Qn	Solution
1	**Vectors**
(a)	$\overrightarrow{BP} = \overrightarrow{OP} - \overrightarrow{OB} = (3\mathbf{a} - 2\mathbf{b}) - \mathbf{b} = 3\mathbf{a} - 3\mathbf{b} = 3(\mathbf{a} - \mathbf{b})$ and $\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA} = (3\mathbf{a} - 2\mathbf{b}) - \mathbf{a} = 2\mathbf{a} - 2\mathbf{b} = 2(\mathbf{a} - \mathbf{b})$ Since $\overrightarrow{BP} = \frac{3}{2}\overrightarrow{AP}$, points *A*, *B* and *P* are collinear. (Shown) $BP : AP = 3 : 2$
(b)	$l_{BP}: \mathbf{r} = \mathbf{b} + \lambda(\mathbf{a} - \mathbf{b}),\ \lambda \in \mathbb{R}$ Since *N* lies on line *BP*, $\overrightarrow{ON} = \mathbf{b} + \lambda(\mathbf{a} - \mathbf{b})$ for some $\lambda \in \mathbb{R}$. Since $\overrightarrow{ON}$ is perpendicular to l_{BP}, $\overrightarrow{ON}$ is perpendicular to $(\mathbf{a} - \mathbf{b})$. $[\mathbf{b} + \lambda(\mathbf{a} - \mathbf{b})] \cdot (\mathbf{a} - \mathbf{b}) = 0$ $[(1-\lambda)\mathbf{b} + \lambda\mathbf{a}] \cdot (\mathbf{a} - \mathbf{b}) = 0$ $(1-\lambda)(\mathbf{b}\cdot\mathbf{a}) - (1-\lambda)\lvert\mathbf{b}\rvert^2 + \lambda\lvert\mathbf{a}\rvert^2 - \lambda(\mathbf{a}\cdot\mathbf{b}) = 0$ Given that $\lvert\mathbf{b}\rvert = 1,\ \lvert\mathbf{a}\rvert = \frac{4}{3},\ \mathbf{a}\cdot\mathbf{b} = \left(\frac{4}{3}\right)(1)\cos\frac{\pi}{3} = \frac{2}{3}$, $(1-\lambda)\left(\frac{2}{3}\right) - (1-\lambda)(1)^2 + \lambda\left(\frac{4}{3}\right)^2 - \lambda\left(\frac{2}{3}\right) = 0$ $(1-\lambda)\left(-\frac{1}{3}\right) + \frac{16}{9}\lambda - \frac{2}{3}\lambda = 0$ $-\frac{1}{3} + \frac{1}{3}\lambda + \frac{10}{9}\lambda = 0$ $\frac{13}{9}\lambda = \frac{1}{3}$ $\lambda = \frac{3}{13}$ $\overrightarrow{BN} = \overrightarrow{ON} - \overrightarrow{OB} = \mathbf{b} + \frac{3}{13}(\mathbf{a} - \mathbf{b}) - \mathbf{b} = \frac{3}{13}\overrightarrow{BA}$ Therefore, $\frac{BN}{BA} = \frac{3}{13}$.

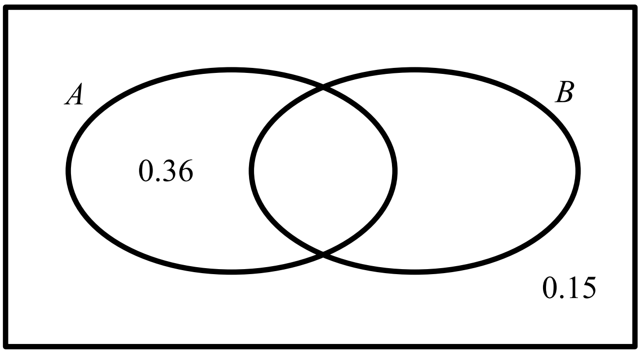
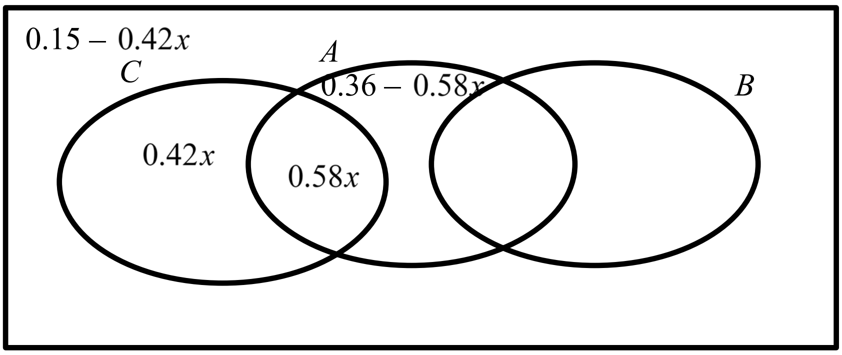
	Alternative Method $\left\|\overrightarrow{BN}\right\| = \left\|\dfrac{\overrightarrow{BO}\bullet\overrightarrow{BA}}{\left\|\overrightarrow{BA}\right\|}\right\|$ $= \left\|\dfrac{-\mathbf{b}\bullet(\mathbf{b}-\mathbf{a})}{\left\|\overrightarrow{BA}\right\|}\right\|$ $-\mathbf{b}\bullet(\mathbf{b}-\mathbf{a}) = -\mathbf{b}\bullet\mathbf{b} + \mathbf{b}\bullet\mathbf{a}$ $= -\|\mathbf{b}\|^2 + \|\mathbf{b}\|\|\mathbf{a}\|\cos\dfrac{\pi}{3}$ $= -1 + \dfrac{4}{3}\left(\dfrac{1}{2}\right)$ $= \dfrac{1}{3}$ $\left\|\overrightarrow{BA}\right\|^2 = \|\mathbf{b}\|^2 + \|\mathbf{a}\|^2 - 2\|\mathbf{b}\|\|\mathbf{a}\|\cos\dfrac{\pi}{3}$ $= 1 + \left(\dfrac{4}{3}\right)^2 - 2(1)\left(\dfrac{4}{3}\right)\left(\dfrac{1}{2}\right)$ $= \dfrac{13}{9}$ $\dfrac{BN}{BA} = \dfrac{\frac{1}{3}}{\frac{13}{9}} = \dfrac{3}{13}$

Qn	Solution
2	**Maclaurin's Series**
(a)(i)	$y=\ln\left(1+2x+3x^2\right)$ $e^y=1+2x+3x^2$ Differentiating with respect to x, $\frac{dy}{dx}e^y=2+6x$ $\frac{d^2y}{dx^2}e^y+\left(\frac{dy}{dx}\right)^2 e^y=6$ When $x=0$, $y=0$ $\frac{dy}{dx}=2$ $\frac{d^2y}{dx^2}=2$ $y=0+(2)x+\frac{2}{2!}x^2+...$ $=2x+x^2+...\ \left(\text{up to } x^2\right)$
(ii)	$y=\ln\left(1+2x+3x^2\right)$ $=2x+3x^2-\frac{\left(2x+3x^2\right)^2}{2}+...\ \left(\text{using standard series}\right)$ $=2x+3x^2-\frac{(2x)^2}{2}+...$ $=2x+x^2+...\ \left(\text{up to } x^2\right)\left(\text{verified}\right)$
(b)	$\frac{x}{\sqrt{4+x}}=x(4+x)^{-\frac{1}{2}}$ $=x(4)^{-\frac{1}{2}}\left(1+\frac{x}{4}\right)^{-\frac{1}{2}}$ $=\frac{x}{2}\left(1+\left(-\frac{1}{2}\right)\frac{x}{4}+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\left(\frac{x}{4}\right)^2+...\right)$ $=\frac{1}{2}x-\frac{1}{16}x^2+\frac{3}{256}x^3\ \left(\text{up to } x^3\right)$

Qn	Solution
3	**Sequences and Series & Complex Numbers**
(a)	$\frac{z_1}{z_2}=\frac{r_1(\cos\theta_1+\mathrm{i}\sin\theta_1)}{r_2(\cos\theta_2+\mathrm{i}\sin\theta_2)}$ $=\frac{r_1(\cos\theta_1+\mathrm{i}\sin\theta_1)}{r_2(\cos\theta_2+\mathrm{i}\sin\theta_2)}\times\frac{(\cos\theta_2-\mathrm{i}\sin\theta_2)}{(\cos\theta_2-\mathrm{i}\sin\theta_2)}$ $=\frac{r_1}{r_2}\frac{\cos\theta_1\cos\theta_2-\mathrm{i}\cos\theta_1\sin\theta_2+\mathrm{i}\sin\theta_1\cos\theta_2-\mathrm{i}^2\sin\theta_1\sin\theta_2}{\cos^2\theta_2-\mathrm{i}^2\sin^2\theta_2}$ $=\frac{r_1}{r_2}\frac{\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2+\mathrm{i}(\sin\theta_1\cos\theta_2-\cos\theta_1\sin\theta_2)}{\cos^2\theta_2+\sin^2\theta_2}$ $=\frac{r_1}{r_2}\left[\cos(\theta_1-\theta_2)+\mathrm{i}\sin(\theta_1-\theta_2)\right].$ $\arg\left(\frac{z_1}{z_2}\right)=\theta_1-\theta_2$
(b)	$\sum_{r=1}^{n}(\arg w_{r-1}-\arg w_r)=\arg w_0-\cancel{\arg w_1}$ $+\cancel{\arg w_1}-\cancel{\arg w_2}$ $+\cancel{\arg w_2}-\cancel{\arg w_3}$ $+\ldots$ $+\cancel{\arg w_{n-2}}-\cancel{\arg w_{n-1}}$ $+\cancel{\arg w_{n-1}}-\arg w_n$ $=\arg w_0-\arg w_n$ $=\arg(0+\mathrm{i})-\arg(n+\mathrm{i})$ $=\frac{\pi}{2}-\arg(n+\mathrm{i})$ $\therefore k=\frac{\pi}{2}.$
(c)	As $n\to\infty$, $\arg(n+\mathrm{i})\to 0$, hence $\sum_{r=1}^{n}\arg\left(\frac{w_{r-1}}{w_r}\right)$ converges. $\sum_{r=1}^{\infty}\arg\left(\frac{w_{r-1}}{w_r}\right)=\frac{\pi}{2}$

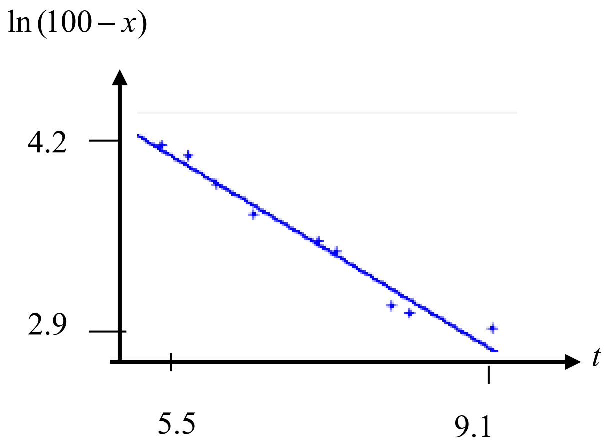
(d)	$\arg\left(\frac{1+\mathrm{i}}{2+\mathrm{i}}\right)^3+\arg\left(\frac{2+\mathrm{i}}{3+\mathrm{i}}\right)^3+\arg\left(\frac{3+\mathrm{i}}{4+\mathrm{i}}\right)^3+\ldots$ $=3\sum_{r=2}^{\infty}\arg\left(\frac{w_{r-1}}{w_r}\right)$ $=3\left(\sum_{r=1}^{\infty}\arg\left(\frac{w_{r-1}}{w_r}\right)-\arg\left(\frac{0+\mathrm{i}}{1+\mathrm{i}}\right)\right)$ $=3\left(\left(\frac{\pi}{2}\right)-\left(\arg(0+\mathrm{i})-\arg(1+\mathrm{i})\right)\right)$ $=3\left(\frac{\pi}{2}-\left(\frac{\pi}{2}-\frac{\pi}{4}\right)\right)$ $=\frac{3\pi}{4}$

Qn	Solution
4	**Applications of Differentiation**
(a)	When $y=0$, $ut\sin\theta-\frac{1}{2}gt^2=0$ $t\left(u\sin\theta-\frac{1}{2}gt\right)=0$ $t=0$ or $t=\frac{2u\sin\theta}{g}$ Since $t\neq 0$, $t=\frac{2u\sin\theta}{g}$. $t=\frac{2u\sin\theta}{g}$ represents the time taken for the projectile to return back to the same height as the origin at which it is launched.
(b)	When $t=\frac{2u\sin\theta}{g}$, $x=u\left(\frac{2u\sin\theta}{g}\right)\cos\theta$ $=\frac{u^2}{g}\sin 2\theta$ (Shown)
(c)	$\frac{\mathrm{d}x}{\mathrm{d}\theta}=\frac{2u^2}{g}\cos 2\theta$ When $\frac{\mathrm{d}x}{\mathrm{d}\theta}=0$, $\frac{2u^2}{g}\cos 2\theta=0$ $\cos 2\theta=0$ $2\theta=\frac{\pi}{2}$ $\theta=\frac{\pi}{4}$ $\left(\text{since } 0<\theta<\frac{\pi}{2}\right)$ $\frac{\mathrm{d}^2x}{\mathrm{d}\theta^2}=-\frac{4u^2}{g}\sin 2\theta$ When $\theta=\frac{\pi}{4}$, $\frac{\mathrm{d}^2x}{\mathrm{d}\theta^2}=-\frac{4u^2}{g}<0$ $\theta=\frac{\pi}{4}$ gives the maximum range of the projectile. $x=\frac{u^2}{g}\sin\left(2\left(\frac{\pi}{4}\right)\right)$ $=\frac{u^2}{g}$ Hence the maximum range of the projectile is $\frac{u^2}{g}$.

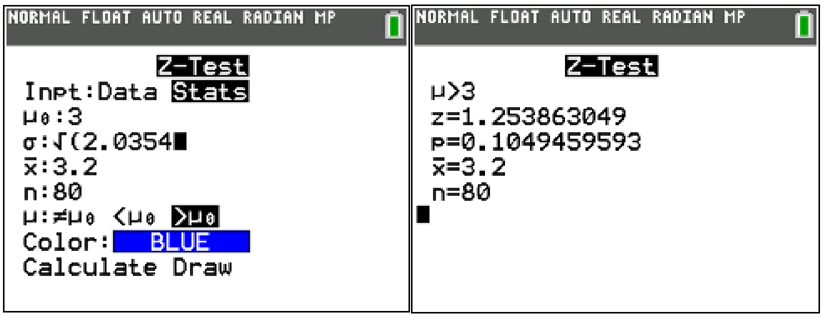
Qn	Solution
5	**Probability**
(a)	$\mathrm{P}(B') = 0.51$ $\mathrm{P}(A' \cap B') = 0.15$ $\mathrm{P}(A \cap B') = 0.51 - 0.15$ $= 0.36$
(b)	 $\mathrm{P}(B \mid A) = \frac{11}{29} \Rightarrow \frac{\mathrm{P}(B \cap A)}{\mathrm{P}(A)} = \frac{11}{29}$ $\frac{\mathrm{P}(A) - \mathrm{P}\left(A \cap B'\right)}{\mathrm{P}(A)} = \frac{11}{29}$ $\frac{\mathrm{P}(A) - 0.36}{\mathrm{P}(A)} = \frac{11}{29}$ $29\mathrm{P}(A) - 10.44 = 11\mathrm{P}(A)$ $18\mathrm{P}(A) = 10.44$ $\mathrm{P}(A) = 0.58$
(c)	Note that $\mathrm{P}(B') = 0.51$. Let $\mathrm{P}\left(C\right) = x$. Since A and C are independent, $\mathrm{P}\left(A \cap C\right) = 0.58x$. Note that B and C are mutually exclusive.  $0.15 - 0.42x \geq 0$ and $0.36 - 0.58x \geq 0$ $x \leq \frac{5}{14}$ and $x \leq \frac{18}{29}$ Therefore, maximum $\mathrm{P}\left(C\right) = \frac{5}{14}$

Qn	Solution
6	**Discrete Random Variable**
(a)	Area of target board = $\pi\left(5^2\right)=25\pi$ Area of region with score 50 $=\pi\left(1^2\right)=\pi$ Probability of dart hitting region with score 50 $=\dfrac{\pi}{25\pi}=\dfrac{1}{25}$ Area of region with score 25 $=\pi\left(3^2\right)-\pi=8\pi$ Probability of dart hitting region with score 25 $=\dfrac{8\pi}{25\pi}=\dfrac{8}{25}$ $\mathrm{P}\left(S=75\right)=\dfrac{8}{25}\times\dfrac{1}{25}\times2!$ $=\dfrac{16}{625}$ $\quad$ (shown)
(b)	Area of region with score 0 $=25\pi-9\pi=16\pi$ Probability of dart hitting region with score 0 $=\dfrac{16\pi}{25\pi}=\dfrac{16}{25}$
	(table below)
(c)	From GC, $\mathrm{E}\left(S\right)=20$ and $\mathrm{Var}\left(S\right)=400$.

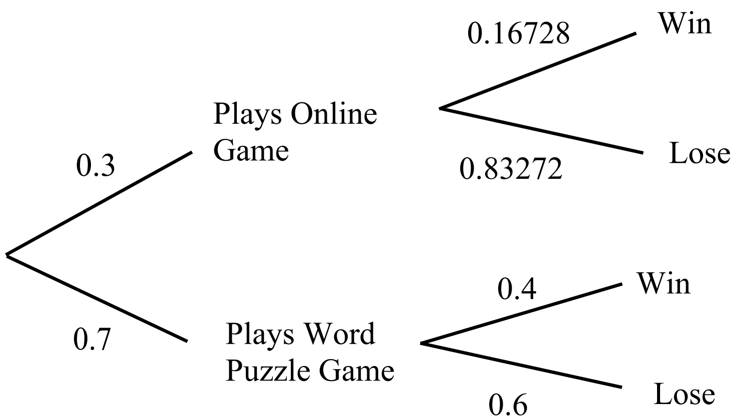
s	0	25	50	75	100
$\mathrm{P}\left(S=s\right)$	$\dfrac{16}{25}\times\dfrac{16}{25}$ $=\dfrac{256}{625}$	$\dfrac{16}{25}\times\dfrac{8}{25}\times2!$ $=\dfrac{256}{625}$	$\dfrac{16}{25}\times\dfrac{1}{25}\times2!$ $+\dfrac{8}{25}\times\dfrac{8}{25}$ $=\dfrac{96}{625}$	$\dfrac{16}{625}$	$\dfrac{1}{25}\times\dfrac{1}{25}$ $=\dfrac{1}{625}$

Qn	Solution
7	**Correlation and Regression**
(a)	No, since product moment correlation coefficient is independent of the scale (or unit) of measurement.
(b)	Using GC, $r = -0.985$. Since r is close to -1, there is a strong negative linear correlation between t and $\ln(100-x)$.
(c)	
(d)	$\ln(100-x) = 5.93932 - 0.33739t$ $\ln(100-x) = 5.94 - 0.337t$ (3 s.f)
(e)	Sub $t = 10$, $\ln(100-x) = 5.93932 - 0.33739(10)$ $100 - x = \mathrm{e}^{2.56541}$ $x = 87.0$ (3 s.f) Since $t = 10$ lies outside of the given data range of t, the linear relation between $\ln(100-x)$ and t may no longer hold. The estimate is not reliable.

Qn	Solution
8	**Normal and Sampling Distribution**
(a)	Let A and S be the mass of a randomly chosen apple from Brand A and Brand S respectively. $S \sim \mathrm{N}\left(78.8, 3.1^2\right)$ and $A \sim \mathrm{N}\left(82.2, 2.2^2\right)$ Required Probability $= \mathrm{P}\left(80 < A < 84\right)$ $= 0.635 \quad \text{(3sf)}$
(b)	Let $T = A_1 + A_2 + \ldots + A_5$. $\mathrm{E}\left(T\right) = 5 \times 82.2 = 411$ $\mathrm{Var}\left(T\right) = 5 \times 2.2^2 = 24.2$ $T \sim \mathrm{N}\left(411, 24.2\right)$ $\mathrm{P}\left(T > 408\right) = 0.729$ (3 s.f.)
(c)	Let $D = S - 0.9A$. $\mathrm{E}\left(D\right) = 78.8 - 0.9\left(82.2\right) = 4.82$ $\mathrm{Var}\left(D\right) = 3.1^2 + 0.9^2\left(2.2^2\right) = 13.5304$ $D \sim \mathrm{N}\left(4.82, 13.5304\right)$ Required Probability $= \mathrm{P}\left(\left\|D\right\| > 1\right)$ $= 1 - \mathrm{P}\left(\left\|D\right\| < 1\right)$ $= 1 - \mathrm{P}\left(-1 < D < 1\right)$ $= 0.907 \quad \text{(3sf)}$

Qn	Solution
9	**Hypothesis Testing**
(a)	Tim can **randomly select 20 Year 1 Boys, 20 Year 1 Girls, 20 Year 2 Boys and 20 Year 2 Girls** to ensure that the sample of students is unbiased and representative of the cohort.
(b)	Tim should conduct a one-tailed test as he believes that students are spending more than 3 hours each day on their homework.
(c)	Unbiased estimate of population mean is $\bar{x} = \frac{16}{80} + 3$ $= 3.2$ Unbiased estimate of population variance is $s^2 = \frac{1}{79}\left(164 - \frac{16^2}{80}\right)$ $= \frac{804}{395}$ or $2.0354430 = 2.04$
(d)	Let X be the time that a randomly chosen student spends on homework each day (in hours). Let μ denote the population mean time that students spend on homework each day (in hours). $H_0 : \mu = 3$ $H_1 : \mu > 3$ Under H_0, since $n = 80$ is large, by Central Limit Theorem, $\overline{X} \sim N\left(3, \frac{2.0354}{80}\right)$ approximately. Test statistics: $Z = \frac{\overline{X} - 3}{\sqrt{\frac{2.0354}{80}}}$ Level of significance: 5% Just choose any suitable level of significance, and conclude accordingly. Reject H_0 if $p-\text{value} < 0.05$ Using GC, $p-\text{value} = 0.105$ (3 s.f)  Since $p-\text{value} = 0.105 > 0.05$ we do not reject H_0 and conclude that there is insufficient evidence at 5% significance level, that the population mean time that students spend on homework each day is more than 3 hours. Thus Tim's belief is not accurate at 5% significance level.

(e)	$H_0 : \mu = 3$ $H_1 : \mu \neq 3$ Under H_0, assuming that $X \sim N\left(3, \sigma^2\right)$, $\overline{X} \sim N\left(3, \dfrac{\sigma^2}{10}\right)$. Test statistics: $Z = \dfrac{\overline{X} - 3}{\sqrt{\dfrac{\sigma^2}{10}}}$ Level of significance: 2% Reject H_0 if $z - \text{value} < -2.3263$ or $z - \text{value} > 2.3263$. Since H_0 is rejected, $\dfrac{3.2 - 3}{\sqrt{\dfrac{\sigma^2}{10}}} < -2.3263$ or $\dfrac{3.2 - 3}{\sqrt{\dfrac{\sigma^2}{10}}} > 2.3263$ $\sigma < -0.27187$ or $\sigma < 0.27187$ (3 s.f.) (rejected) $0 < \sigma < 0.27187$ $(\text{since } \sigma > 0)$ Therefore, for teachers' claim to be rejected at 2% level of significance, $0 < \sigma < 0.271$.

Qn	Solution
10	**Binomial Distribution**
(a)	$X=8$ means there are 8 steps forward and 2 steps backward. Hence the counter ends up 6 steps ahead of point S. $X \sim \mathrm{B}\left(10, \frac{3}{5}\right)$ $\mathrm{P}(X=8)=0.121 \quad (3\text{ s.f.})$
(b)	$\mathrm{P}(\text{Serene wins the game})=\mathrm{P}(X \geq 8)$ $=1-\mathrm{P}(X \leq 7)$ $=0.16728$ ≈ 0.167 (3 s.f.) (shown)
(c)	Let W be the number of games, out of 20, that Serene wins. $W \sim \mathrm{B}(20, 0.16728)$ $\mathrm{P}(W=3)=0.23753 \approx 0.238$ (3 s.f.)
(d)	 $\mathrm{P}(\text{Plays Word Puzzle} \mid \text{Wins})$ $=\dfrac{(0.7)(0.4)}{(0.7)(0.4)+(0.3)(0.16728)}$ $=0.84801$ ≈ 0.848 (3 s.f.)