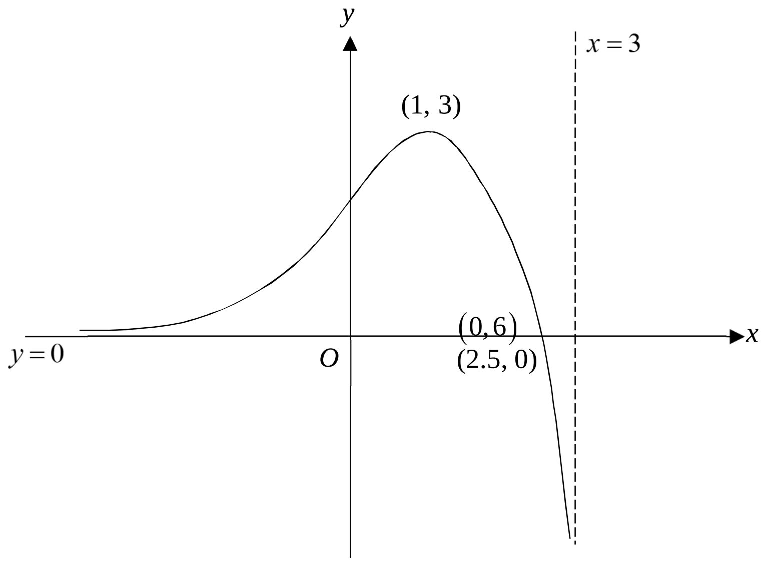
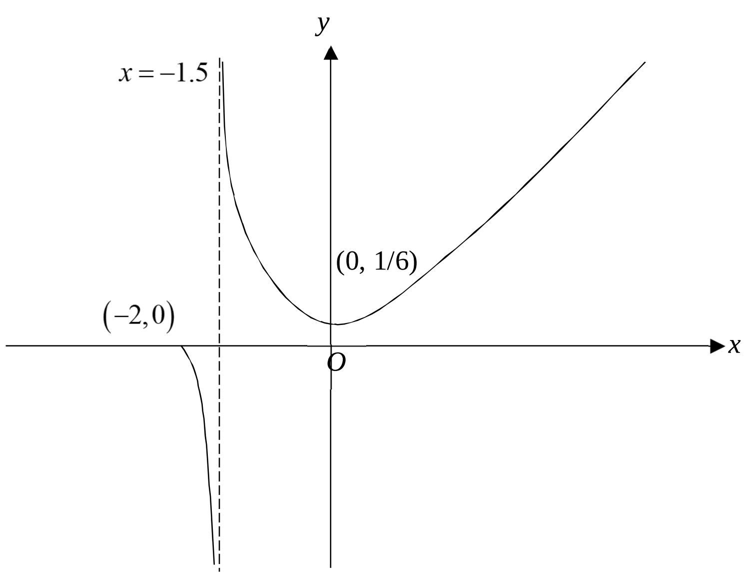
Section A: Pure Mathematics [40 marks]

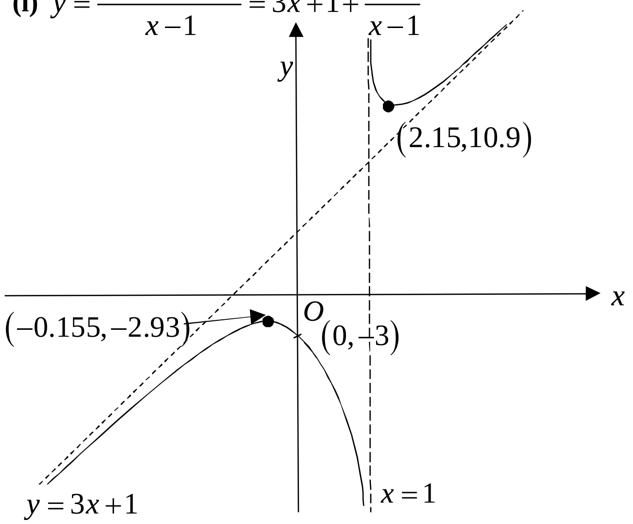
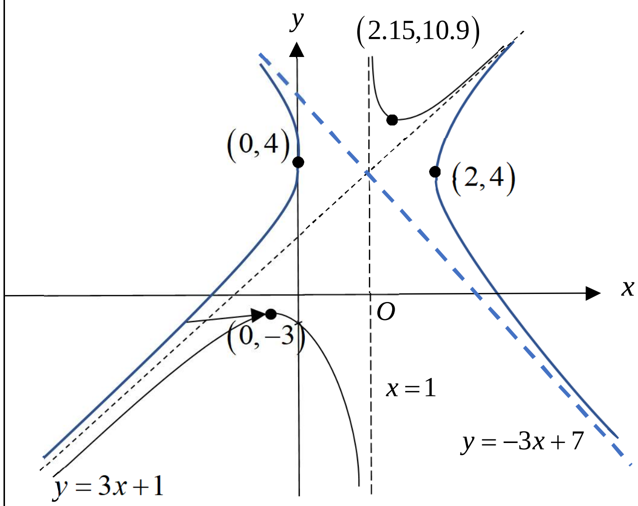
1

Suggested solution	
	
Suggested solution	
 a)	

2 The curve C_1 has equation $y=\dfrac{3x^2-2x+3}{x-1}$. The curve C_2 has equation $(x-1)^2-\dfrac{(y-4)^2}{b^2}=1$, where $b>0$.

(i) Sketch C_1, stating the equations of any asymptotes and the coordinates of any turning points and points where the curve crosses the axes. [4]

(ii) Hence, find the range of values of b such that there is no point of intersection between C_1 and C_2 . Using the maximum value of b found, sketch C_2 on the same diagram as **part (i)**. [3]

Suggested solution	
(i) $y=\dfrac{3x^2-2x+3}{x-1}=3x+1+\dfrac{4}{x-1}$ 	
(ii) C_2 is a horizontal hyperbola with center $(1,4)$. Asymptotes: $y=\pm b(x-1)+4$ For no point of intersection between C_1 and C_2, the asymptote of hyperbola must be as steep or less steep than the oblique asymptote of C_1. Note that the gradient of oblique asymptote of C_1 is 3. $\therefore 0<b\le 3$ For $b=3$, Asymptotes: $y=\pm 3(x-1)+4=3x+1,-3x+7$ 	

3

A curve C has parametric equations

$$x=1+\cos\theta,\quad y=\sin\theta+\theta+1,\qquad \text{for } 0<\theta<\frac{\pi}{2}.$$

(i) The point P on C has parameter p. Show that the normal to C at P crosses the y-axis at a point Q with coordinates $(0,\ p+1)$. [5]

(ii) The normal to C at P crosses the x-axis at point R. Given that point S is the midpoint of QR, find a cartesian equation of the curve traced by S as p varies. [3]

	Suggested Solution	
(i)	$\frac{\mathrm{d}x}{\mathrm{d}\theta}=-\sin\theta$ $\frac{\mathrm{d}y}{\mathrm{d}\theta}=\cos\theta+1$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{\cos\theta+1}{\sin\theta}$ Gradient of normal = $\frac{\sin\theta}{\cos\theta+1}$ Equation of normal at point P $y-\sin p-p-1=\frac{\sin p}{\cos p+1}\left[x-\left(1+\cos p\right)\right]$ $y=\frac{\sin p}{\cos p+1}x-\sin p+\sin p+p+1$ $y=\frac{\sin p}{\cos p+1}x+p+1$ When $x=0, y=p+1$ (Shown)	
(ii)	Since equation of normal is $y=\frac{\sin p}{\cos p+1}x+p+1$ when $y=0$ $x=-\frac{(p+1)(1+\cos p)}{\sin p}$ Midpoint of QR $x=-\frac{(p+1)(1+\cos p)}{2\sin p}$ $y=\frac{p+1}{2}$ Cartesian equation of the curve traced by S $x=-\frac{y\left[1+\cos(2y-1)\right]}{\sin(2y-1)}$	

4 Vectors **a** and **b** are such that the modulus of **a** is 2 and **b** is a unit vector perpendicular to **a**.

(i) State the value of $\mathbf{a}\cdot(2\mathbf{a}\times3\mathbf{b})$ and explain your answer briefly. [1]

(ii) Find the numerical area of the parallelogram with adjacent sides defined by $2\mathbf{a}+\mathbf{b}$ and $5\mathbf{a}-7\mathbf{b}$. [3]

A vector **c** is such that $\mathbf{b}\times\mathbf{c}=2\mathbf{a}\times3\mathbf{b}$.

(iii) Show that $\mathbf{c} = \lambda\mathbf{b} - 6\mathbf{a}$, where λ is a constant. [2]

(iv) State the geometrical meaning of $|\mathbf{b} \cdot \mathbf{c}|$ and find the possible values of λ if $|\mathbf{b} \cdot \mathbf{c}| = 5$. [2]

	Suggested solution	
(i)	$\mathbf{a} \cdot (2\mathbf{a} \times 3\mathbf{b}) = 0$ since $2\mathbf{a} \times 3\mathbf{b}$ is a vector that is perpendicular to $2\mathbf{a}$, and hence it is also perpendicular to **a**.	
(ii)	Area of the parallelogram $= \lvert(2\mathbf{a}+\mathbf{b}) \times (5\mathbf{a}-7\mathbf{b})\rvert$ $= \lvert 10(\mathbf{a}\times\mathbf{a}) - 7(\mathbf{b}\times\mathbf{b}) + 5(\mathbf{b}\times\mathbf{a}) - 14(\mathbf{a}\times\mathbf{b})\rvert$ $= \lvert 10(\mathbf{0}) - 7(\mathbf{0}) + 19(\mathbf{b}\times\mathbf{a})\rvert$ $= 19\lvert\mathbf{b}\times\mathbf{a}\rvert$ $= 19\lvert\mathbf{b}\rvert\lvert\mathbf{a}\rvert\sin 90°$ $= 19(1)(2)(1)$ $= 38$	
(iii)	$\mathbf{b}\times\mathbf{c} = 2\mathbf{a}\times 3\mathbf{b}$ $\Rightarrow (\mathbf{b}\times\mathbf{c}) - (2\mathbf{a}\times 3\mathbf{b}) = \mathbf{0}$ $\Rightarrow (\mathbf{b}\times\mathbf{c}) + 6(\mathbf{b}\times\mathbf{a}) = \mathbf{0}$ $\Rightarrow \mathbf{b}\times(\mathbf{c}+6\mathbf{a}) = \mathbf{0}$ $\therefore$ $\mathbf{c}+6\mathbf{a}$ is parallel to **b** and so $\mathbf{c}+6\mathbf{a} = \lambda\mathbf{b}$, where λ is a constant. $\Rightarrow \mathbf{c} = \lambda\mathbf{b} - 6\mathbf{a}$	
(iv)	$\lvert\mathbf{b}\cdot\mathbf{c}\rvert = \lvert\mathbf{c}\cdot\mathbf{b}\rvert$ is the length of projection of **c** onto **b**. $\lvert\mathbf{b}\cdot\mathbf{c}\rvert = \lvert\mathbf{b}\cdot(\lambda\mathbf{b}-6\mathbf{a})\rvert$ $= \lvert\lambda(\mathbf{b}\cdot\mathbf{b}) - 6(\mathbf{b}\cdot\mathbf{a})\rvert$ $= \lvert\lambda\lvert\mathbf{b}\rvert^2 - 6(0)\rvert$ $= \lvert\lambda\rvert$ Since $\lvert\mathbf{b}\cdot\mathbf{c}\rvert = 5$, $\lambda = \pm 5$.	

5 On 1 Jan 2019, Emma takes up a study loan of \$30000 from a bank that offers two repayment plans. Plan A charges a monthly instalment of \$600 at the beginning of every month and increases by \$50 for every subsequent month. There is no interest charged; instead, the bank charges a one-time administrative fee of \$2500 which is to be added to the study loan.

(i) **(a)** Show that, after the n^{th} payment, she would have paid a total amount of $\$\left(25n^2 + 575n\right)$. [2]

(b) How many payments will it take for Emma to fully repay her loan? [2]

Plan B charges a fixed monthly instalment of \$*x* which is to be paid at the beginning of each month, starting from 1 Jan 2019. There is no interest charged for the first month only. Thereafter, interest is charged at the end of each month on the outstanding amount at 1% per month, (i.e. interest will be first charged on 28 February 2019.)

(ii) **(a)** Show that the amount she owes after the fourth payment is $(1.01)^2(30000)-(1.01)^2(2x)-1.01x-x$. [2]

(b) Use the formula for the sum of a geometric progression to find an expression, in terms of n and x, the amount she owes after the n^{th} payment, for $n \geq 2$. [2]

(c) If Emma intends to repay the loan fully after 32 payments, find the least value of x. [2]

(iii) Using the value of x in **part (ii)(c)**, determine which plan will be cheaper for Emma. [1]

<table>
<tr><td></td><td>Suggested solution</td><td></td></tr>
<tr><td>(i)(a)</td><td>Total amount paid = 600 + 650 + 700 + ….

$a=600, d=50$
$S_n=\frac{n}{2}\left[2(600)+(n-1)50\right]$
$=n(600+25n-25)$
$=25n^2+575n$ (Shown)

Total paid after nth payment $=\$\left(25n^2+575n\right)$</td><td></td></tr>
<tr><td>(i)(b)</td><td>To complete fully repay her study loan,
Amount paid $\geq 30000+2500$
$25n^2+575n-32500\geq 0$
From GC, $n\leq -49.345$ or $n\geq 26.345$

It will take Emma 27 payments to fully repay her study loan.</td><td></td></tr>
<tr><td>(ii)(a)</td><td>
<table>
<tr><td>Mth</td><td>Amount owed at start of the month</td><td>Amount owed at end of the month</td></tr>
<tr><td>Jan (n=1)</td><td>$30000-x$</td><td>$30000-x$</td></tr>
<tr><td>Feb (n=2)</td><td>$30000-2x$</td><td>$30000(1.01)-2x(1.01)$</td></tr>
<tr><td>Mar (n=3)</td><td>$30000(1.01)-2x(1.01)-x$</td><td>$30000(1.01^2)-2x(1.01^2)$
$-x(1.01)$</td></tr>
<tr><td>Apr</td><td>$30000(1.01^2)-2x(1.01^2)$
$-x(1.01)$
$-x$</td><td>$30000(1.01^3)-2x(1.01^3)$
$-x(1.01^2)$
$-x(1.01)$</td></tr>
<tr><td>…</td><td>…</td><td>…</td></tr>
<tr><td>n</td><td>$30000(1.01^{n-2})-2x(1.01^{n-2})$
$-x(1.01^{n-3})$
$-x(1.01^{n-4})$
...
$-x$</td><td></td></tr>
</table>
Amount owed after 4 payments
$=30000(1.01^2)-2x(1.01^2)-x(1.01)-x$</td><td></td></tr>
<tr><td>(ii)(b)</td><td>After n^{th} payments, amount owed</td><td></td></tr>
</table>

	$=30000(1.01)^{n-2}-2x(1.01)^{n-2}-(1.01)^{n-3}x-(1.01)^{n-4}x-...-x$ $=30000(1.01)^{n-2}-2x(1.01)^{n-2}-x\left[(1.01)^{n-3}+(1.01)^{n-4}+(1.01)^{n-5}+...+(1.01)^{0}\right]$ $=30000(1.01)^{n-2}-2x(1.01)^{n-2}-x\left[\frac{1.01^{n-2}-1}{1.01-1}\right]$ $=30000(1.01)^{n-2}-2x(1.01)^{n-2}-100x(1.01^{n-2}-1)$ **Alternative** After n^{th} payments, amount owed $=30000(1.01)^{n-2}-2x(1.01)^{n-2}-(1.01)^{n-3}x-(1.01)^{n-4}x-...-(1.01)^{1}x-x$ $=30000(1.01)^{n-2}-2x(1.01)^{n-2}-\left[(1.01)^{n-3}x+(1.01)^{n-4}x+...+(1.01)^{1}x\right]-x$ $=30000(1.01)^{n-2}-2x(1.01)^{n-2}-x\left[\frac{1.01(1.01^{n-3}-1)}{1.01-1}\right]-x$ $=30000(1.01)^{n-2}-2x(1.01)^{n-2}-101x(1.01^{n-3}-1)-x$	
(ii)(c)	For Emma to clear the debt, the amount that she owed after n^{th} payments ≤ 0 When $n=32$, $30000(1.01)^{n-2}-2x(1.01)^{n-2}-100x(1.01^{n-2}-1)\leq 0$ $30000(1.01)^{32-2}-2x(1.01)^{32-2}-100x(1.01^{32-2}-1)\leq 0$ $30000(1.01)^{30}-2x(1.01)^{30}-100x(1.01^{30}-1)\leq 0$ $\Rightarrow x\geq \frac{30000(1.01)^{30}}{2(1.01)^{30}+100(1.01^{30}-1)}$ $x\geq 1078.837557$ Least value of x is \$1078.84	
(iii)	Under plan A, total amount repaid $=30000+2500=32500$ Under plan B, total amount repaid $=1078.84\times 32=34522.88$ Therefore, it is cheaper for Emma to take up Plan A	

Section B: Probability and Statistics [60 marks]

6 Nine letter tiles that spell the word PRINCIPAL are used.

(i) Find the number of ways to arrange the nine tiles in a row such that the vowels are together. [2]

(ii) The tiles are shuffled and placed in a row randomly. Find the probability that the tiles spell PRINCIPAL. [2]

(iii) Eight of the nine tiles are taken and arranged in a circle. Find the number of ways that this can be done. [3]

Suggested solution	
(i) Treat the vowels I,I,A as one unit, and the consonants as 6 units. No of ways to arrange these 7 units is $\frac{7!}{2!}$. No of ways to arrange the vowels is $\frac{3!}{2!}$. Total no of ways= $\frac{7!}{2!}\times\frac{3!}{2!}=7560$ (ii) Total no of ways to arrange without restriction $=\frac{9!}{2!2!}=90720$ Note that all 90720 are equally likely and distinct arrangements. PRINCIPAL is one of these arrangements. Thus Prob required $=\frac{1}{90720}$ (iii) Case 1: 2 sets of 2 repeated letters (i.e. P, P, I, I and 4 other distinct letters) No of ways $=\frac{{}^{5}C_{4}\times(8-1)!}{2!2!}=6300$ Case 2: 1 set of repeated letters (i.e. PP or II and P/I, R,N,C,A,L) No. of ways $=\frac{{}^{2}C_{1}\times(8-1)!}{2!}=5040$ Total no. of ways= $6300+5040=11340$	

7 For events A and B, it is given that $\mathrm{P}(A\cap B')=0.32$ and $\mathrm{P}(B|A)=\frac{7}{23}$.

(i) Show that $\mathrm{P}(A\cap B)=0.14$. [2]

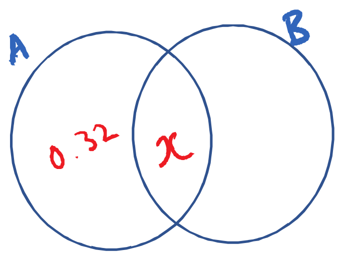
(ii) Given further that $\mathrm{P}(A|B)=\frac{7}{19}$, find $\mathrm{P}(A\cup B)$. [2]

It is given that $\mathrm{P}(C)=0.5$, where C is an event such that events A and C are independent and B and C are independent.

(iii) Find the greatest and least possible values of $\mathrm{P}(A'\cap B'\cap C)$. [4]

Suggested solution

(i)



Let $P(A\cap B)=x$.

$$P(B|A)=\frac{7}{23}$$

$$\Rightarrow \frac{P(A\cap B)}{P(A)}=\frac{7}{23}\Rightarrow \frac{x}{0.32+x}=\frac{7}{23}$$

$$\Rightarrow 23x=7x+2.24\Rightarrow x=0.14$$

(ii)

$$P(A|B)=\frac{7}{19}$$

$$\Rightarrow \frac{P(A\cap B)}{P(B)}=\frac{7}{19}$$

$$\Rightarrow \frac{0.14}{P(B)}=\frac{7}{19}$$

$$\Rightarrow P(B)=0.38$$

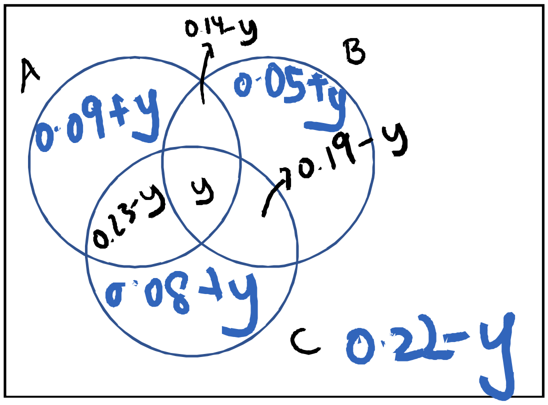
$$P(A\cup B)=P(B)+P(A\cap B')$$
$$=0.38+0.32=0.7$$

(iii)

Since A, C are independent, $P(A\cap C)=0.5(0.46)=0.23$

Since B, C are independent, $P(B\cap C)=0.5(0.38)=0.19$

Let $P(A\cap B\cap C)=y$



The values of *y* can range from 0 to 0.14.

When $y = 0$,

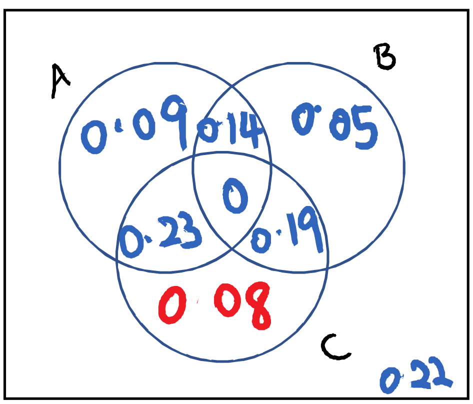
$P(A' \cap B' \cap C) = 0.5 - (0.19 + 0.23) = 0.08$ **[Min value]**

When $y = 0.14$,

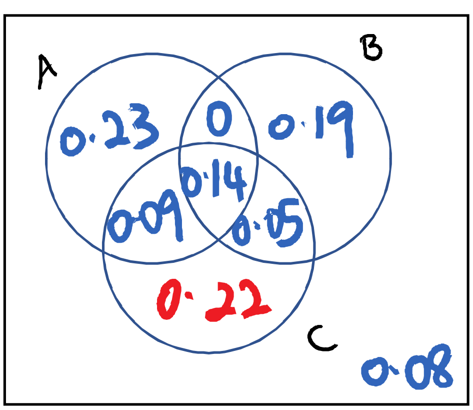
$P(A' \cap B' \cap C) = 0.5 - (0.19 + 0.23 - 0.14) = 0.22$ **[Max value]**

For a complete solution, we would still need to verify if the remaining regions are non-negative for this range of *y*.

For y=0



For *y=0.14*



8 The time spent per day, in minutes, by Eunice on social media Toktik is normally distributed with mean 20.2 and standard deviation 4.5. Due to an unfortunate event, Eunice is confined to home for two months. During this period, the time spent per day by Eunice on Toktik is recorded for 20 randomly chosen days, which averages 22.4 minutes.

It is assumed that the population variance remains unchanged throughout.

(i) Test, at 1% significance level, if there has been a change in the average time spent per day by Eunice on Toktik. [4]

(ii) Without carrying out another test, find the range of α for which a test at α % level of significance suggests an **increase** in the average time spent per day by Eunice on Toktik. [2]

Another set of 15 timings is taken. Using only this set of 15 timings, find the range of values within which the mean of this set of readings must lie such that the claim that there has been a change in the average time spent by Eunice on Toktik is supported at 5% level of significance. [3]

Suggested solution	
(i) Let X be the time spent on Toktik (in minutes). $X \sim \mathrm{N}(20.2,\ 4.5^2)$ $\mathrm{H}_0:\ \mu = 20.2$ $\mathrm{H}_1:\ \mu \neq 20.2$ Level of significance: 1% Under H_0, $\overline{X} \sim \mathrm{N}\left(20.2,\ \dfrac{4.5^2}{20}\right)$ From GC, p-value $= 0.028788 = 0.0288$ (3 s.f.) Since p-value $= 0.0288 > 0.01$, <u>do not reject H_0</u>. There is <u>insufficient evidence</u> at the <u>1% level of significance</u> to conclude that there is <u>a change in the average time</u> spent per day by Eunice on Toktik.	
(ii) From 2-tailed test in (i), p-value = 0.028788. For a 1-tailed test, new p-value $= \dfrac{1}{2}(0.028788) = 0.014394$. For H_0 to be rejected, $p\text{-value} < \dfrac{\alpha}{100}$. $\Rightarrow 0.014394 < \dfrac{\alpha}{100}$ $\Rightarrow \alpha > 1.44$ (3 s.f.)	

(iii) *** Method 1: Under H_0, $\bar{X} \sim \mathrm{N}\left(20.2,\ \frac{4.5^2}{15}\right)$ Test Statistic: $Z = \frac{\bar{X} - 20.2}{\sqrt{\frac{4.5^2}{15}}} \sim \mathrm{N}(0,\ 1)$ To reject H_0 at 5% level of significance, $z < -1.95996$ or $z > 1.95996.$ $\Rightarrow \frac{\bar{x} - 20.2}{\sqrt{\frac{4.5^2}{15}}} < -1.95996$ or $\frac{\bar{x} - 20.2}{\sqrt{\frac{4.5^2}{15}}} > 1.95996$ $\therefore\ \bar{x} < 17.9$ or $\bar{x} > 22.5$ *** Method 2: Under H_0, $\bar{X} \sim \mathrm{N}\left(20.2,\ \frac{4.5^2}{15}\right)$ To reject H_0 at 5% level of significance, p-value < 0.05. Using GC, $\bar{x} < 17.923$ or $\bar{x} > 22.477$. $\therefore\ \bar{x} < 17.9$ or $\bar{x} > 22.5$	

9 A computer chip generates a random number from the set $\{1, 2, 3, 4\}$. The probability distribution of the number generated, X, is given by

$$\mathrm{P}(X = x) = \begin{cases} p & x = 1 \\ \dfrac{1-p}{3} & x = 2, 3, 4 \end{cases}$$

where $0 < p < 1$.

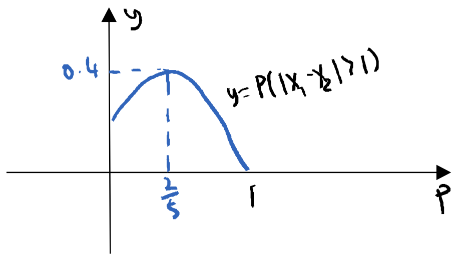
(i) Find E(X) in terms of p and hence show that $\mathrm{Var}(X) = \frac{2}{3}(1-p)(1+6p)$. [3]

(ii) X_1 and X_2 are two independent observations of X. Find $P(|X_1 - X_2| > 1)$ in terms of p. Hence, find the maximum possible value of this probability as p varies. [5]

It is given instead that $p = \frac{1}{2}$.

(iii) Find the probability that the sum of 50 independent observations of X exceeds 110. [3]

Suggested solution	
(i) Let X be the number randomly generated by the computer chip. $\mathrm{E}(X) = p + \frac{2(1-p)}{3} + \frac{3(1-p)}{3} + \frac{4(1-p)}{3} = 3 - 2p$ $\mathrm{Var}(X) = \mathrm{E}(X^2) - (\mathrm{E}(X))^2$ $= (1^2)p + (2^2 + 3^2 + 4^2)\frac{(1-p)}{3} - (3-2p)^2$ $= \frac{2}{3} + \frac{10}{3}p - 4p^2 = \frac{2}{3}(1-p)(1+6p)$	
(ii) Consider (X_1, X_2) as the values for the r.v. X_1 and X_2 $P(\lvert X_1 - X_2 \rvert > 1) = P((1,3),(3,1),(1,4),(4,1),(2,4),(4,2))$ $= p \times \frac{(1-p)}{3} \times 2 + p \times \frac{(1-p)}{3} \times 2 + \left[\frac{(1-p)}{3}\right]^2 \times 2$ $= \frac{4}{3}p(1-p) + \frac{2}{9}(1-p)^2$ $= \frac{2}{9}(1-p)\left[6p + (1-p)\right]$ $= \frac{2}{9}(1-p)(5p+1)$	



Max value of $P\left(\left|X_1 - X_2\right| > 1\right)$ occurs when $p = \dfrac{-\frac{1}{5}+1}{2} = \dfrac{2}{5}$

Max value of $P\left(\left|X_1 - X_2\right| > 1\right) = \dfrac{2}{9}\left(1 - \dfrac{2}{5}\right)(2+1) = 0.4$

(iii)

Required probability = $P\left(X_1 + X_2 + \cdots + X_{50} > 110\right)$

By Central Limit Theorem, since n=50 is large

$$\overline{X} = \frac{X_1 + X_2 + \cdots + X_{50}}{50} \sim N\left(2, \frac{4/3}{50}\right) \text{approximately}$$

Hence,

$$P\left(X_1 + X_2 + \cdots + X_{50} > 110\right) = P\left(\overline{X} > 2.2\right) = 0.11034 \approx 0.110$$

Alternative solution:

By Central Limit Theorem, since n=50 is large

$$\Rightarrow 50\overline{X} = X_1 + X_2 + \cdots + X_{50} \sim N\left(100, \frac{200}{3}\right) \text{approximately}$$

Hence,

$$P\left(X_1 + X_2 + \cdots + X_{50} > 110\right)$$
$$= 0.11034 \approx 0.110$$

10 A manufacturer produces disposable masks. On average, 6% of the masks are faulty. The masks are sold in boxes of 25.

(i) State, in context, two assumptions needed for the number of faulty masks in a box to be well modelled by a binomial distribution. [2]

Assume now that the number of faulty masks in a box has a binomial distribution.

(ii) Find the probability that a box of 25 of these masks contains at least 3 faulty masks. [2]

(iii) Find the probability that a box of 25 of these masks contains at most 5 faulty masks given that it contains at least 3 faulty masks. [2]

(iv) A store bought 100 boxes of these masks. Find the probability that at least 3 faulty masks are found in no more than $\frac{1}{5}$ of the boxes. [2]

The manufacturer also produces bottles of hand sanitisers which are sold in boxes of n bottles each. The number of faulty bottles in a box is denoted by the random variable F. It is known that F follows a binomial distribution, with each bottle having a 10% probability of being faulty.

(v) Given that the most likely value of F is 4, find all possible values of n, without using a calculator. [4]

	Suggested solution	
(i)	Each mask has the same probability of being faulty of 0.06. The event that a mask is faulty is independent of any other mask.	
(ii)	Let X be the number of faulty masks in a box of 25. $X \sim \mathrm{B}(25, 0.06)$ Probability $= P(X \geq 3)$ $= 1 - P(X \leq 2) = 0.18711 \quad (5 \text{ s.f.}) = 0.187 \quad (3 \text{ s.f.})$	
(iii)	Probability $= \mathrm{P}(X \leq 5 \mid X \geq 3)$ $= \dfrac{\mathrm{P}(X \leq 5 \cap X \geq 3)}{\mathrm{P}(X \geq 3)} = \dfrac{\mathrm{P}(3 \leq X \leq 5)}{\mathrm{P}(X \geq 3)}$ $= \dfrac{\mathrm{P}(X \leq 5) - \mathrm{P}(X \leq 2)}{\mathrm{P}(X \geq 3)} = \dfrac{0.99694 - 0.81289}{0.18711}$ $= 0.98359 \quad (5 \text{ s.f.}) = 0.984 \quad (3 \text{ s.f.})$	
(iv)	Let Y be the number of boxes out of 100 which have at least 3 faulty masks. $Y \sim \mathrm{B}(100, 0.18711)$ Probability $= \mathrm{P}(Y \leq 20)$ $= 0.68423 \quad (5 \text{ s.f.}) = 0.684 \quad (3 \text{ s.f.})$	
(v)	$F \sim \mathrm{B}(n, 0.1)$ Given that mode is 4 (i.e. $P(F=4)$ has the highest probability) $\mathrm{P}(F=3) < \mathrm{P}(F=4)$ and $\mathrm{P}(F=4) > \mathrm{P}(F=5)$ $\binom{n}{3}(0.1)^3(0.9)^{n-3} < \binom{n}{4}(0.1)^4(0.9)^{n-4}$; $\binom{n}{4}(0.1)^4(0.9)^{n-4} > \binom{n}{5}(0.1)^5(0.9)^{n-5}$ $\dfrac{n!}{3!(n-3)!}(0.1)^3(0.9)^{n-3} < \dfrac{n!}{4!(n-4)!}(0.1)^4(0.9)^{n-4}$; $\dfrac{n!}{4!(n-4)!}(0.1)^4(0.9)^{n-4} > \dfrac{n!}{5!(n-5)!}(0.1)^5(0.9)^{n-5}$ $\dfrac{4!}{3!}\dfrac{(0.9)^{n-3}}{(0.9)^{n-4}} < \dfrac{(n-3)!}{(n-4)!}\dfrac{(0.1)^4}{(0.1)^3}$; $\dfrac{5!}{4!}\dfrac{(0.9)^{n-4}}{(0.9)^{n-5}} < \dfrac{(n-4)!}{(n-5)!}\dfrac{(0.1)^5}{(0.1)^4}$ $4(0.9) < (n-3)(0.1)$; $5(0.9) > (n-4)(0.1)$ $n > 39$; $n < 49$ $\therefore 39 < n < 49$ Since n is an integer, $\Rightarrow n = 40, 41, 42, 43, 44, 45, 46, 47$ or 48	

11 The masses, in pounds, of apples and pears have independent normal distributions with means and standard deviations as shown in the following table.

	Mean	Standard deviation
Apple	0.33	σ
Pear	0.45	0.08

(i) The probability of a randomly chosen apple weighing less than t pounds and that weighing between t and 0.35 pounds is 0.34 each. Find the values of σ and t. [4]

(ii) Three pears are chosen at random. Find the probability that one of them weighs less than the population mean mass and each of the other two pears has a mass within one standard deviation of the population mean mass. [2]

The apples cost \$2.20 per pound and the pears cost \$3.10 per pound.

(iii) Find the probability that the total cost of 3 randomly chosen apples and 2 randomly chosen pears is more than \$5. [3]

(iv) A fruit seller gives p% discount on the cost of a fruit pack containing 3 apples and 2 pears. Find the smallest value of p if the fruit seller wants to be at least 99% sure that the cost of the pack is below \$4.50 after discount. [4]

	Suggested solution	
(i)	Let X be the mass of an apple (in pounds). $X \sim \mathrm{N}\left(0.33, \sigma^2\right)$ Let Y be the mass of a pear (in pounds). $Y \sim \mathrm{N}\left(0.45, 0.08^2\right)$ $\mathrm{P}(X > 0.35) = 1 - 0.34 - 0.34 = 0.32$ $\mathrm{P}\left(Z > \frac{0.35-0.33}{\sigma}\right) = 0.32$ $\frac{0.02}{\sigma} = 0.46770$ $\sigma = 0.042763 = 0.0428$ (3sf) $\mathrm{P}(X < t) = 0.34$ By GC, $t = 0.31236 = 0.312$ (3sf)	
(ii)	Req'd prob $= {}^3C_1(0.5)\left[\mathrm{P}(0.37 < Y < 0.53)\right]^2$ $= 0.699$ (3sf) OR Req'd prob $= {}^3C_1(0.5)\left[0.68269\right]^2$ $= 0.699$ (3sf)	

(iii)	$T = 2.2(X_1 + X_2 + X_3) + 3.1(Y_1 + Y_2)$ $T \sim \mathrm{N}(2.2\times3\times0.33 + 3.1\times2\times0.45, 2.2^2\times3\times0.042763^2 + 3.1^2\times2\times0.08^2)$ $T \sim \mathrm{N}(4.968, 0.14956)$ $\mathrm{P}(T > 5) = 0.46703 = 0.467$ (3sf)	
(iv)	$W = \left(1-\frac{p}{100}\right)T \sim \mathrm{N}\left(\left(1-\frac{p}{100}\right)(4.968), \left(1-\frac{p}{100}\right)^2(0.14956)\right)$ $\mathrm{P}(W < 4.5) \geq 0.99$ $\mathrm{P}\left(Z < \frac{4.5-\left(1-\frac{p}{100}\right)(4.968)}{\sqrt{\left(1-\frac{p}{100}\right)^2(0.14956)}}\right) \geq 0.99$ $\frac{4.5-\left(1-\frac{p}{100}\right)(4.968)}{\sqrt{\left(1-\frac{p}{100}\right)^2(0.14956)}} \geq 2.3263$ By GC, $p \geq 23.308$ $\therefore p = 23.3$ (3sf) Alternative solution: $W = \left(1-\frac{p}{100}\right)T$ $\mathrm{P}(W < 4.5) \geq 0.99 \Rightarrow \mathrm{P}\left(\left(1-\frac{p}{100}\right)T < 4.5\right) \geq 0.99$ $\Rightarrow \mathrm{P}\left(T < \frac{450}{100-p}\right) \geq 0.99$ $\mathrm{P}\left(Z < \frac{\frac{450}{100-p}-(4.968)}{\sqrt{(0.14956)}}\right) \geq 0.99$ $\frac{\frac{450}{100-p}-(4.968)}{\sqrt{(0.14956)}} \geq 2.3263$ By GC, $p \geq 23.308$ $\therefore p = 23.3$ (3sf)	