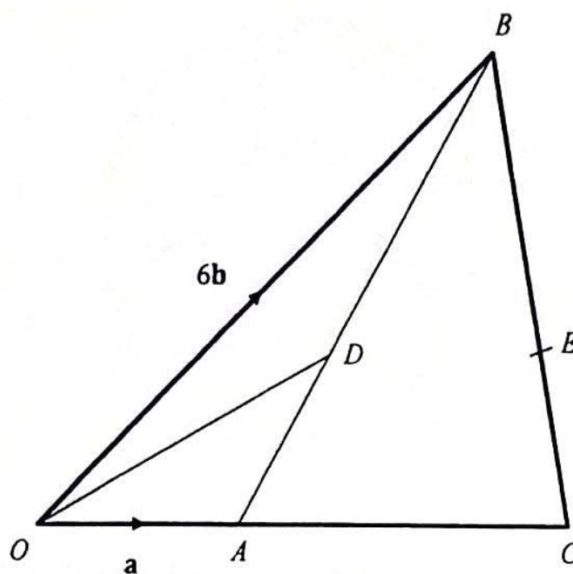
Answer: (d)(i)  $\frac{\text{Area of } \triangle OBX}{\text{Area of } \triangle ABC} =$  \_\_\_\_\_

4.  $PQRS$  is a parallelogram such that  $\overrightarrow{PQ} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$  and  $\overrightarrow{PS} = \begin{pmatrix} 12 \\ 5 \end{pmatrix}$ .

Calculate  $|\overrightarrow{PR}|$ .

Answer:  $|\overrightarrow{PR}| =$  \_\_\_\_\_ units

5.



In the diagram,  $\overrightarrow{OA} = \mathbf{a}$ ,  $\overrightarrow{OB} = 6\mathbf{b}$  and  $\overrightarrow{OA} = \frac{1}{3}\overrightarrow{OC}$ .

$D$  is a point on  $AB$  such that  $3AD = 2DB$  and  $E$  is a point on  $BC$  such that  $CE : EB = 4 : 5$ .

(a) Express in terms of  $\mathbf{a}$  and/or  $\mathbf{b}$ , as simply as possible,

(i)  $\overrightarrow{BA}$ ,

Answer: (a)(i)  $\overrightarrow{BA} =$  \_\_\_\_\_

(ii)  $\overrightarrow{OD}$ ,

Answer: (a)(ii)  $\overrightarrow{OD} =$  \_\_\_\_\_

(iii)  $\overrightarrow{CB}$ ,

Answer: (a)(iii)  $\overrightarrow{CB} =$  \_\_\_\_\_

(iv)  $\overrightarrow{AE}$ ,

Answer: (a)(iv)  $\overrightarrow{AE} =$  \_\_\_\_\_

(b) Write down the relationship between  $OD$  and  $AE$ . Explain your answer.

(c) Find the ratio of

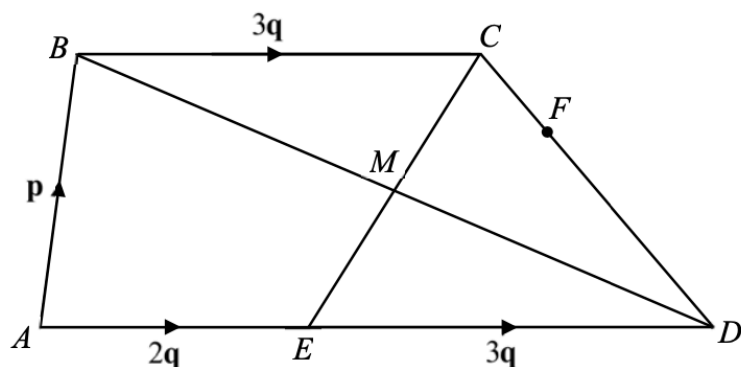
(i) area of triangle  $CAE$  : area of triangle  $AOD$ ,

Answer: (c)(i) \_\_\_\_\_

(ii) area of triangle  $CAE$  : area of triangle  $AOB$ .

Answer: (c)(ii) \_\_\_\_\_

6.



$ABCD$  is a quadrilateral and  $E$  is a point on  $AD$ .

$M$  is the point of intersection of  $BD$  and  $CE$ .

$\overrightarrow{AB} = \mathbf{p}$ ,  $\overrightarrow{AE} = 2\mathbf{q}$  and  $\overrightarrow{BC} = \overrightarrow{ED} = 3\mathbf{q}$ .

(a) Show that triangles  $BMC$  and  $DME$  are congruent. Give a reason for each statement you make.

(b) Express in terms of  $\mathbf{p}$  and/or  $\mathbf{q}$ , as simply as possible,

(i)  $\overrightarrow{AC}$ ,

Answer: (a)(i)  $\overrightarrow{AC} =$  \_\_\_\_\_

(ii)  $\overrightarrow{BD}$ ,

Answer: (a)(ii)  $\overrightarrow{BD} =$  \_\_\_\_\_



(iii)  $\overrightarrow{AM}$ ,

Answer: (a)(iii)  $\overrightarrow{AM} =$  \_\_\_\_\_

(c)  $F$  is a point on  $CD$  such that  $CF : FD = 2 : 5$ .

(i) Explain why  $A$ ,  $M$  and  $F$  lie on a straight line.

(ii) Find the ratio of area of triangle  $AME$  : area of triangle  $FMD$ .

Answer: (c)(ii) \_\_\_\_\_

7. Given that  $ABC$  is a triangle where  $\overrightarrow{AB} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$  and  $\overrightarrow{AC} = \begin{pmatrix} -1 \\ 8 \end{pmatrix}$ .

(a) Find  $\overrightarrow{BC}$ .

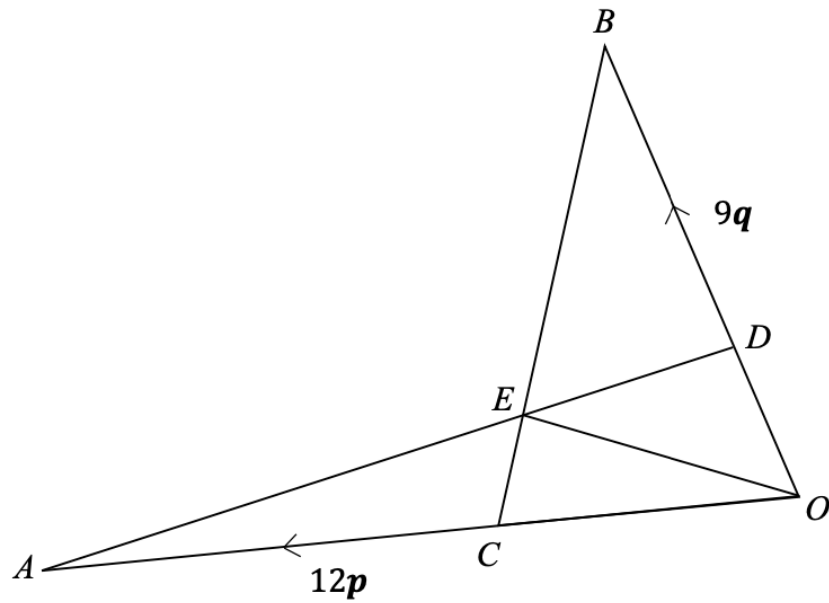
Answer: (a)  $\overrightarrow{BC} =$  \_\_\_\_\_

(b) Hence, or otherwise, show that  $\angle BAC = 108.4^\circ$ .

(c) Hence, calculate the area of  $\triangle ABC$ .

Answer: (c) area of  $\triangle ABC =$  \_\_\_\_\_  $\text{units}^2$

8. In the diagram,  $\overrightarrow{OA} = 12\mathbf{p}$  and  $\overrightarrow{OB} = 9\mathbf{q}$   
 It is given that  $3DB = 2OB$  and  $OA = 3OC$ .



- (a) Express in terms of  $\mathbf{p}$  and/or  $\mathbf{q}$ , as simply as possible,

(i)  $\overrightarrow{BC}$ ,

Answer: (a)(i)  $\overrightarrow{BC} =$  \_\_\_\_\_

(ii)  $\overrightarrow{DA}$ ,

Answer: (a)(ii)  $\overrightarrow{DA} =$  \_\_\_\_\_

(b) Given that  $\frac{\text{area of } \triangle ODE}{\text{area of } \triangle ODA} = \frac{1}{4}$ , find  $\overrightarrow{OE}$  in terms of  $\mathbf{p}$  and  $\mathbf{q}$ .

Answer: (b)  $\overrightarrow{OE} =$  \_\_\_\_\_

(c) Find the value of  $\frac{\text{area of } \triangle BDE}{\text{area of quadrilateral } EDOC}$ .

Answer: (c)  $\frac{\text{area of } \triangle BDE}{\text{area of quadrilateral } EDOC} =$  \_\_\_\_\_