Tutorial 18

ALTERNATING CURRENTS

Self-Check Questions

S1. What is meant by period, frequency, peak value and root-mean square current as applied to an alternating current?

S2. A sinusoidal current is represented by the equation $I = I_o \sin(\omega t)$. What is its period, frequency, peak value and root-mean square value?

S3. Deduce that the mean power in a resistive load is half the maximum power for a sinusoidal alternating current.

S4. What is the relationship between r.m.s. and peak values for a sinusoidal current?

S5. Explain the principle of operation of a simple iron-core transformer.

S6. For an ideal transformer, what is the relationship between the turns ratio of windings, the input and output potential differences, and the primary and secondary currents?

S7. Explain how a diode is used for rectification of an alternating current.

Self-Practice Questions

SP1. (a) What is meant by the r.m.s. value of an alternating current?

(b) Calculate the peak value of a 240 V mains electricity supply.

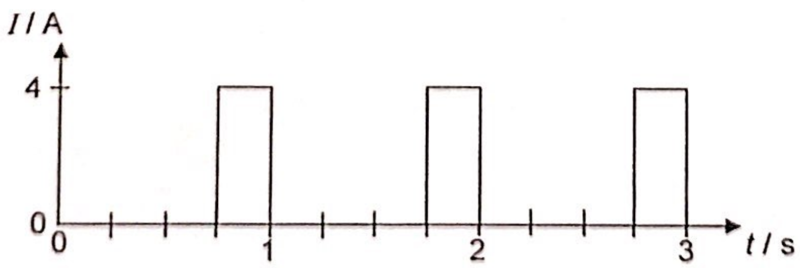
(c) A sinusoidal alternating current of r.m.s. value 5.0 A passes through a 4.0 Ω resistor. Calculate

(i) the peak value of the current,

(ii) the mean power in the resistor,

(iii) the maximum power in the resistor.

SP2.



(a) The figure above shows the variation with time t of a periodic current I. Determine

(i) the average value of the current,

(ii) the root-mean-square current.

(b) The periodic current passes through a resistor, producing heat at a certain rate. State the steady current, when passed through the same resistor, would have an identical heating effect. (J83/I/10)

SP3. A mains electricity supply has a r.m.s. voltage of 240 V and a peak voltage of 340 V. When connected to this supply, a heater dissipates energy at a rate of 1000 W. The heater is then connected to a 340 V d.c. supply and its resistance remains the same. At what rate does the heater now dissipate energy? (N2000/I/21)

SP4. The primary coil of a transformer is connected to an alternating voltage supply. The secondary coil is connected across a variable resistor. Suggest ways in which you can decrease the p.d. across the secondary coil.

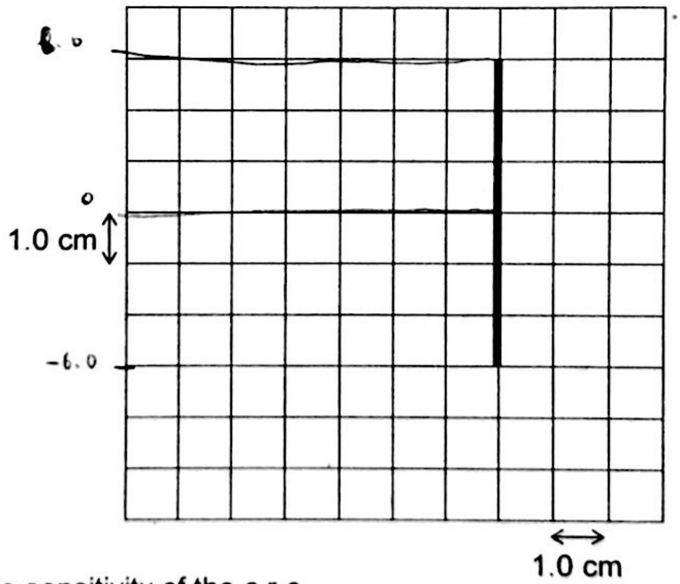
SP5. In a laboratory experiment to test a transformer, a student obtained the following results.

V_p / V	I_p / mA	N_p turns	V_s / V	I_s / mA	N_s turns
240	2.0	?	?	50	50

Assuming the transformer is 100% efficient, what are the missing entries?

(Modified from J93/I/20)

SP6. The diagram shows the display on a cathode-ray oscilloscope (c.r.o.) when a sinusoidal p.d. is applied to the Y-input. The r.m.s. value of the applied p.d. is 4.24 V. The time-base is switched off.

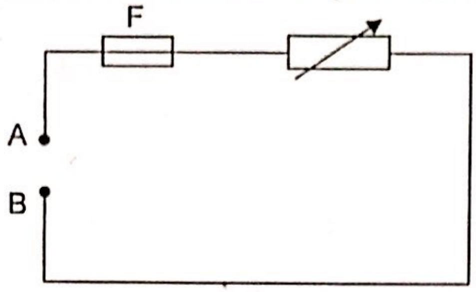


Calculate the Y-plate sensitivity of the c.r.o.

Discussion Questions

Characteristics of Alternating Currents

D1. When an a.c. supply of 240 V r.m.s. is connected to the terminals AB in the circuit shown below, the fuse F breaks the circuit if the current just exceeds 13 A r.m.s.



When the a.c. supply is replaced with a 120 V d.c. source, an identical fuse breaks the circuit if the current just exceeds

A $\frac{13}{2}$ A **B** $\frac{13}{\sqrt{2}}$ A **C**

(J88/I/23)

D2. A sinusoidal potential V_1 shown in Fig. 2a is applied across a resistor R which produces heat at a mean rate P.

Determine the mean rate of heat production when the square-wave of potential V_2 shown in Fig. 2b is applied across the same resistor.

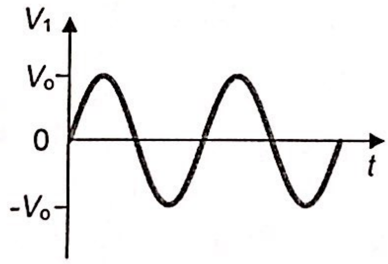


Fig. 2a

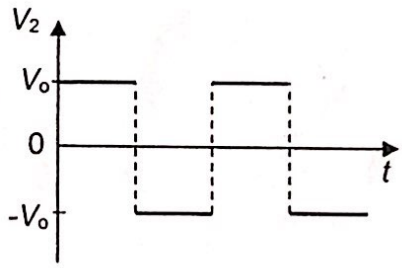
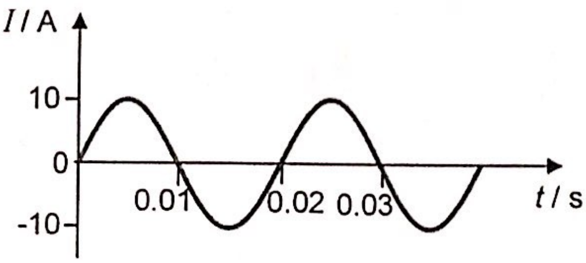


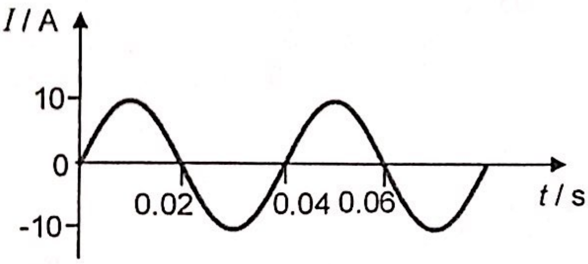
Fig. 2b

(J82/II/18) [2]

D3. **(a)** Calculate the r.m.s. current of (i) and (ii). [2]



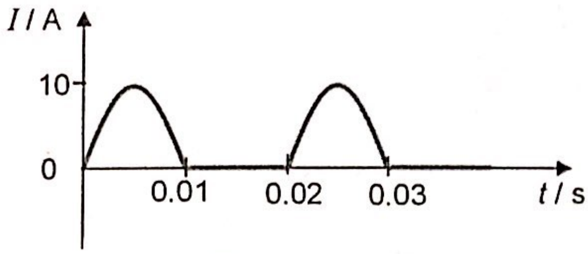
(i)



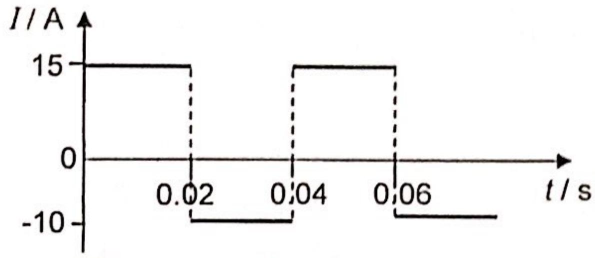
(ii)

(b) What can you conclude from your answers to (i) and (ii)? [1]

(c) Determine the r.m.s. current of (iii) and (iv). [4]

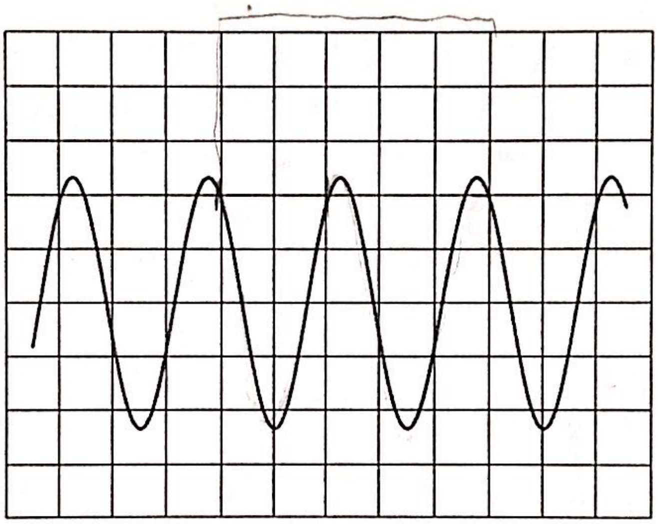


(iii)



(iv)

D4. A cathode-ray oscilloscope (c.r.o.) is connected across the output of a transformer and the screen is as shown below. The squares on the screen have sides of one centimetre.



The Y-plate sensitivity is set at 2.0 V cm^{-1} and the timebase is set so that the horizontal deflection is 0.50 ms cm^{-1}.

For the alternating potential difference applied to the Y-plates, deduce the values of the following quantities:

(a) period, [1]

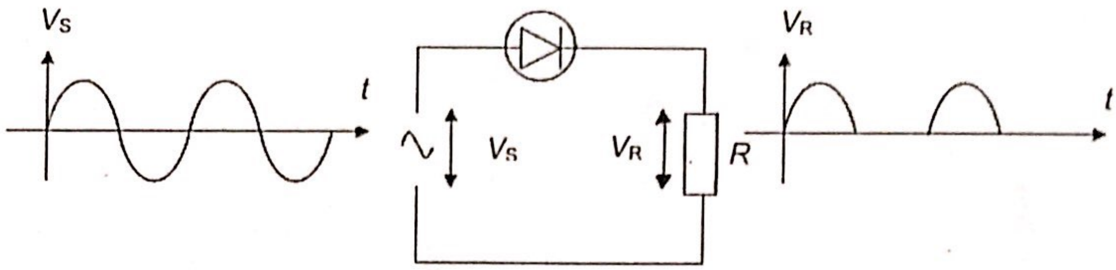
(b) frequency, [1]

(c) peak value of potential difference, [1]

(d) root-mean-sqaure value of potential difference. [1]

Rectification

D7. An alternating supply is applied to a resistor R with a diode connected in series with it. The time t variation of the supply voltage V_S and the p.d. V_R across the resistor are shown in the diagram.



Which of the following statements is *incorrect*?

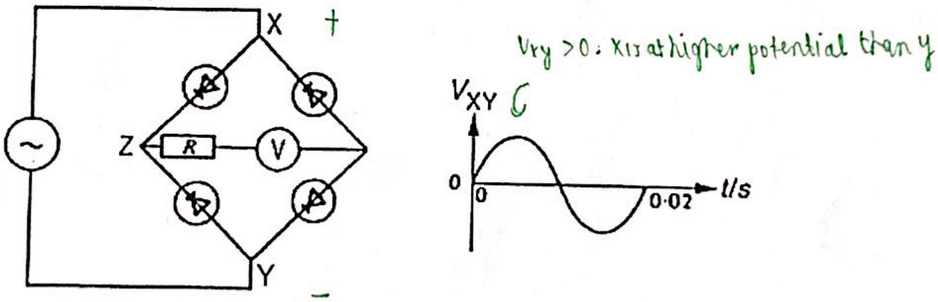
A Half-wave rectification takes place so that the current supplied to the resistor is one-directional.

B The period of the V_R-t graph is half of that of the V_S-t graph.

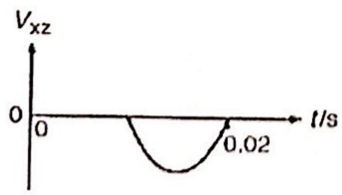
C The time intervals corresponding to zero voltage across resistor R in the graph represent the stages when the diode is reverse-biased.

D Root-mean-square voltage across resistor R is half of the peak voltage applied across it.

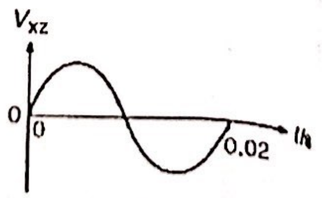
D8. The circuit represents a bridge rectifier arrangement. The graph below shows the variation over one cycle of the potential of X with respect to Y.



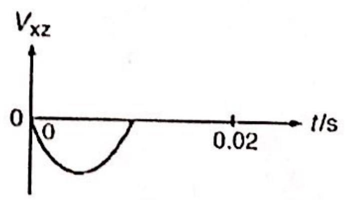
Which one of the graphs best represents the corresponding variation of the potential of X with respect to Z?



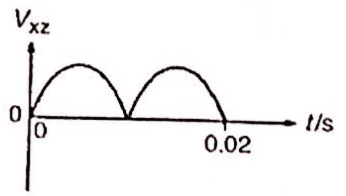
A



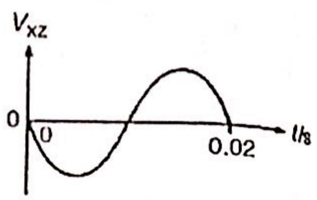
B



E



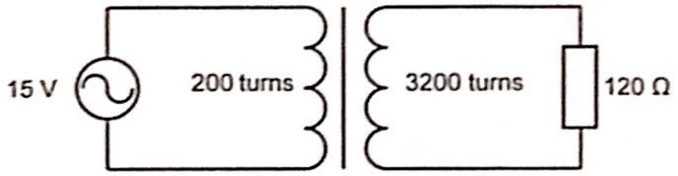
C



D

The transformer

D5. The primary of an ideal transformer has 200 turns and is connected to a 15 V supply. The secondary has 3200 turns and is connected to a resistor of resistance 120 Ω, as shown in the diagram.

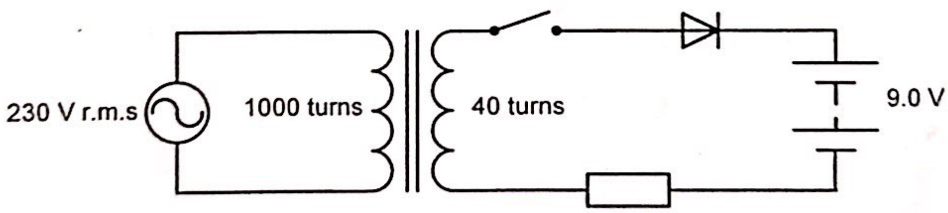


What are possible values of the secondary voltage, the secondary current and the mean power dissipated in the resistor?

	secondary voltage / V r.m.s.	secondary current / A r.m.s.	resistor power / W
A	24	0.020	4.8
B	24	0.20	48
C	240	0.50	120
D	240	2.0	480

(N2011/1/33)

D6. The primary coil of a transformer has 1000 turns and is connected to a 230 V r.m.s. supply. The secondary coil has 40 turns and may be connected, through a switch and a diode, to a 9.0 V rechargeable battery, as illustrated in the figure below.



(a) Initially the switch is open. Considering both the transformer and the diode to be ideal, calculate

(i) the r.m.s. potential difference across the secondary, [1]

(ii) the peak potential difference across the secondary. [1]

(b) The switch is now closed so that the battery is being recharged.

(i) Suggest why the diode is necessary in the secondary circuit. [1]

(ii) Suggest why the resistor is necessary in the circuit. [1]

(J2000/II/5)

Challenging Questions

C1. Two air-core solenoids P and Q which are identical in length and area of cross-section, have 10 and 50 turns respectively. They are placed near each other.

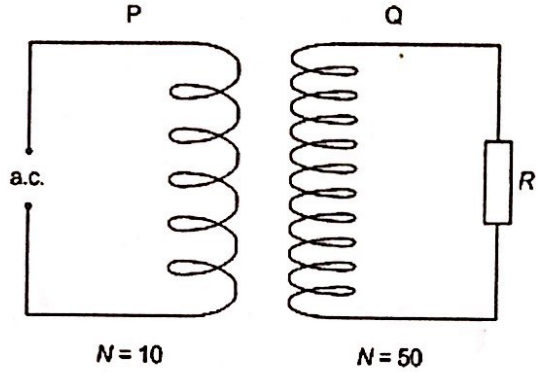


Fig 23.1

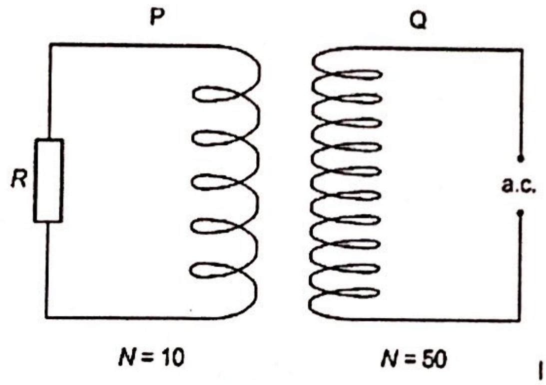


Fig 23.2

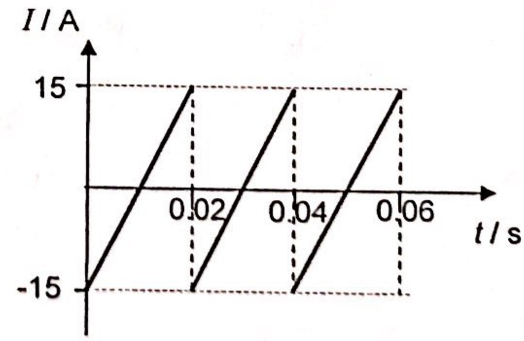
In Fig. 23.1, when the current in P changes at a rate of 5.0 A s^{-1}, an e.m.f. of 2.0 mV is induced in Q. The current in P is then switched off . In Fig. 23.2, a current which is changing at 2.0 A s^{-1} is then sent through Q. What e.m.f. will be induced in P?

[Magnetic flux density in a solenoid, $B = \mu_0 nI$ where μ_0 is the permeability of free space, n is the number of turns per unit length and I is the current in the solenoid.]

*Knowledge of mutual inductance is required in this question.

(RJC 2004 Prelim P1 Q23)

C2. Determine the r.m.s. current shown in the figure.



Ans:

SP1. 339 V, 7.07 A, 100 W, 200 W
SP2. 1.0 A, 2.0 A, 2.0 A
SP3. 2000 W
SP5. 1250, 9.6 V
SP6. 2.0 V cm^{-1}
D1. C
D2. 2*P*
D3. 7.07 A, 5.00 A, 12.7 A
D4. 1.25 ms, 800 Hz, 4.7 V, 3.3 V
D5. D
D6. 9.2 V, 13.0 V
D7. B
D8. A
C1: 8.0×10^{-4} V
C2: 8.66 A

TUTORIAL 18 SUGGESTED SOLUTIONS

SELF-CHECK QUESTIONS:

S1.

Term	Definition
Period, T	of an alternating current is the time taken for one complete cycle.
Frequency, f	of an alternating current is the number of complete cycles per unit time.
Peak current, I_0	of an alternating current is the amplitude of the current.
Root-mean square current, $I_{r.m.s.}$	of the alternating current is that value of direct current that would produce thermal energy at the same rate in a resistor.

S2. Period $T = \frac{2\pi}{\omega}$

Frequency $f = \frac{1}{T} = \frac{\omega}{2\pi}$

Peak value is I_0

Root-mean square value $I_{r.m.s.} = \frac{I_0}{\sqrt{2}}$

S3. The instantaneous power delivered to R at time t is

$$P = IV_R = (I_0 \sin\omega t)(V_0 \sin\omega t) = I_0 V_0 \sin^2 \omega t$$

The mean power is given by

$$\langle P \rangle = \langle I_0 V_0 \sin^2 \omega t \rangle = I_0 V_0 \langle \sin^2 \omega t \rangle = I_0 V_0 \left(\frac{1}{2}\right) = \frac{P_0}{2}$$

i.e. mean power in a resistive load is half the maximum power for a sinusoidal a.c.

*Proof for yourself that $\langle \sin^2 \omega t \rangle = \frac{1}{2}$

S4.

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

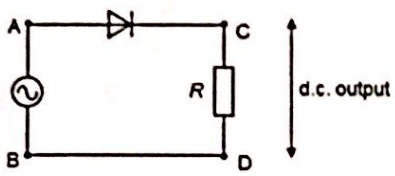
S5. The principle of operation of a simple iron-core transformer:

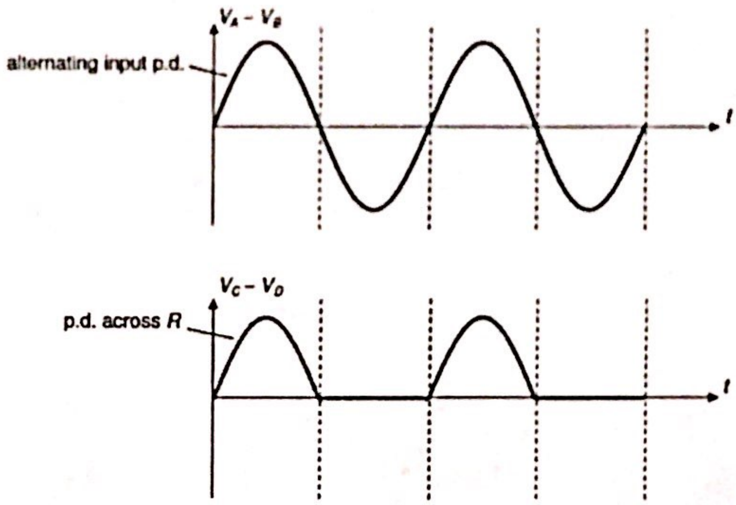
- a transformer works by electromagnetic induction
- the a.c. source causes an alternating current to flow in the primary coil, which sets up an alternating magnetic flux in the iron core
- this induces an alternating e.m.f. in the secondary coil, in accordance with Faraday's law of electromagnetic induction
- the induced e.m.f. in the secondary coil give rise to an alternating current which delivers energy to the device to which the secondary is connected
- all currents and e.m.f.s have the same frequency as the a.c. source

S6. For an ideal transformer:

$$\frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{I_p}{I_s}$$

S7.
1. Connect a diode in circuit as shown below.
2. Diode allows current to flow during one-half of a cycle when it is forward biased.
3. When it is reverse bias, no current flows through R.





SELF-PRACTICE QUESTIONS:

SP1. **(a)** The r.m.s. value of the alternating current is the value of the direct current that would produce thermal energy at the same rate in a resistor.

(b) For a 240 V mains electricity supply, $V_{rms} = 240$ V.

The peak value of the 240 V mains, $V_0 = \sqrt{2}\ V_{rms} = \sqrt{2}(240\text{ V}) = 339\text{ V}$.

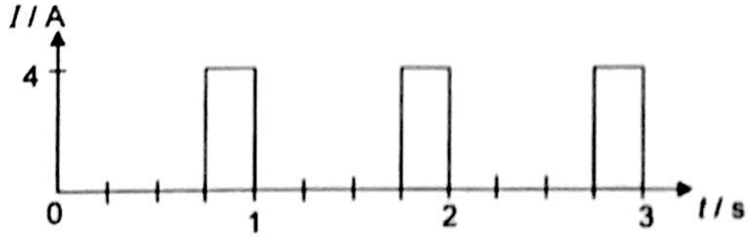
(c) A sinusoidal alternating current of r.m.s. value 5.0 A passes through a 4.0 Ω resistor.

(i) Peak value of the current $I_0 = \sqrt{2}\ I_{rms} = \sqrt{2}(5.0\text{ A}) = 7.07\text{ A}$

(ii) Mean power in the resistor $\langle P \rangle = I_{rms}{}^2 R = 5.0^2 \times 4.0 = 100\text{ W}$

(iii) Maximum power $P_0 = 2\langle P \rangle = 2 \times 100 = 200\text{ W}$

SP2.



(a)(i) Average value of the current $= \frac{0 \times 0.75 + 4 \times 0.25}{1} = 1.0$ A

(ii) Root-mean-square current $= \sqrt{\frac{0 \times 0.75 + 4^2 \times 0.25}{1}} = 2.0$ A

(b) A steady current of 2.0 A will produce an identical heating effect as the alternating current. This is according to the definition of r.m.s. value of an alternating current.

SP3.

$$\langle P \rangle = \frac{V_{rms}^{\ 2}}{R} \Rightarrow R = \frac{V_{rms}^{\ 2}}{\langle P \rangle} = \frac{240^2}{1000}$$

Power dissipated in 340 V d.c. source $= \frac{V_{d.c.}^{\ 2}}{R} = \frac{340^2}{\left(\frac{240^2}{1000}\right)} = 2000$ W

<u>OR</u> we can treat 340 V d.c. as V_o, then $P_o = 2\langle P \rangle = 2000$ W

SP4.

Since $\frac{V_s}{V_p} = \frac{N_s}{N_p}$, to decrease the p.d. V_s across the secondary coil, we can

- increase N_p, the number of turns in the primary coil
- decrease N_s, the number of turns in the secondary coil
- decrease V_p, the supply voltage that is connected to the primary coil

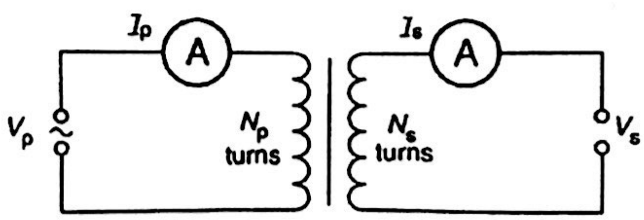
*changing the resistance of the variable resistor will not affect the p.d. across the secondary coil.

SP5.

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s} = \frac{2.0}{50}$$

$$N_p = \frac{50}{2.0} \times N_s = \frac{50}{2.0} \times 50 = 1250$$

$$V_s = \frac{2.0}{50} \times V_p = \frac{2.0}{50} \times 240 = 9.6 \text{ V}$$



SP6.

For sinusoidal p.d.,

$V_0 = \sqrt{2}\ V_{r.m.s.} = \sqrt{2}(4.24 \text{ V}) = 6.0$ V

From the diagram,

peak-to-peak p.d. has a span of 6.0 cm

$\Rightarrow V_0$ has a span of 3.0 cm

$\Rightarrow$ 6.0 V has a span of 3.0 cm

$\therefore$ Y-plate sensitivity $= \frac{6.0 \text{ V}}{3.0 \text{ cm}} = 2.0 \text{ V cm}^{-1}$