2023 Sec 4 Express Additional Mathematics Paper 1 Preliminary Examinations Marking Scheme

No.	Solution	Marks	AO
1(i)	$\operatorname{cosec}\theta$ $=\dfrac{1}{\sin\theta}$ $=\dfrac{1}{\left(\dfrac{\sqrt{7}}{4}\right)}$ $=\dfrac{4}{\sqrt{7}}$	B1	AO 1
1(ii)	$\cos 30°(\tan 45° + \sin 60°)$ $=\dfrac{\sqrt{3}}{2}\left(1+\dfrac{\sqrt{3}}{2}\right)$ $=\dfrac{\sqrt{3}}{2}+\dfrac{3}{4}$ or $\dfrac{2\sqrt{3}+3}{4}$	B1 (special angle for cos 30 and sin 60) B1	AO 1
2	Sub. $y = ax - 3$ into $y = 2x^2 + 7$ $2x^2 + 7 = ax - 3$ $2x^2 - ax + 10 = 0$ Let $b^2 - 4ac < 0$, $(-a)^2 - 4(2)(10) < 0$ $a^2 - 80 < 0$ $\left(a-4\sqrt{5}\right)\left(a+4\sqrt{5}\right)<0$ or $\left(a-\sqrt{80}\right)\left(a+\sqrt{80}\right)<0$ $-4\sqrt{5} < a < 4\sqrt{5}$ or $-\sqrt{80} < a < \sqrt{80}$	M1 (Form quadratic equation) M1 (Discriminant is negative) M1 (Factorise using surds) A1	AO 1

3	$y = Ae^{\frac{1}{2}x} + Be^{-\frac{1}{2}x}$ $\frac{dy}{dx} = \frac{1}{2}Ae^{\frac{1}{2}x} - \frac{1}{2}Be^{-\frac{1}{2}x}$ $\frac{dy}{dx} + \frac{3}{2}y = 2e^{\frac{1}{2}x} - 5e^{-\frac{1}{2}x}$ $\frac{3}{2}\left(Ae^{\frac{1}{2}x} + Be^{-\frac{1}{2}x}\right) = 2e^{\frac{1}{2}x} - 5e^{-\frac{1}{2}x} - \frac{1}{2}Ae^{\frac{1}{2}x} + \frac{1}{2}Be^{-\frac{1}{2}x}$ $\frac{3}{2}Ae^{\frac{1}{2}x} + \frac{3}{2}Be^{-\frac{1}{2}x} = \left(2 - \frac{1}{2}A\right)e^{\frac{1}{2}x} + \left(-5 + \frac{1}{2}B\right)e^{-\frac{1}{2}x}$ By comparing coefficients, $\frac{3}{2}A = 2 - \frac{1}{2}A$ $2A = 2$ $A = 1$ $\frac{3}{2}B = -5 + \frac{1}{2}B$ $B = -5$	B1 (Differentiate y correctly) M1 M1 (Compare coeff. for A or B correctly. FT from previous M1) A1 A1	AO 2
4	$V = \frac{3}{2}\left(h^2 + 8h\right)$ Sub. $V = 13.5$, $13.5 = \frac{3}{2}\left(h^2 + 8h\right)$ $3h^2 + 24h - 27 = 0$ $(3h + 27)(h - 1) = 0$ $h = -9$ (reject) or 1 $\frac{dV}{dh} = \frac{3}{2}\left(2h + 8\right)$ $= 3h + 12$ Sub. $h = 1$, $\frac{dV}{dh} = 3(1) + 12$ $= 15$ $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ $= \frac{1}{15} \times -8$ $= -\frac{8}{15}$ cm/s or -0.533 cm/s (to 3 s.f.)	M1 (Simplify to get quadratic equation) A1 B1 (Differentiate correctly) M1 (Substitute into Chain Rule. FT for dV/dh.) A1	AO 2

5(i)	$A = A_0 e^{kt}$ $0.841A_0 = A_0 e^{k(400)}$ $400k = \ln 0.841$ $k \simeq -0.00043290$ $= -0.000433$ (to 3 s.f.) (Shown)	M1 (Sub. A correctly) AG1	AO 3
5(ii)	$0.5A_0 = A_0 e^{-0.00043290t}$ $-0.00043290t = \ln 0.5$ $t = \frac{\ln 0.5}{-0.00043290}$ $\simeq 1601.2$ = 1601 years (to nearest whole number) OR $0.5A_0 = A_0 e^{-0.000433t}$ $-0.000433t = \ln 0.5$ $t = \frac{\ln 0.5}{-0.000433}$ $\simeq 1600.8$ = 1601 years (to nearest whole number)	M1 (Sub. A correctly) A1	AO 2
5(iii)	$A = 100e^{-0.00043290(3200)}$ $\simeq 25.025$ = 25.0 grams (to 3 s.f.) OR $A = 100e^{-0.000433(3200)}$ $\simeq 25.017$ = 25.0 grams (to 3 s.f.)	M1 (Substitute correctly) A1	AO 2
6(i)	$\angle DAR = \angle ABD$ (alternate segment theorem) $\angle CBD = \angle ABD$ (BSD bisects angle ABC) $\angle CBD = \angle CAD$ (angles in the same segment) $\therefore \angle DAR = \angle CAD$ (proven)	B1 (2 statements correct) B1 (3 statements correct) AG1	AO 3
6(ii)	$\angle ARD = \angle ARC$ (common angle) $\angle RAD = \angle RCA$ (Alternate segment theorem) $\therefore \Delta RAD$ is similar to ΔRCA (AA similarity or 2 pairs of corresponding angles are equal)	M1 (Both statements correct) AG1 (Similarity test must be stated)	AO 3

6(iii)	$\frac{RA}{RC}=\frac{RD}{RA}$ $\therefore RA^2 = RC\times RD$ (proven)	Form proportional ratios and conclude AG1	AO 3
7(i)	For $\left(x^3-\frac{1}{x^2}\right)^8$, $T_{r+1}=\binom{8}{r}\left(x^3\right)^{8-r}\left(-\frac{1}{x^2}\right)^r$ $=\binom{8}{r}(-1)^r x^{24-5r}$ For constant term, $24-5r=0$ $r=4.8$ (N.A.) Hence, there is no constant term because *r* must be a positive integer/whole number.	M1 (Form r +1 term) AG1 (Show that power of x is not 0 and conclude accordingly)	AO 3
7(ii)	For $\left(x^3-\frac{1}{x^2}\right)^8\left(1+x^5\right)$, $24-5r=-6$ $r=\frac{30}{5}$ $=6$ and $24-5r+5=-6$ $r=\frac{35}{5}$ $=7$ $\binom{8}{6}(-1)^6 x^{-6}(1)+\binom{8}{7}(-1)^7 x^{-11}\left(x^5\right)$ $=(28-8)x^{-6}$ $=20x^{-6}$ Hence the coefficient of x^{-6} is 20 (Shown)	M1 M1 M1 AG1	AO 3

8(i)	$2x^2-4x+9$ $=2(x^2-2x)+9$ $=2\left(x^2-2x+1-1\right)+9$ $=2\left[(x-1)^2-1\right]+9$ $=2(x-1)^2-2+9$ $=2(x-1)^2+7$ Stationary point is (1, 7).	B1(completed sq) B1	AO 1
8(ii)	Sub. $y=3x+3$ into $y=2x^2-4x+9$: $2x^2-4x+9=3x+3$ $2x^2-7x+6=0$ $(2x-3)(x-2)=0$ $x=1.5$ or $x=2$ $y=7.5$ $\quad y=9$ $AB=\sqrt{(2-1.5)^2+(9-7.5)^2}$ $=\sqrt{\frac{5}{2}}$ or $\sqrt{2.5}$ $\therefore h=\frac{5}{2}$ or 2.5	M1 (Equate and factorise) A1 (For both coordinates) M1 (Apply distance formula) A1	AO 2
9(i)	$y=x-\frac{2x+1}{1-2x}$ Since $1-2x\neq 0$ $x\neq\frac{1}{2}$ y is not defined at $x=\frac{1}{2}$.	B1	AO 1

9(ii)	$y = x - \frac{2x+1}{1-2x}$ $\frac{dy}{dx} = 1 - \frac{(1-2x)(2)-(2x+1)(-2)}{(1-2x)^2}$ $= 1 - \frac{2-4x+4x+2}{(1-2x)^2}$ $= 1 - \frac{4}{(1-2x)^2}$ or $\frac{(2x+1)(2x-3)}{(1-2x)^2}$ or $1 - \frac{4x+2}{(1-2x)^2} + \frac{2}{1-2x}$ $\frac{d^2y}{dx^2} = 8(1-2x)^{-3}(-2)$ $= -\frac{16}{(1-2x)^3}$ or $\frac{16}{(2x-1)^3}$	[Quotient Rule M1 –correct] M2 – *dy/dx* fully correct] A1 B1	AO 1
9(iii)	For stationary points, $\frac{dy}{dx} = 0.$ $1 - \frac{4}{(1-2x)^2} = 0$ $(1-2x)^2 = 4$ or $4x^2 - 4x - 3 = 0$ $1-2x = \pm 2$ $(2x-3)(2x+1) = 0$ $x = \frac{3}{2}, \quad -\frac{1}{2}$ $y = \frac{7}{2}, \quad -\frac{1}{2}$ Coordinates of stationary points are $\left(\frac{3}{2}, \frac{7}{2}\right)$ and $\left(-\frac{1}{2}, -\frac{1}{2}\right)$ (or equivalent)	M1 FT (Equate derivative to 0) A1, A1	AO 1
9(iv)	$\frac{d^2y}{dx^2} = -\frac{16}{(1-2x)^3}$ Sub. $x = \frac{3}{2}$, $\frac{d^2y}{dx^2} = 2 > 0$ Sub. $x = -\frac{1}{2}$, $\frac{d^2y}{dx^2} = -2 < 0$ Hence, *y* has a <u>minimum point</u> at $x = \frac{3}{2}$ and a <u>maximum point</u> at $x = -\frac{1}{2}$.	<u>Note: By 2nd derivative test only</u> M1 FT (Find second derivative <u>value</u> for either point) A1 A1	AO 1

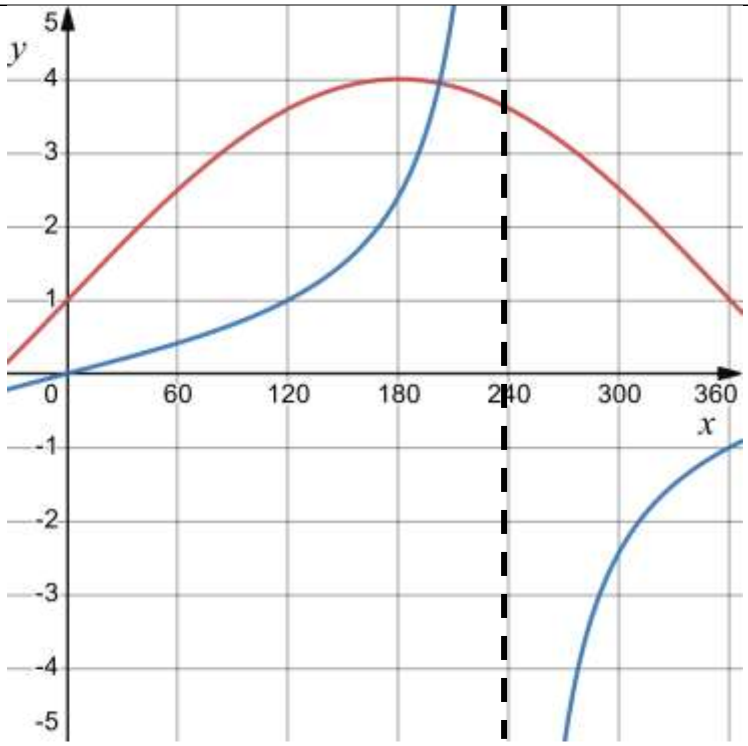
10(i)	$m_{BD} = \frac{-3-0}{-3-1.5}$ $= \frac{2}{3}$ Equation of *BD* is $y-(-3) = \frac{2}{3}(x-(-3))$ $y+3 = \frac{2}{3}x+2$ $y = \frac{2}{3}x-1$ or $3y = 2x-3$ Equation of *AD*: $y = -\frac{1}{2}x+\frac{5}{2}$ $\frac{2}{3}x-1 = -\frac{1}{2}x+\frac{5}{2}$ $\frac{7}{6}x = \frac{7}{2}$ $x = 3$ $y = 1$ $\therefore$ coordinates of *D* are (3, 1).	B1 M1(FT for value of gradient) M1 (FT from equation of BD) A1 (no FT)	AO 2
10 (ii)	$m_{AC} = -\frac{3}{2}$ Equation of *AC* is $y-2 = -\frac{3}{2}(x-1)$ $y = -\frac{3}{2}x+\frac{7}{2}$ or $2y+3x = 7$	B1 (FT from gradient of BD) B1 (no FT)	AO 2
10 (iii)	Note *CD* is not parallel to the *y*-axis. Let *E* be the mid-point of *AC*. *AC*: $y = -\frac{3}{2}x+\frac{7}{2}$ ----- (1) *BD*: $y = \frac{2}{3}x-1$ ----- (2) $(1)-(2): -\frac{3}{2}x+\frac{7}{2}-\left(\frac{2}{3}x-1\right) = 0$ $-\frac{13}{6}x = -\frac{9}{2}$ $x = \frac{27}{13}$	M1 (FT from eqn. of AC and BD) OE	AO 2

	$\therefore y=\frac{2}{3}\left(\frac{27}{13}\right)-1$ $=\frac{5}{13}$ $\therefore E$ is $\left(\frac{27}{13},\frac{5}{13}\right)$. Let C be (x, y), $\left(\frac{1+x}{2},\frac{2+y}{2}\right)=\left(\frac{27}{13},\frac{5}{13}\right)$ $\frac{1+x}{2}=\frac{27}{13}$ and $\frac{2+y}{2}=\frac{5}{13}$ $x=\frac{41}{13}$ $\quad y=-\frac{16}{13}$ $\therefore C$ is $\left(\frac{41}{13},-\frac{16}{13}\right)$ Hence, area of $ABCD=\frac{1}{2}\begin{vmatrix}1 & -3 & \frac{41}{13} & 3 & 1\\ 2 & -3 & -\frac{16}{13} & 1 & 2\end{vmatrix}$ $=\frac{1}{2}\left\|-3+\frac{48}{13}+\frac{41}{13}+6-(-6-\frac{123}{13}-\frac{48}{13}+1)\right\|$ $=14\text{ units}^2$	A1 (no FT) M1 (evaluate the 'shoelace'. FT for coordinates of C and D) A1 (no FT)	
11(a)	$9^{x-1}=3^x-8$ $\frac{3^{2x}}{9}-3^x=-8$ Let 3^x be y, $\frac{y^2}{9}-y=-8$ $y^2-9y+72=0$ $y=\frac{-(-9)\pm\sqrt{(-9)^2-4(1)(72)}}{2(1)}$ OR b^2-4ac $=\frac{9\pm\sqrt{-207}}{2}$ $\quad=(-9)^2-4(1)(72)$ $\quad=-207$ OR	M1 (or equivalent method) M1 (solve for y or discriminant)	AO 3

	$\frac{1}{9}y^2 - y + 8 = 0$ $y = \frac{1 \pm \sqrt{-\frac{23}{9}}}{\frac{2}{9}}$ or $b^2 - 4ac = -\frac{23}{9}$ Since the discriminant is negative, there is <u>no solution</u> for the equation.	AG1 (mention "<u>no solution</u>")	
11 (b) (i)	Since $\angle XYZ = 90°$ (angle in a semi-circle), by Pythagoras' Theorem, $XZ^2 = \left(\sqrt{50}+\sqrt{2}\right)^2 + \left(\sqrt{28}-\sqrt{2}\right)^2$ $= 50 + 2\left(5\sqrt{2}\right)\left(\sqrt{2}\right) + 2 + 28 - 2\left(2\sqrt{7}\right)\left(\sqrt{2}\right) + 2$ $= 102 - 4\sqrt{14}$ (Shown)	B1 (state circle property for angle XYZ) M1 (apply Pythagoras' Theorem) AG0	AO 3
11 (b) (ii)	Gradient = $\tan \angle YXZ$ $= \frac{\sqrt{28}-\sqrt{2}}{\sqrt{50}+\sqrt{2}}$ $= \frac{2\sqrt{7}-\sqrt{2}}{5\sqrt{2}+\sqrt{2}} \times \frac{5\sqrt{2}-\sqrt{2}}{5\sqrt{2}-\sqrt{2}}$ $= \frac{10\sqrt{14}-2\sqrt{14}-10+2}{(25)(2)-2}$ $= \frac{8\sqrt{14}-8}{48}$ $= \frac{\sqrt{14}}{6} - \frac{1}{6}$	B1 (tangent ratio) M1 (rationalise denominator) A1	AO 2

12(i)	$v = pt - qt^2$ Sub. $v = 48$ when $t = 2$, $p(2) - q(2)^2 = 48$ $2p - 4q = 48$ $p - 2q = 24$ ---------(1) $\frac{dv}{dt} = p - 2qt$ At max. speed, $a = 0$ when $t = 2$, $p - 2q(2) = 0$ $p - 4q = 0$ -----------(2) $(1) - (2)$: $2q = 24$ $q = 12$ Sub. $q = 12$ into (1): $p - 2(12) = 24$ $p = 48$ $\therefore$ $p = 48$ and $q = 12$ (Shown)	M1 (Form equation 1) M1 (Differentiate v) M1 (Form equation 2) M1 (solve simultaneously) AG1	AO 3
12 (ii)	At instantaneous rest, $v = 0$, $48t - 12t^2 = 0$ $12t(4 - t) = 0$ $t = 0$ or $t = 4$ The <u>giraffe changes direction/ moves in the opposite direction at $t = 4$</u>. So the total distance travelled by the giraffe in the interval $t = 0$ to $t = 7$ is not obtained by finding the value of s when $t = 7$.	B1 (Mention underlined phrase)	AO 3

12 (iii)	$s = \int (48t - 12t^2)dt$ $= \frac{48t^2}{2} - \frac{12t^3}{3} + c$ $= 24t^2 - 4t^3 + c$ When $t = 0$, $s = 0$, $\therefore c = 0$. $\therefore s = 24t^2 - 4t^3$ When $t = 4$, $s = 24(4)^2 - 4(4)^3$ $= 128$ cm When $t = 7$, $s = 24(7)^2 - 4(7)^3$ $= -196$ cm Total distance travelled = $128 + 128 + 196$ = 452 cm	M1 (Integrate with + c) A1 (Find value of c) M1 (FT from s obtained) M1 (FT from s obtained) A1	AO 1
13(i) (a)	$\mathrm{f}(x) = 3\sin\left(\frac{x}{2}\right) + 1$ Greatest value = 3 + 1 = 4 Least value $= -3 + 1 = -2$	B1 B1	AO 1
13(i) (b)	Period = $\frac{360°}{1/2} = 720°$ or 4π	B1	AO 1
13 (ii)	$\mathrm{g}(x) = \tan ax$ $a = \frac{180}{480}$ $= \frac{3}{8}$	B1	AO 1

13 (iii)		For $f(x) = 3\sin\left(\frac{x}{2}\right) + 1$, S1 (shape correct and above x-axis) P1 (passes through (0,1), (180,4), (360,1) (FT from (i)(a) greatest value) For $g(x) = \tan\left(\frac{3x}{8}\right)$, S1 (shape correct and on both sides of the asymptote $x = 240°$) P1 (passes through $(0,0)$ and symptote labelled at 240°)	AO 1
13 (iv)	1 solution	B1	AO 1