Centre Number	Index Number	Name	Class
S3016			

RAFFLES INSTITUTION
2024 Preliminary Examination

PHYSICS (Higher 3) **9814/01**
Paper 1 **24 September 2024**
3 hours

Candidates answer on the Question Paper. No additional materials are required.

READ THESE INSTRUCTIONS FIRST
Write your index number, name and class in the spaces provided at the top of this page.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams, graphs or rough working.
Do not use staples, paper clips, glue or correction fluid.
The use of approved scientific calculator is expected, where appropriate.

Section A
Answer **all** questions.
You are advised to spend about 1 hour 50 minutes on Section A.

Section B
Answer **two** questions only.
You are advised to spend about 35 minutes on each question in Section B.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

For Examiner's Use		
1	(a)	/ 9
	(b)	/ 6
2		/ 6
3		/ 6
4		/ 6
5		/ 6
6		/ 8
7		/ 6
8		/ 7
9		/ 20
10	(a)	/ 10
	(b)	/ 10
11		/ 20
Total		/100

This document consists of **33** printed pages.

Data

speed of light in free space	c	=	3.00×10^{8} m s^{-1}
permeability of free space	μ_0	=	$4\pi \times 10^{-7}$ H m^{-1}
permittivity of free space	ε_0	=	8.85×10^{-12} F m^{-1}
		=	$(1/(36\pi)) \times 10^{-9}$ F m^{-1}
elementary charge	e	=	1.60×10^{-19} C
the Planck constant	h	=	6.63×10^{-34} J s
unified atomic mass constant	u	=	1.66×10^{-27} kg
rest mass of electron	m_e	=	9.11×10^{-31} kg
rest mass of proton	m_p	=	1.67×10^{-27} kg
molar gas constant	R	=	8.31 J K^{-1} mol^{-1}
the Avogadro constant	N_A	=	6.02×10^{23} mol^{-1}
the Boltzmann constant	k	=	1.38×10^{-23} J K^{-1}
gravitational constant	G	=	6.67×10^{-11} N m^{2} kg^{-2}
acceleration of free fall	g	=	9.81 m s^{-2}

Formulae

uniformly accelerated motion	s	=	$ut + \frac{1}{2}at^2$
	v^2	=	$u^2 + 2as$
moment of inertia of rod through one end	I	=	$\frac{1}{3}ML^2$
moment of inertia of hollow cylinder through axis	I	=	$\frac{1}{2}M\left(r_1^2 + r_2^2\right)$
moment of inertia of solid sphere through centre	I	=	$\frac{2}{5}MR^2$
moment of inertia of hollow sphere through centre	I	=	$\frac{2}{3}MR^2$
work done on/by a gas	W	=	$p\Delta V$
hydrostatic pressure	p	=	ρgh
gravitational potential	ϕ	=	$-Gm/r$
Kepler's third law of planetary motion	T^2	=	$\frac{4\pi^2 a^3}{GM}$
temperature	T/K	=	$T/°\text{C} + 273.15$
pressure of an ideal gas	p	=	$\frac{1}{3}\frac{Nm}{V}\langle c^2\rangle$
mean translational kinetic energy of an ideal gas molecule	E	=	$\frac{3}{2}kT$
displacement of particle in s.h.m.	x	=	$x_0 \sin \omega t$
velocity of particle in s.h.m.	v	=	$v_0 \cos \omega t = \pm\omega\sqrt{x_0^2 - x^2}$

electric current	I	=	$Anvq$
resistors in series	R	=	$R_1 + R_2 + \ldots$
resistors in parallel	$1/R$	=	$1/R_1 + 1/R_2 + \ldots$
capacitors in series	$1/C$	=	$1/C_1 + 1/C_2 + \ldots$
capacitors in parallel	C	=	$C_1 + C_2 + \ldots$
energy in a capacitor	U	=	$\frac{1}{2}CV^2$
electric potential	V	=	$\frac{Q}{4\pi\varepsilon_0 r}$
electric field strength due to a long straight wire	E	=	$\frac{\lambda}{2\pi\varepsilon_0 r}$
electric field strength due to a large sheet	E	=	$\frac{\sigma}{2\varepsilon_0}$
alternating current/voltage	x	=	$x_0 \sin \omega t$
magnetic flux density due to a long straight wire	B	=	$\frac{\mu_0 I}{2\pi d}$
magnetic flux density due to a flat circular coil	B	=	$\frac{\mu_0 N I}{2r}$
magnetic flux density due to a long solenoid	B	=	$\mu_0 n I$
energy in an inductor	U	=	$\frac{1}{2}L I^2$
RL series circuits	τ	=	$\frac{L}{R}$
RLC series circuits (underdamped)	ω	=	$\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$
radioactive decay	x	=	$x_0 \exp(-\lambda t)$
decay constant	λ	=	$\frac{\ln 2}{t_{\frac{1}{2}}}$

Section A

Answer **all** questions in this section.
You are advised to spend about 1 hour 50 minutes on this section.

1 (a) Fig. 1.1 shows a ball A of mass m_A, moving with velocity u_A, makes a head-on and perfectly elastic collision with a ball B of mass m_B, which is initially at rest.

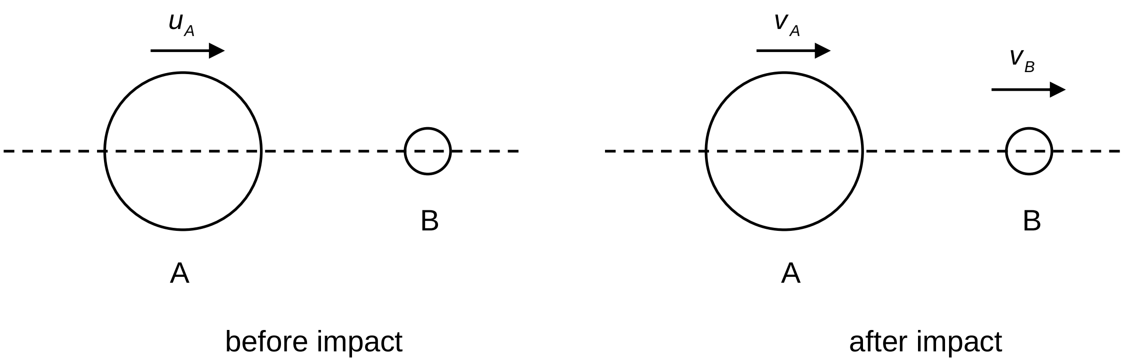

Fig. 1.1

After the impact, balls A and B move off with velocities v_A and v_B respectively along the line of centres of the balls.

(i) Show that, no matter how small the mass of ball B, its speed after the impact cannot exceed $2u_A$

[3]

(ii) Write down an expression, in terms of m_A and m_B, for the fraction f of the initial kinetic energy of ball A which is transferred to ball B.

[1]

(iii) In another experiment shown in Fig 1.2, ball A is being projected again with velocity u_A, towards ball B which is stationary and in contact with another ball C of mass m_C.

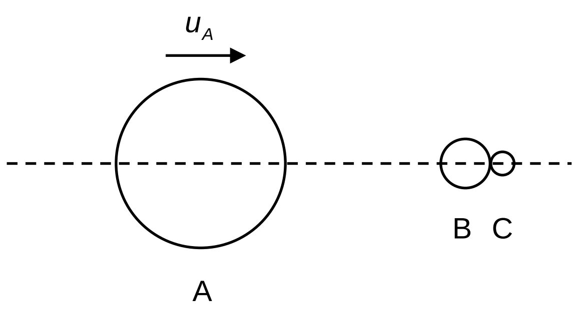

Fig. 1.2

Ball A strikes ball B, which then knocks forward ball C. All collisions are perfectly elastic.

1. Using your answer to (a)(ii), obtain an expression in terms of m_A, m_B and m_C, for the fraction F of the initial kinetic energy of ball A which is transferred to ball C.

[1]

2. Hence, in terms of m_A and m_C, find the value of m_B which will provide the largest fraction *F*.

[4]

Total [9]

(b) Fig 1.3 shows two rigid spheres, P and Q, approaching each other with speeds u_1 and u_2 respectively in the laboratory frame. Both spheres have the same mass *m*.

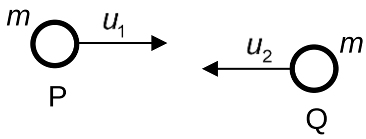

Fig. 1.3

(i) Show that the total momentum of the two spheres is zero in the centre-of-mass frame before the collision.

[2]

(ii) After colliding with each other elastically, they move off to the right with speeds v_1 and v_2 at the same angle θ to the horizontal in the laboratory frame, as shown in Fig. 1.4.

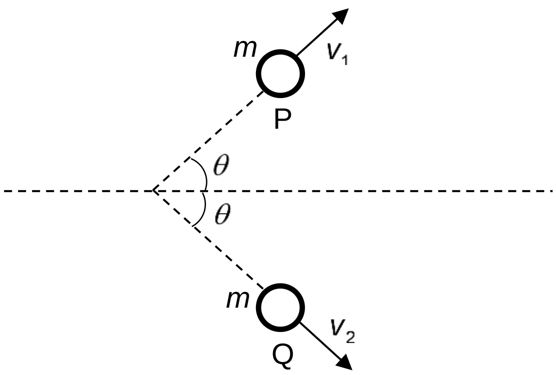

Fig. 1.4

1. Show that $v_1 = v_2$.

[1]

2. Hence, by considering the velocities of the spheres in the centre-of-mass frame, show that for $\theta = 60°$, the ratio of the initial speeds of the spheres can be expressed as

$$\frac{u_1}{u_2} = \frac{\alpha + 1}{\alpha - 1}.$$

Determine the value of α.

α = ... [3]

Total [6]

2 Fig 2.1 shows a uniform stick of mass m and length L resting against a rough circular hoop of radius R. The stick makes an angle θ with the horizontal and is tangent to the hoop at its upper end at A. Both the hoop and lower end of the stick are resting on the rough ground at points B and C respectively. Point O is the centre of the hoop.

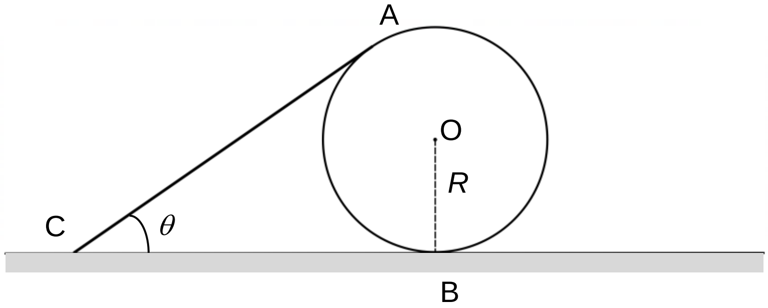

Fig. 2.1

(a) Draw and label the normal contact forces N_A and N_B acting on the hoop by the stick and the ground respectively.

Draw and label also, the frictional forces f_A and f_B acting on the hoop by the stick and the ground respectively.

[1]

(b) Show that the magnitudes of the frictional forces f_A and f_B are equal.

[1]

(c) Show that the normal contact force N_A is given by the expression

$$N_A = \frac{1}{2} mg \cos\theta$$

where g is the acceleration due to gravity.

Explain your working clearly.

[2]

(d) Hence or otherwise, derive an expression for the frictional force f_B in terms of m, L, R, θ and g.

$$\left[\text{Hint: } \tan\frac{\theta}{2} = \frac{\sin\theta}{(1+\cos\theta)}\right]$$

[2]

Total [6]

3 A string is attached to the bottom corners of a rear-view mirror in a car and a smooth ring is threaded through the string. When the car makes a left turn along a horizontal circle of radius 250 m at a constant speed *v*, the position of the ring is as shown in Fig. 3.1.

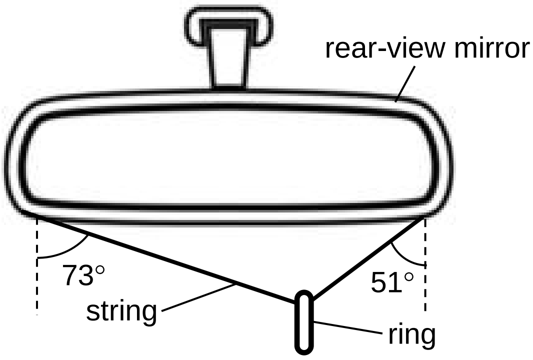

Fig. 3.1

(a) Determine the speed *v* of the car.

v = ... m s^{-1} [3]

(b) Calculate the centripetal acceleration of the car.

centripetal acceleration = ... m s^{-2} [1]

(c) Explain why there is a resultant force acting on the car when it is turning and this force does not cause its kinetic energy to change.

....

....

....

.... [2]

Total [6]

4 A thin horizontal platform of mass m is suspended by three vertical light springs between two fixed supports. A ball, of negligible mass, is placed on the platform at the centre as shown in Fig. 4.1.

The two springs above the platform are identical. Each spring has unstretched length L and force constant k_1.

The spring below the platform also has unstretched length L but has force constant k_2, where $k_2 < 2k_1$.

All three springs have the same extension when the set-up in Fig. 4.1 is in equilibrium.

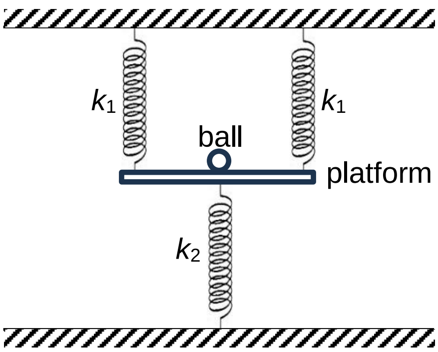

Fig. 4.1

(a) The platform is displaced vertically downwards from the equilibrium position by a distance y_0 and released. All the springs remain in tension and the ball is always in contact with the platform as it oscillates vertically.

The displacement of the oscillating platform from the equilibrium position is y.

Show, with explanations, that the platform and the ball oscillate in simple harmonic motion.

[2]

(b) When the platform is at the lowest point of its oscillation, the spring below the platform breaks.

(i) Show that the new amplitude of oscillation y_0' of the platform in the two-springs system is

$$y'_0 = mg\left(\frac{1}{2k_1 - k_2} - \frac{1}{2k_1} + \frac{y_0}{mg}\right)$$

where g is the acceleration of free fall.

[2]

(ii) Hence, in terms of m, k_1, k_2, y_0 and g, derive an expression for the maximum speed v_{max} of the platform.

[1]

(iii) Show that for the ball to lose contact with the platform during the oscillation,

$$\frac{k_1}{2k_1 - k_2} + \frac{k_1 y_0}{mg} \geq 1$$

[1]

Total [6]

5 A metallic conductor, of resistivity ρ, is used to make a resistor in the shape of a truncated cone as shown in Fig. 5.1. The length of the cone is L, and the radii of the circular cross-sections at the ends are $2r$ and r.

If the taper is small, it can be assumed that the current density is constant across any cross section.

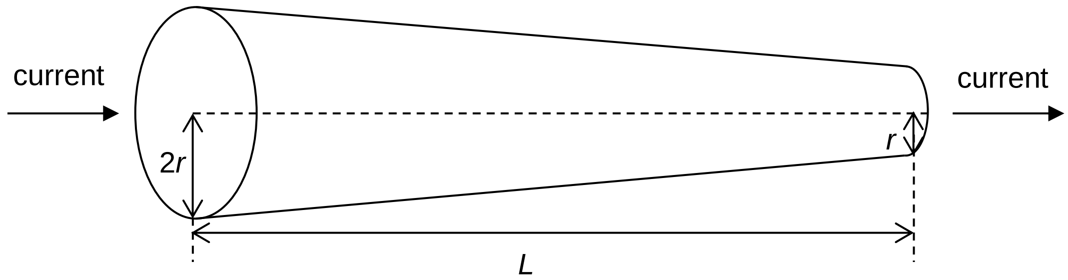

Fig. 5.1

Volume of a right circular cone of base radius r and height h is $\frac{1}{3}\pi r^2 h$.

(a) Show that the resistance R_i across the ends of the resistor is $R_i = \frac{\rho L}{2\pi r^2}$.

[3]

(b) The resistor is then melted and re-molded into a uniform hollow cylinder of inner radius *r* and outer radius 2*r* and length *L*' as shown in Fig. 5.2.

The resistance across the ends of the hollow cylinder is R_f.

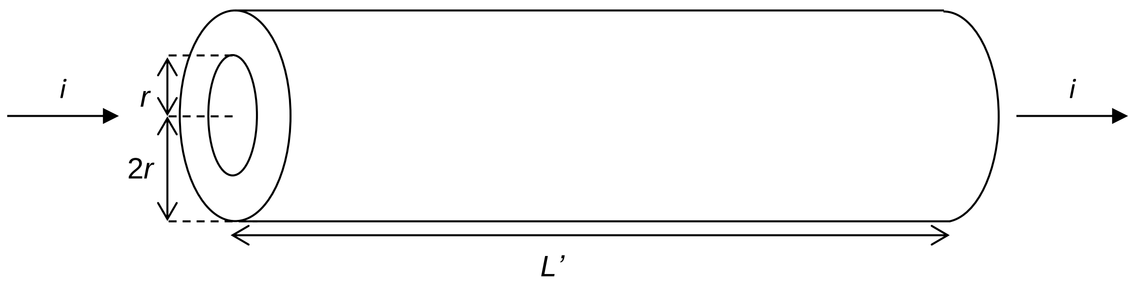

Fig. 5.2

Determine the ratio $\frac{R_i}{R_f}$.

$\frac{R_i}{R_f}$ = ... [3]

Total [6]

6 A linear particle accelerator accelerates charged particles or ions to a high speed by subjecting them to a series of oscillating electric potential differences along a linear beamline.

Fig. 6.1 shows the principle of operation of a linear accelerator used to accelerate charged particles to high speeds so that they can initiate nuclear reactions.

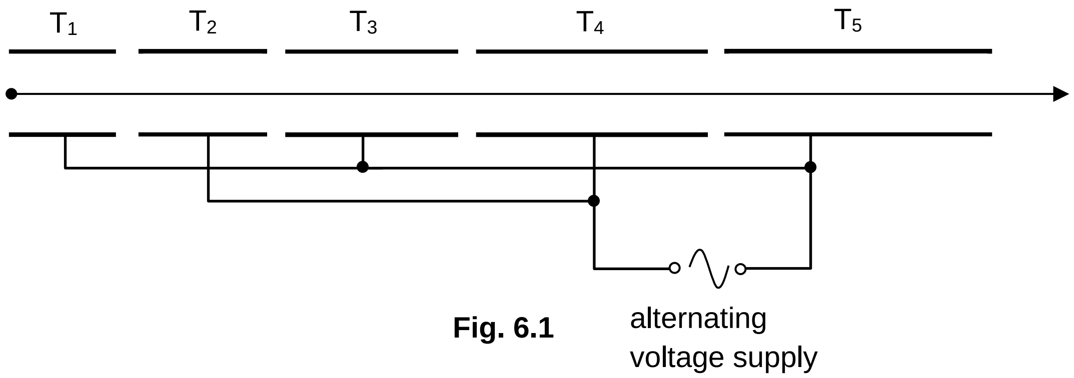

Fig. 6.1

The particles are injected along the axis of a series of cylindrical metal tubes T_1, T_2, … T_5. Alternate tubes are connected, and the two sets are connected to a high-frequency alternating voltage supply. Inside a tube, the particles travel at constant speed, but when it reaches a gap between two tubes, it is accelerated across by the applied voltage to the next tube.

(a) Explain why the tubes shown in Fig. 6.1 are not all of the same length.

....

....

....

....

....

.... [3]

(b) In a particular accelerator, there are eleven tubes. Inside the first, the particle's speed is 2.0×10^6 m s^{-1} and, inside the eleventh, it is 1.0×10^7 m s^{-1}, having been accelerated by a potential difference of 5.0×10^4 V between each of the tubes.

Determine whether the particle is an electron, a proton or an alpha particle.

[4]

(c) Magnets are usually used to focus the beam of charged particles as they travel through the accelerator.

Suggest why there is a need to focus the beam.

....

.... [1]

Total [8]

7 Consider an atom of mass m that is initially at rest. It emits a photon of wavelength λ.

(a) In terms of m, λ and any other relevant constants, write down an expression for the recoil speed of the atom.

Assume non-relativistic conditions.

[1]

(b) The atom recoils with kinetic energy K and the energy of the photon emitted is E.

If $\frac{K}{E} << 1$, the recoil of the atom may be ignored in the emission process.

In terms of m and λ, state and explain two conditions for which the recoil of the atom may be ignored.

....

....

....

.... [2]

(c) Determine the ratio $\frac{K}{E}$ for a hydrogen atom of mass 1.67×10^{-27} kg that emits a photon of ultraviolet light of energy 10.2 eV.

$\frac{K}{E}$ = ... [2]

Explain whether the recoil of the atom may be ignored.

....

.... [1]

Total [6]

8 Body P of mass m is revolving about body Q of mass M in an elliptical orbit. At an instant of time P is at a distance r from the centre of Q and is moving with a velocity v as shown in Fig. 8.1. v_r and v_t are the radial and transverse components of the velocity v respectively.

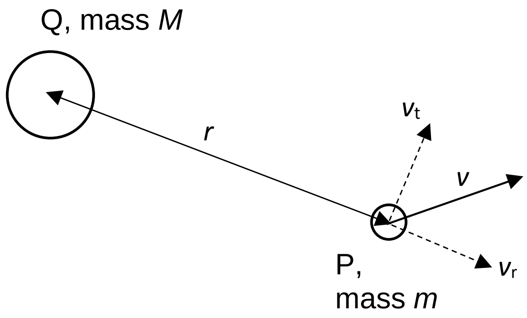

Fig. 8.1

The total energy E of the system is given by the expression,

$$E = \frac{1}{2}mv_r^{\ 2} + U_{\text{eff}}$$

where U_{eff} is the effective radial potential energy.

(a) Show that,

$$U_{eff} = -\frac{GMm}{r} + \frac{L^2}{2mr^2}$$

where L is the angular momentum of P about the centre of Q.

[3]

(b) State, with a reason, the equation relating E and U_{eff} when P is at an apse of the elliptical orbit.

....

.... [1]

(c) Hence, show that the total energy E is,

$$E = -\frac{GMm}{2a}$$

where $2a$ is the length of the major axis of the elliptical orbit.

[3]

Total [7]

Section B

Answer **two** questions from this section.

You are advised to spend about 35 minutes on each question.

9 **(a)** Fig. 9.1 shows a thin triangular plate of base length b and height h. Its mass per unit area is σ.

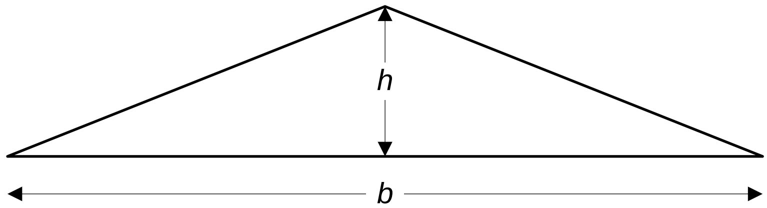

Fig. 9.1

(i) In terms of b, h and σ, write down an expression for its mass m.

[1]

(ii) From first principles, show that the distance of the centre of mass of the plate from its base is $h/3$.

Your answer should include a clearly labelled diagram.

[3]

(iii) **1.** Show that the moment of inertia I of the triangular plate about the axis perpendicular to its plane and passing through the mid-point of its base is

$$I = \frac{1}{24} m \cdot \left(b^2 + 4h^2\right)$$

[3]

2. Hence, in terms of m, b and h, derive an expression for the moment of inertia I_{CM} of the plate about an axis perpendicular to its plane and passing through its centre of mass.

[2]

(b) A uniform rod of mass *m* and length *L* is placed horizontally over the edge of a table with its centre of mass at a distance $L/4$ beyond the edge as shown in Fig. 9.2. The rod is perpendicular to the edge A of the table.

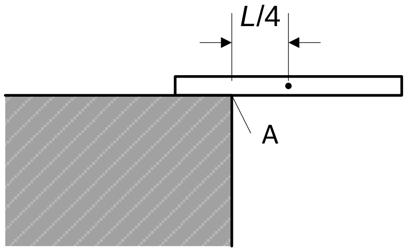

Fig. 9.2

The rod begins to rotate about the edge A. When the rod is at an angle of θ below the horizontal as shown in Fig. 9.3, it begins to slide. The coefficient of friction between the edge of the table and the rod is μ.

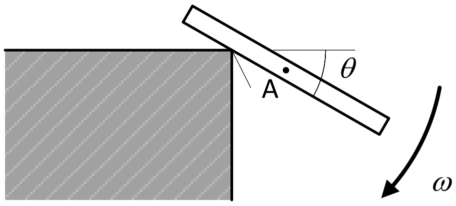

Fig. 9.3

(i) On Fig. 9.3, draw and label all the forces acting on the rod. [1]

(ii) In terms of *m* and *L*, write down an expression for the moment of inertia I_{rod} of the rod about edge A.

[1]

(iii) By considering the energy changes during the motion of the rod, derive an expression for the angular speed ω of the centre of mass of the rod about edge A, in terms of θ, *L* and *g*, the acceleration of free fall.

[1]

(iv) By considering the torque acting on the rod about edge A, obtain an expression for the angular acceleration α of the rod, in terms of θ, L and g.

[1]

(v) **1.** By considering the forces acting on the rod and parallel to it, write down an equation for the net force F_{net} in terms of f, m, g, and θ where f is the frictional force.

[1]

2. Hence, write down the equation for the centripetal force acting on the rod in terms of F_{net}, m, L and ω.

[1]

(vi) By considering the forces acting perpendicularly to the rod, write down an equation for the linear acceleration a of the centre of mass of the rod, in terms of m, g and θ.

[1]

(vii) Hence, or otherwise, determine in terms of μ, the angle θ when the rod begins to slip.

[4]

Total [20]

10 **(a)** **(i)** An infinitely long straight wire is uniformly charged with a linear charge density λ. By using Gauss's Law, determine the electric field strength E at a distance r from the wire in terms of λ.

Explain your working clearly.

[5]

(ii) A second identical wire with a linear charge density of $-\lambda$, is placed parallel to the first at a distance d from it as shown Fig. 10.1.

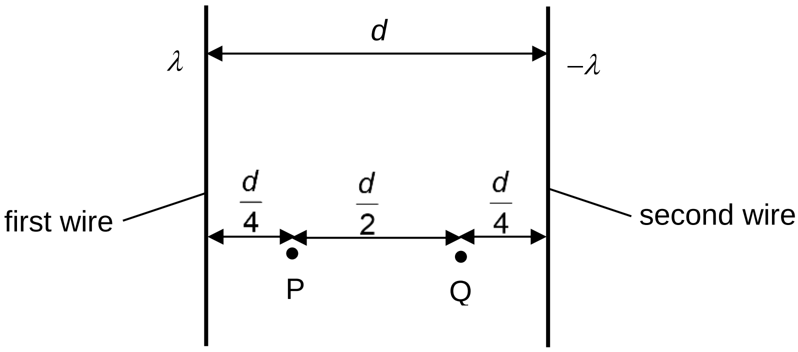

Fig. 10.1

P and Q are two points between the two wires and at a distance $\frac{d}{4}$ from the first and second wires respectively.

1. Write down an expression for the electric field strength at a point between the two wires that is a distance r away from the first wire.

[1]

2. Determine the potential difference between P and Q in terms of λ.

[4]

Total [10]

10 (b) (i) State Faraday's law of electromagnetic induction.

....

....

....

.... [2]

(ii) A helicopter has four main rotor blades, each of length 8.0 m, as shown in Fig. 10.2 and Fig. 10.3.

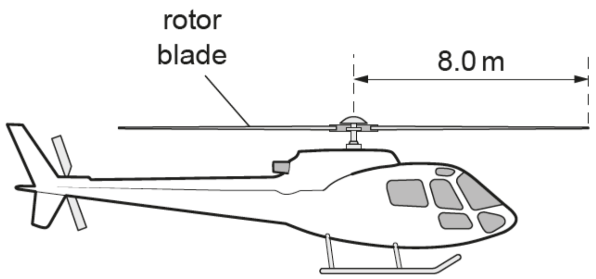

Fig. 10.2
(view from the side)

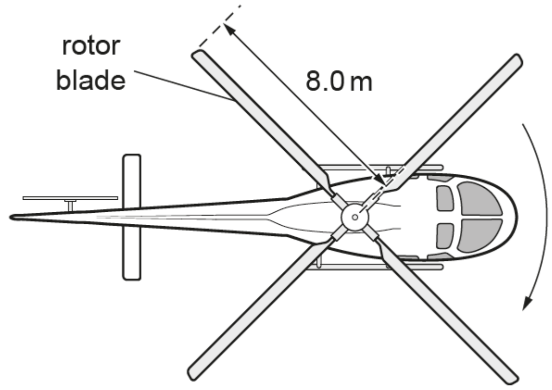

Fig. 10.3
(view from above)

The rotor blades are horizontal and rotating at 200 revolutions per second. This causes the helicopter to remain stationary in equilibrium in the air. Viewed from above, the direction of rotation is clockwise.

The Earth's magnetic field through the rotor blades is uniform. It has a magnitude of 4.7×10^{-5} T and is in a direction downwards at an angle of 50° to the horizontal, as shown in Fig. 10.4.

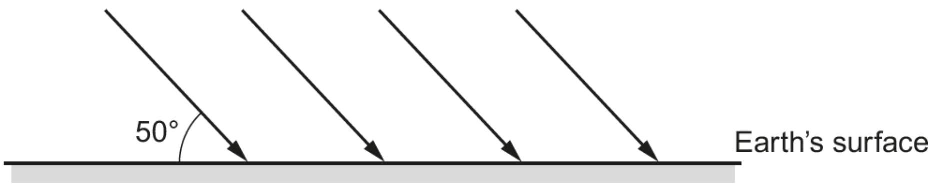

Fig. 10.4

1. Calculate the magnetic flux cut by one rotor blade during one complete revolution.

magnetic flux = ... Wb [2]

2. Calculate the e.m.f. induced across the two ends of this rotor blade.

e.m.f. = ... [2]

3. State, with a reason, which end of the rotor blade (the inner end or the outer end) is at the higher potential.

....

....

....

.... [2]

4. State, with a reason, the potential difference between the outer ends of two rotor blades that are directly opposite to each other.

....

....

....

.... [2]

Total [10]

11 A capacitor of capacitance C = 1.0 μF is charged to 50 V. At time t = 0 s, the fully charged capacitor is connected to an inductor of inductance L = 10 mH to form an oscillating LC circuit by closing the switch S in Fig. 11.1.

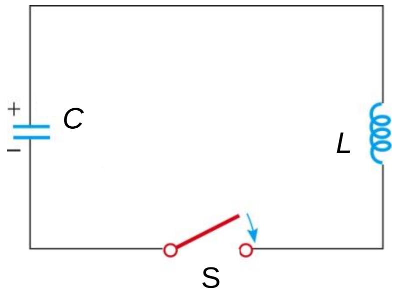

Fig. 11.1

(a) Determine

(i) the period T of oscillation of charge in the circuit,

T = ... s [1]

(ii) the initial charge Q_0 stored in the capacitor,

Q_0 = ... C [2]

(iii) The initial total energy stored in the capacitor.

total energy = ... J [1]

(b) Use your answers in **(a)** to sketch, on the axes of Fig. 11.2, the variation with time t of the potential energy U_C stored in the capacitor and the potential energy U_L stored in the inductor over one period T. Label the graphs and axes appropriately.

Fig. 11.2

[3]

(c) At the instant when the capacitor is fully discharged, the current in the circuit is a maximum.

Determine

(i) the maximum current in the circuit,

maximum current = ... A [2]

(ii) the time at which the maximum current first occurs.

time = ... s [1]

(iii) Explain why a current continues to flow in the circuit after the capacitor is discharged.

....

....

....

.... [2]

(d) Determine the times, within the first period $t = T$, at which the capacitor and the inductor have the same amount of energy stored in them.

times = ... s [3]

(e) Fig. 11.1 is modified by connecting another 1.0 μF capacitor to the circuit.

(i) Two new resonant frequencies can be produced.

Determine the ratio of these two frequencies.

ratio = ... [2]

(ii) The second 1.0 μF capacitor is next removed from the circuit and the original circuit shown in Fig. 11.1 is restored.

At time t = 0 s, when the capacitor is fully charged, a 200 Ω resistor is connected in series with the capacitor and the inductor. The switch is closed.

On Fig. 11.3, sketch the variation with time t of the charge Q in the capacitor.

Fig. 11.3

Explain the shape of your graph.

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Total [20]

End of Paper