Class	Register No.

Candidate Name ______________________________

PEIRCE SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2021
SECONDARY 4 EXPRESS

ADDITIONAL MATHEMATICS **4049/01**
Paper 1 **13 September 2021**
2 hour 15 minutes

Additional Materials:
Plain Paper (for rough work)

INSTRUCTIONS TO CANDIDATES
Candidates answer on the Question Paper.

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

PARENT'S SIGNATURE

For Examiner's Use	
Total	

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots\ldots\ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\mathrm{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Solve the equation $10^{2x+1} - 80\left(10^{x}\right) = -70$. [6]

2 The polynomial $f(x) = x^2 - 5x + 7$ leaves the same remainder whether divided by $x - b$ or $x - c$, where $b \neq c$.

(i) Show that $b + c = 5$. [3]

(ii) Given further that $4bc = 21$ and $b > c$, find the value of b and of c. [4]

3 **(i)** Find the first four terms in the expansion, in the ascending powers of *x*, of $\left(3-\frac{x}{2}\right)^8$. [3]

(ii) Hence obtain the coefficient of x in the expansion of $\left(x-\frac{5}{x}\right)^2\left(3-\frac{x}{2}\right)^8$. [3]

4 The mass M (in grams), of a radioactive substance is given by $M = 24e^{-kt}$, where k is a constant and t is measured in days after the substance is first being observed. It takes 35 days for the radioactive substance to be reduced to half of its original mass. Calculate

(i) the value of k, [3]

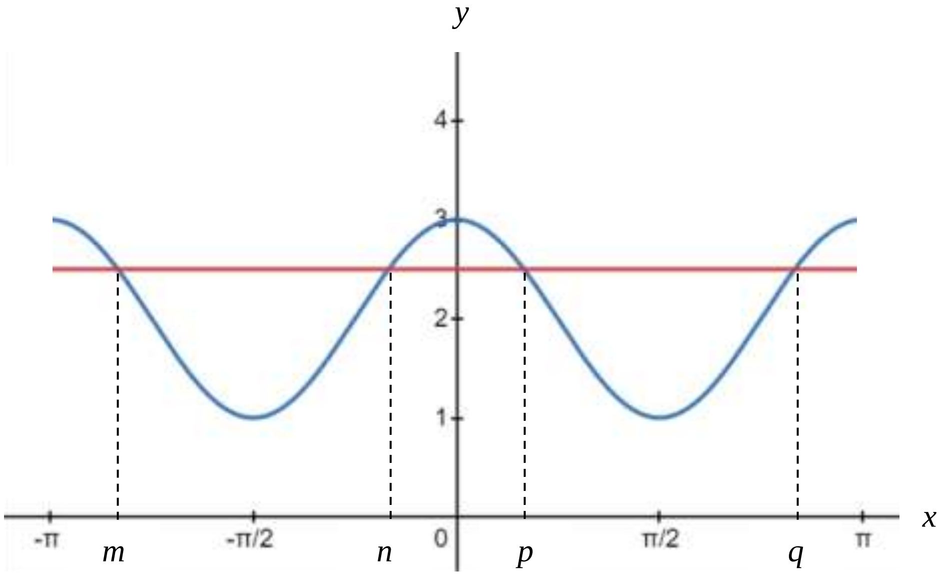
(ii) the amount of radioactive substance which indicates an observation period of one year. [2]

5 The equation of a curve is $y = \frac{3x}{2x-3},\ x \neq 1.5$.

(i) Find the coordinates of the points on the curve where the tangent is parallel to the line $9x + 4y = 3$. [5]

(ii) Determine whether the curve is an increasing or decreasing function for all values of *x*, except *x* = 1.5. [2]

6



The diagram above shows the graph of $y = a + \cos bx$ for $-\pi \le x \le \pi$.

(i) State the value of *a* and of *b*. [2]

(ii) Hence, given that m, n, p and q are solutions to the equation $a + \cos bx = k$, where $k > a$ and for $-\pi \le x \le \pi$, express *m* in terms of *p*. [1]

(iii) State the range(s) of k such that there is no solution to the equation $a+\cos bx=k$. [1]

(iv) Explain how a suitable straight line can be added to the graph of $y=a+\cos bx$ to obtain the solutions to the equation $x-\pi\cos bx=\pi(a-1)$ for $-\pi\le x\le\pi$.
Hence, draw this straight line on the diagram and state the number of solutions. [3]

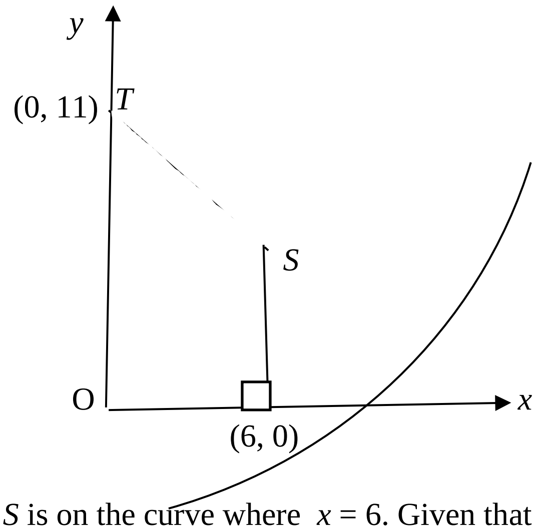
7 **(i)** Given that $y = e^x(\cos x - \sin x)$, show that $\dfrac{dy}{dx} = -2e^x \sin x$. [2]

(ii) Hence, without using a calculator, find the value of *a* and of *b* for which

$$\int_0^{\frac{\pi}{3}} e^x \sin x \, dx = \left(\frac{a\sqrt{3}-1}{4}\right)e^{\frac{\pi}{3}} + b$$

. [4]

8 The diagram shows part of the curve $y = \frac{x^2}{9} + \frac{x}{6} + k$, where k is a constant.



(i) The point S is on the curve where $x = 6$. Given that the normal to the curve at the point S meets y-axis at T (0, 11), show that $k = 2$. [6]

(ii) Find the area of the shaded region. [4]

9 As part of a mechanism in a retractable hanger, the rods *ST* and *SU* are hinged at *S* perpendicularly with lengths 12 cm and 5 cm respectively as shown in Diagram 1. *ST* and *SU* are able to move about S as shown in Diagrams 2 and 3. Moving either rod will move the other rod such that angle *STQ* = angle *PSU* = θ, where θ is a variable and $0^o < \theta < 90^o$.
UP and *TQ* are perpendicular to the horizontal straight line *SPQ*.

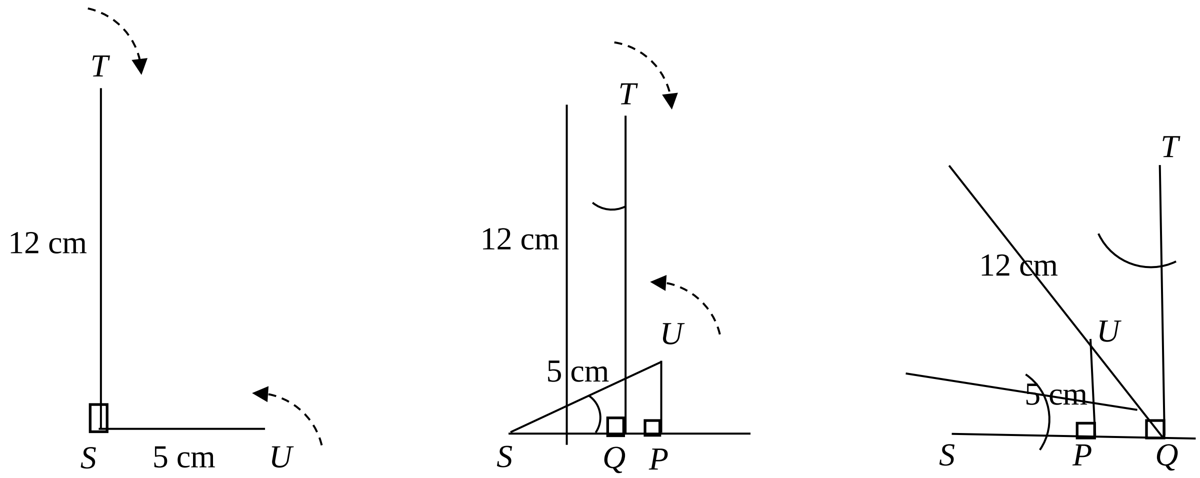


Diagram 1 Diagram 2 Diagram 3

(i) By expressing *SP* and *SQ* in terms of θ, determine the value of θ when *SP* is equal to *SQ*. [4]

(ii) Given that P has to be between S and Q, obtain an expression for PQ, in terms of θ.

Hence, express PQ in the form of $R\sin(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [4]

(iii) Find the range of values of θ for which P lies between S and Q. [1]

10 A particle moves in a straight line so that t seconds after passing through O, its velocity v cm s^{-1}, is given by $v = t^2 - 6t + 5$. The particle comes to instantaneous rest, firstly at A and then at B.

(i) Find an expression, in terms of t, for the displacement of the particle from O at time t. [2]

(ii) Calculate the distance AB. [4]

(iii) Calculate the average speed of the particle as it travelled from A to B. [2]

(iv) It is given that C is the point at which the particle has zero acceleration. Determine, with clear working, whether C is nearer to O or to B. [4]

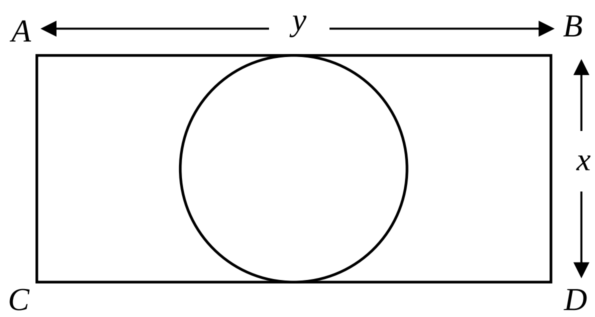
11 The equation of a circle, $C_{1,}$ is $x^2 + y^2 - 10x - 12y + 52 = 0$.

(i) Find the coordinates of its centre and the length of its radius. [3]

(ii) Explain why the line $x = 2$ is a tangent to the circle, C_1. [2]

(iii) The circle, $C_{2,}$ is a reflection of C_1 about the line $x = 2$.
Find the equation of the circle, C_2. [2]

12 A landscape designer uses 40 m of fencing for a rectangular plot of garden *ABCD*. Within the rectangle, there is a circle inscribed such that it is always in contact with the two sides of the rectangle. The circle is reserved for a fountain feature while the remaining area is for two flower beds, one on each side of the fountain.



(i) Show that the total area, A m^2, of the two flower beds is given by

$$A = 20x - \left(\frac{4+\pi}{4}\right)x^2 .$$

[3]

(ii) Given that x can vary, find the value of x which gives a stationary value of A. [3]

(iii) Find the nature of this stationary value and explain why the landscape designer might be delighted. [2]

End of Paper