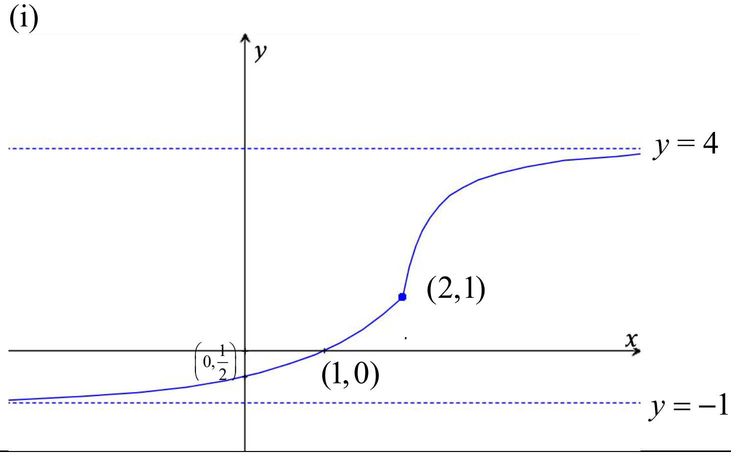
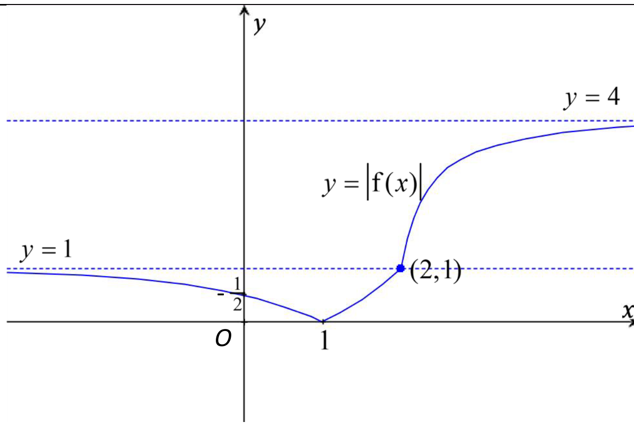
2023 H2 Math Prelim Paper 1 Solutions

No	Solutions	
1	$f(x)=ax^3+bx^2+cx+d$ $f(-1)=-a+b-c+d=-15 \quad -(1)$ $f(2)=8a+4b+2c+d=3 \quad -(2)$ $f'(x)=3ax^2+2bx+c$ $f'(1)=3a+2b+c=0 \quad -(3)$ $\int_0^2 f(x)\ dx=6 \Rightarrow \int_0^2 (ax^3+bx^2+cx+d)\ dx=5$ $\left[\frac{1}{4}ax^4+\frac{1}{3}bx^3+\frac{1}{2}cx^2+dx\right]_0^2=5$ $4a+\frac{8}{3}b+2c+2d=5 \quad -(4)$ Using GC to solve (1), (2), (3), (4) $a=1.5,\ b=-6,\ c=7.5,\ d=0$	
2(a)	$u_n=S_n-S_{n-1}$ $=\left(n^2+n\right)-\left[\left(n-1\right)^2+\left(n-1\right)\right]$ $=\left(n^2-(n-1)^2\right)+\left(n-(n-1)\right)$ $=\left(n+(n-1)\right)(n-(n-1))+1$ $=2n-1+1$ $=2n$ The general term $u_n=2n$ $u_n-u_{n-1}=2n-\left(2(n-1)\right)$ $=2\ \left(\text{constant}\right)$ Since u_n-u_{n-1} is a constant independent of n, hence $\{u_n\}$ forms a GP.	
2(b)	Let a denote the first term of the geometric progression. Let b and d denote the first term and common difference of the arithmetic progression. $\therefore \quad ar^2=b+6d \quad \ldots(1)$ $ar^4=b+12d \quad \ldots(2)$ $ar^6=b+24d \quad \ldots(3)$ (2) – (1): $ar^4-ar^2=6d \quad \ldots(4)$ (3) – (2): $ar^6-ar^4=12d \quad \ldots(5)$ (4)/(5): $\frac{ar^2\left(r^2-1\right)}{ar^4\left(r^2-1\right)}=\frac{6d}{12d}$	

	$\frac{r^2}{r^4}=\frac{1}{2}$ $\frac{1}{r^2}=\frac{1}{2}$ $r=\pm\sqrt{2}$ Since $r>0,\ r=\sqrt{2}$ Since $\lvert r\rvert>1$, the geometric progression is not convergent.	
3(i)	Let the height of the isosceles triangle be a cm. $a^2+\frac{x^2}{4}=k^2$ $a^2=k^2-\frac{x^2}{4}$ Therefore, height of the pyramid $=\sqrt{k^2-\frac{x^2}{4}-\frac{x^2}{4}}$ $=\sqrt{k^2-\frac{x^2}{2}}$ Volume of pyramid, V $=\frac{1}{3}\times\text{base area}\times\text{height}$ $=\frac{1}{3}x(x)\sqrt{k^2-\frac{x^2}{2}}$ $=\frac{x^2}{3}\sqrt{k^2-\frac{x^2}{2}}$ Hence $V^2=\frac{x^4}{9}\left(k^2-\frac{x^2}{2}\right)$.	
(ii)	$V^2=\frac{x^4}{9}\left(k^2-\frac{x^2}{2}\right)=\frac{k^2x^4}{9}-\frac{x^6}{18}$ Differentiating with respect to x, $2V\frac{\mathrm{d}V}{\mathrm{d}x}=\frac{4k^2x^3}{9}-\frac{6x^5}{18}=\frac{4k^2x^3}{9}-\frac{3x^5}{9}=\frac{1}{9}x^3\left(4k^2-3x^2\right)$ When $\frac{\mathrm{d}V}{\mathrm{d}x}=0$,	

	$\frac{1}{9}x^3\left(4k^2-3x^2\right)=0$ Since $x\neq 0$, $4k^2-3x^2=0$ $x=\frac{2\sqrt{3}}{3}k$ or $x=-\frac{2\sqrt{3}}{3}k$ (rejected $\because x>0$) $2V\frac{\mathrm{d}V}{\mathrm{d}x}=\frac{4k^2x^3}{9}-\frac{3x^5}{9}$ Differentiating with respect to *x*, $2\left(V\frac{\mathrm{d}^2V}{\mathrm{d}x^2}+\left(\frac{\mathrm{d}V}{\mathrm{d}x}\right)^2\right)=\frac{12k^2x^2}{9}-\frac{15x^4}{9}=\frac{1}{3}x^2\left(4k^2-5x^2\right)$ When $x=\frac{2\sqrt{3}}{3}k$, $\frac{\mathrm{d}V}{\mathrm{d}x}=0$, $2\left(V\frac{\mathrm{d}^2V}{\mathrm{d}x^2}\right)=\frac{1}{3}\left(\frac{4}{3}k^2\right)\left(4k^2-5\left(\frac{4}{3}k^2\right)\right)=\left(\frac{4}{9}k^2\right)\left(-\frac{8}{3}k^2\right)$ $\frac{\mathrm{d}^2V}{\mathrm{d}x^2}=-\frac{1}{V}\frac{16k^2}{27}$ Since $V>0$, then $\frac{\mathrm{d}^2V}{\mathrm{d}x^2}<0$ when $x=\frac{2\sqrt{3}}{3}k$. Therefore $x=\frac{2\sqrt{3}}{3}k$ will maximise the volume of the pyramid.	
4	(i)  	
	(ii)	



maximum a =1

(iii)

$x<1 \Rightarrow \mathrm{g}(x) = -\left[\frac{1}{2}\left(2^x\right)-1\right]$

$\mathrm{g}(x) = 1-\frac{1}{2}\left(2^x\right)$

Let $y = 1-\frac{1}{2}\left(2^x\right)$

$2^x = 2(1-y)$

$x = \log_2\left(2(1-y)\right)$

$x = 1+\log_2(1-y)$

Since $x = g^{-1}(y)$,

$\therefore \mathrm{g}^{-1}(y) = 1+\log_2(1-y)$

$\therefore \mathrm{g}^{-1}(x) = 1+\log_2(1-x)$

$D_{g^{-1}} = R_g = (0,1)$

(iv)

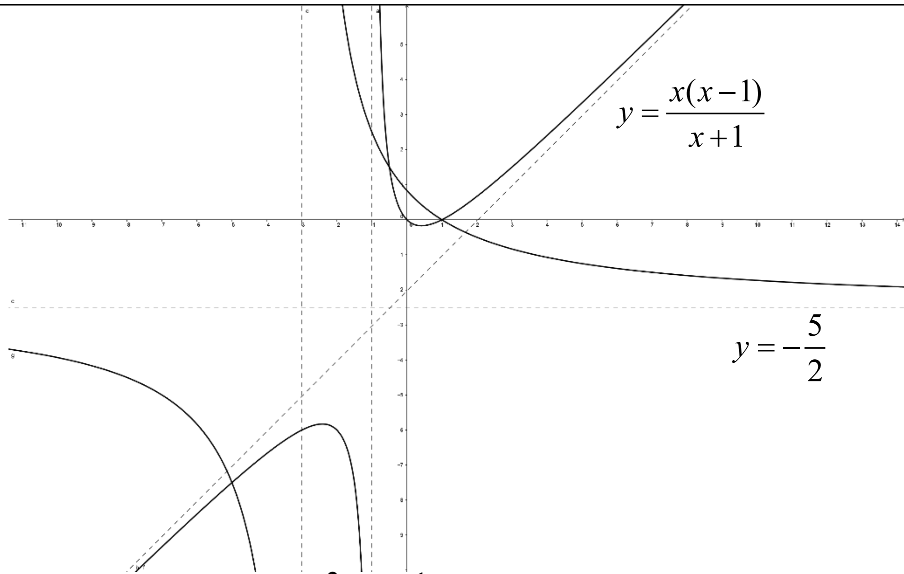
$\mathrm{f}(x) = 2$

$\Rightarrow 4-\frac{3}{3x-5} = 2$

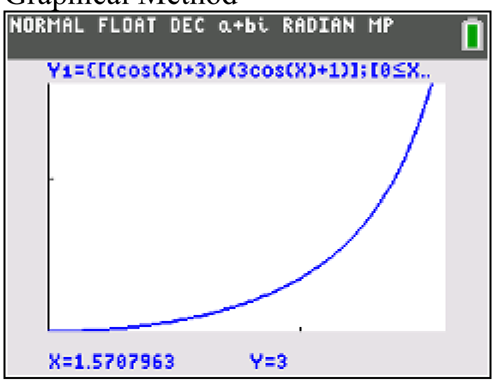
$\Rightarrow x = \frac{13}{6}$

	$$\mathrm{f}^2(x)=\begin{cases}\frac{1}{2}\left(2^{\frac{1}{2}\left(2^x\right)-1}\right)-1 & \text{for } x\in\mathbb{R},\ x<2,\\ \frac{1}{2}\left(2^{4-\frac{3}{3x-5}}\right)-1 & \text{for } x\in\mathbb{R},\ 2\le x<\frac{13}{6},\\ 4-\frac{3}{3\left(4-\frac{3}{3x-5}\right)-5} & \text{for } x\in\mathbb{R},\ x\ge\frac{13}{6}.\end{cases}$$ $$=\begin{cases}2^{2^x-2}-1 & \text{for } x\in\mathbb{R},\ x<2,\\ 2^{3-\frac{3}{3x-5}}-1 & \text{for } x\in\mathbb{R},\ 2\le x<\frac{13}{6},\\ 4-\frac{9x-15}{21x-44} & \text{for } x\in\mathbb{R},\ x\ge\frac{13}{6}.\end{cases}$$	
5	$(x-4)^2+y^2=9$ $(x-4)^2=9-y^2$ $x-4=\pm\sqrt{9-y^2}$ $x=4\pm\sqrt{9-y^2}$ Since $x<4$, $x=4-\sqrt{9-y^2}$ $y=-\sqrt{5}\ x+3\sqrt{5}$ $-\sqrt{5}\ x=y-3\sqrt{5}$ $x=\dfrac{y-3\sqrt{5}}{-\sqrt{5}}$ $V=\pi\int_0^{\sqrt{5}}\left(\frac{y-3\sqrt{5}}{-\sqrt{5}}\right)^2-\left(4-\sqrt{9-y^2}\right)^2\,\mathrm{d}y$ $=\pi\int_0^{\sqrt{5}}\frac{\left(y-3\sqrt{5}\right)^2}{5}-\left[16-8\sqrt{9-y^2}+(9-y^2)\right]\mathrm{d}y$ $=\pi\int_0^{\sqrt{5}}\frac{\left(y-3\sqrt{5}\right)^2}{5}-\left(25-y^2-8\sqrt{9-y^2}\right)\mathrm{d}y$ Using GC: $V=31.899$ units^3 (correct to 3 d.p.)	
6(i)	Consider $y=k$, k is a constant	

	$\frac{x(x-a)}{x+a}=k$ $x^2-ax=xk+ak$ $x^2-(a+k)x-ak=0$ For the range of y can take, the line $y=k$ and the curve C should have point(s) of intersection. $(a+k)^2+4ak\geq 0$ $a^2+2ak+k^2+4ak\geq 0$ $a^2+6ak+k^2\geq 0$ Consider $k^2+6ak+a^2=0$ $k=\frac{-6a\pm\sqrt{36a^2-4a^2}}{2}=\frac{-6a\pm\sqrt{32a^2}}{2}=\left(-3\pm 2\sqrt{2}\right)a$ $\therefore k\geq\left(-3+2\sqrt{2}\right)a$ or $k\leq\left(-3-2\sqrt{2}\right)a$ Hence, $y\geq\left(-3+2\sqrt{2}\right)a$ or $y\leq\left(-3-2\sqrt{2}\right)a$	
(ii)	$y=\frac{x(x-1)}{x+1}$ and $y=-\frac{5}{2}+\frac{10}{x+3}$ $\frac{x(x-1)}{x+1}=-\frac{5}{2}+\frac{10}{x+3}$ $2x(x-1)(x+3)=(-5(x+3)+20)(x+1)$ $2x(x-1)(x+3)=(5-5x)(x+1)$ $2x(x-1)(x+3)=-5(x-1)(x+1)$ $(x-1)(2x(x+3)+5(x+1))=0$ $(x-1)(2x^2+11x+5)=0$ $(x-1)(2x+1)(x+5)=0$ $x=1$ or $x=-\frac{1}{2}$ or $x=-5$	
(iii)	$y=\frac{x(x-1)}{x+1}$ Sketch the curve $y=-\frac{5}{2}+\frac{10}{x+3}$ Coordinates of intersection $\left(-\frac{1}{2},\frac{3}{2}\right)$ and $\left(1,0\right)$ and (-5, -15/2)	

	 Note: the minimum and maximum y values can be found from (i): $(0.414,\ 2\sqrt{2}-3)$ and $(-2.41,\ -2\sqrt{2}-3)$	
(iii)	Area $A=\int_{-\frac{1}{2}}^{1}\left(\left(-\frac{5}{2}+\frac{10}{x+3}\right)-\frac{x(x-1)}{x+1}\right)\mathrm{d}x$ $=\int_{-\frac{1}{2}}^{1}\left(-\frac{5}{2}+\frac{10}{x+3}-(x-2)-\frac{2}{x+1}\right)\mathrm{d}x$ $=\int_{-\frac{1}{2}}^{1}\left(-\frac{1}{2}-x-\frac{2}{x+1}+\frac{10}{x+3}\right)\mathrm{d}x$ $=\left[-\frac{1}{2}x-\frac{x^2}{2}-2\ln\lvert x+1\rvert+10\ln\lvert x+3\rvert\right]_{-\frac{1}{2}}^{1}$ $=-\frac{1}{2}-\frac{1^2}{2}-2\ln\lvert 2\rvert+10\ln\lvert 4\rvert-\left(\frac{1}{4}-\frac{\left(-\frac{1}{2}\right)^2}{2}-2\ln\left\lvert\frac{1}{2}\right\rvert+10\ln\left\lvert\frac{5}{2}\right\rvert\right)$ $=-\frac{9}{8}-2\ln 2+20\ln 2-2\ln 2-10\ln 5+10\ln 2$ $=26\ln 2-10\ln 5-\frac{9}{8}$ $a=26,\ b=-10,\ c=-\frac{9}{8}$	

7(i)	Since Q lies on the line passing through OB, OQ is parallel to OB. Hence Q has position vector in the form $\lambda\mathbf{b}$ where λ is a real constant. [OR] Equation of line OB: $\mathbf{r} = \mathbf{0} + \lambda\mathbf{b},\ \lambda \in \mathbb{R}$ $\Rightarrow \mathbf{r} = \lambda\mathbf{b}$ Since Q lies on the line, it has position vector in the form $\lambda\mathbf{b}$ where λ is a real constant.	
(ii)	 By Ratio Theorem, $\overrightarrow{OP} = \frac{1}{4}(3\mathbf{a} + \mathbf{b})$ $\overrightarrow{OQ} = \lambda\mathbf{b}$ $\overrightarrow{AQ} = \lambda\mathbf{b} - \mathbf{a}$ $AQ \perp OP$ $\Rightarrow \frac{1}{4}(3\mathbf{a} + \mathbf{b}) \cdot (\lambda\mathbf{b} - \mathbf{a}) = 0$ $3\lambda\mathbf{a} \cdot \mathbf{b} - 3\mathbf{a} \cdot \mathbf{a} + \lambda\mathbf{b} \cdot \mathbf{b} - \mathbf{a} \cdot \mathbf{b} = 0$ $3\lambda\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta - 3\lvert\mathbf{a}\rvert^2 + \lambda\lvert\mathbf{b}\rvert^2 - \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta = 0$ $3\lambda\cos\theta - 3 + \lambda - \cos\theta = 0$ $(3\cos\theta + 1)\lambda = 3 + \cos\theta$ $\lambda = \frac{3 + \cos\theta}{3\cos\theta + 1}$	
(iii)	Analytical method $\lambda = \frac{3 + \cos\theta}{3\cos\theta + 1} = \frac{1}{3} + \frac{\frac{8}{3}}{3\cos\theta + 1}$	

	$0<\theta<\frac{\pi}{2}$ $0<\cos\theta<1$ $0<3\cos\theta<3$ $1<3\cos\theta+1<4$ $\frac{1}{4}<\frac{1}{3\cos\theta+1}<1$ $\frac{2}{3}<\frac{\frac{8}{3}}{3\cos\theta+1}<\frac{8}{3}$ $1<\frac{1}{3}+\frac{\frac{8}{3}}{3\cos\theta+1}<3$ $1<\lambda<3$ From GC. Graphical Method  $\therefore 1<\lambda<3$ *Q* lies on *OB* produced. Hence, the point *Q* does not lie between *O* and *B*.	
8 (i)	$w=\frac{i-1}{i+i}=\frac{i-1}{2i}=\frac{1}{2}+\frac{1}{2}i$ $\lvert w\rvert=\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2}=\frac{1}{\sqrt{2}}$ $\arg w=\frac{\pi}{4}$ $w=\frac{1}{\sqrt{2}}e^{i\frac{\pi}{4}}$	

	$w^{14} = \left(\frac{1}{\sqrt{2}}\right)^{14}\left(e^{i\frac{14\pi}{4}}\right)$ $= \frac{1}{2^7}\left(e^{i\frac{7\pi}{2}}\right)$ $= \frac{1}{2^7}\left(e^{-i\frac{\pi}{2}}\right)$ $= -\frac{\mathrm{i}}{128}$	
(ii)	$w = \frac{\cos\theta + \mathrm{i}\sin\theta - 1}{\cos\theta + \mathrm{i}\sin\theta + \mathrm{i}}$ $= \frac{e^{i\theta} - 1}{e^{i\theta} + e^{i\left(\frac{\pi}{2}\right)}}$ $= \frac{e^{i\left(\frac{\theta}{2}\right)}\left(e^{i\left(\frac{\theta}{2}\right)} - e^{-i\left(\frac{\theta}{2}\right)}\right)}{e^{i\left(\frac{\theta}{2}+\frac{\pi}{4}\right)}\left(e^{i\left(\frac{\theta}{2}-\frac{\pi}{4}\right)} + e^{-i\left(\frac{\theta}{2}-\frac{\pi}{4}\right)}\right)}$ $= e^{-i\left(\frac{\pi}{4}\right)}\frac{2i\sin\left(\frac{\theta}{2}\right)}{2\cos\left(\frac{\theta}{2}-\frac{\pi}{4}\right)}$ $= e^{i\left(\frac{\pi}{2}-\frac{\pi}{4}\right)}\frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{\pi}{4}\right)+\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{\pi}{4}\right)}$ $= e^{i\left(\frac{\pi}{4}\right)}\frac{\sin\left(\frac{\theta}{2}\right)}{\frac{1}{\sqrt{2}}\left(\cos\left(\frac{\theta}{2}\right)+\sin\left(\frac{\theta}{2}\right)\right)}$ $= \frac{\sqrt{2}e^{i\left(\frac{\pi}{4}\right)}}{\cot\left(\frac{\theta}{2}\right)+1}$ $k = \sqrt{2}e^{i\left(\frac{\pi}{4}\right)}$	

9(i)	$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=\begin{pmatrix}-4\\0\\-2\end{pmatrix}$ $\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=\begin{pmatrix}-2\\4\\1\end{pmatrix}$ $\overrightarrow{AB}\times\overrightarrow{AC}=\begin{pmatrix}8\\8\\-16\end{pmatrix}$ Take the normal vector to the plane as $\begin{pmatrix}1\\1\\-2\end{pmatrix}$. Equation of the plane *ABDC* is given by $\underset{\sim}{r}\bullet\begin{pmatrix}1\\1\\-2\end{pmatrix}=\begin{pmatrix}2\\0\\3\end{pmatrix}\bullet\begin{pmatrix}1\\1\\-2\end{pmatrix}=-4$ The cartesian equation of the plane *ABDC* is $x+y-2z=-4.$ (Ans)	
(ii)	Let the acute angle be θ . $\cos\theta=\dfrac{\left\|\begin{pmatrix}1\\1\\-2\end{pmatrix}\bullet\begin{pmatrix}0\\0\\1\end{pmatrix}\right\|}{\sqrt{6}}=\dfrac{2}{\sqrt{6}}=\dfrac{\sqrt{6}}{3}$ $\theta=35.3^o$ (1 dec place) or 0.615 radians	
(iii)	Since *D* lies on plane *ABDC*, from (i) $x+y-2z=-4$ Substitute $D(-2,4,k)$, $-2+4-2k=-4$ $\Rightarrow -2k=-6$ $k=3$ (Ans)	

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$\overrightarrow{BD}=\begin{pmatrix}0\\4\\2\end{pmatrix}, \overrightarrow{AC}=\begin{pmatrix}-2\\4\\1\end{pmatrix}$

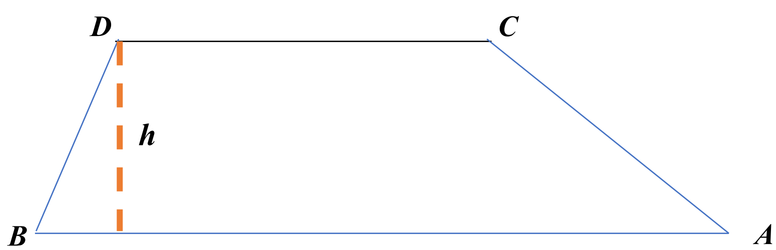
Since $\overrightarrow{BD}$ cannot be expressed as $\overrightarrow{BD}=k\overrightarrow{AC}$, where k is a constant, hence $\overrightarrow{BD}$ and $\overrightarrow{AC}$ are NOT parallel.

$\overrightarrow{AB}=\begin{pmatrix}-4\\0\\-2\end{pmatrix}, \overrightarrow{CD}=\begin{pmatrix}-2\\0\\-1\end{pmatrix}$

$\overrightarrow{AB}=2\overrightarrow{CD}$

Hence $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel.

Since $ABDC$ has one pair of parallel sides and one pair of non-parallel sides, hence $ABDC$ is a trapezium.



To find height of the trapezium $ABDC$, we use

$h=\left|\overrightarrow{BD}\times\frac{\overrightarrow{BA}}{\left|\overrightarrow{BA}\right|}\right|=\frac{\left|\begin{pmatrix}0\\4\\2\end{pmatrix}\times\begin{pmatrix}4\\0\\2\end{pmatrix}\right|}{\sqrt{20}}=\left|\frac{1}{\sqrt{20}}\begin{pmatrix}8\\8\\-16\end{pmatrix}\right|=\frac{4}{5}\sqrt{30}$

$CD=\sqrt{5}$

$AB=2\sqrt{5}$

Area of trapezium $ABCD$

$=\frac{1}{2}\left(CD+AB\right)h$

$=\frac{1}{2}(3\sqrt{5})\left(\frac{4\sqrt{30}}{5}\right)$

$=6\sqrt{6}=6^{\frac{3}{2}}$ units^2

$\therefore a=6$ (Ans)

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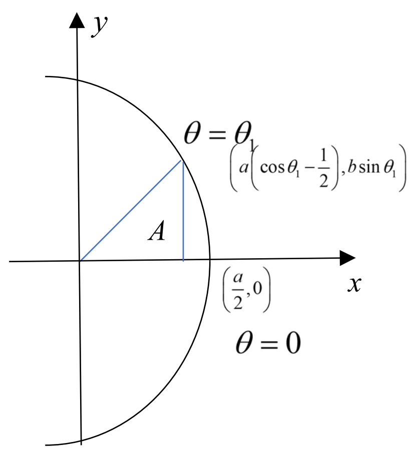
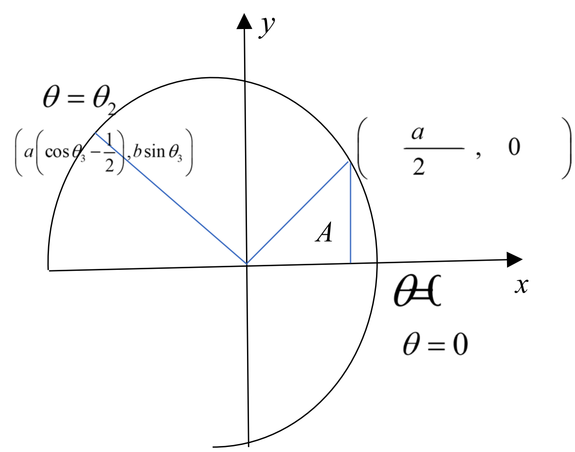
	Alternative: Trapezium *ABDC* = Area of triangle *ABD* + Area of triangle *ACD* $=\frac{1}{2}\left\|\overrightarrow{BD}\times\overrightarrow{BA}\right\|+\frac{1}{2}\left\|\overrightarrow{CD}\times\overrightarrow{CA}\right\|$ $=\frac{1}{2}\left\|\overrightarrow{BD}\times\overrightarrow{BA}\right\|+\frac{1}{2}\left\|\overrightarrow{CD}\times\overrightarrow{CA}\right\|$ $=\frac{1}{2}\left\|\begin{pmatrix}0\\4\\2\end{pmatrix}\times\begin{pmatrix}4\\0\\2\end{pmatrix}\right\|+\frac{1}{2}\left\|\begin{pmatrix}-2\\0\\-1\end{pmatrix}\times\begin{pmatrix}2\\-4\\-1\end{pmatrix}\right\|$ $=\frac{1}{2}\left\|\begin{pmatrix}8\\8\\-16\end{pmatrix}\right\|+\frac{1}{2}\left\|\begin{pmatrix}-4\\-4\\8\end{pmatrix}\right\|$ $=4\sqrt{1^2+1^2+(-2)^2}+2\sqrt{1^2+1^2+(-2)^2}$ $=6\sqrt{6}$ $=6^{\frac{3}{2}}$ units^2	
10(i)	$\frac{\mathrm{dQ_{in}}}{\mathrm{d}t}\propto Q$ $\frac{\mathrm{dQ_{in}}}{\mathrm{d}t}=\frac{a}{Q}, \qquad \frac{\mathrm{dQ}_{out}}{\mathrm{d}t}=bQ \qquad a,\ b>0.$ Rate of change of amount of glucose,	

	$\frac{\mathrm{d}Q}{\mathrm{d}t}=\frac{\mathrm{dQ_{in}}}{\mathrm{d}t}-\frac{\mathrm{dQ}_{out}}{\mathrm{d}t}$ $\frac{\mathrm{d}Q}{\mathrm{d}t}=\frac{a}{Q}-bQ$ When $Q=4, \frac{\mathrm{d}Q}{\mathrm{d}t}=0$ $\frac{a}{4}=4b$ $a=16b$ $\therefore \frac{\mathrm{d}Q}{\mathrm{d}t}=\frac{16b}{Q}-bQ$ $=b\left(\frac{16-Q^2}{Q}\right)$ $=k\left(\frac{16-Q^2}{Q}\right)$ $(shown)$ where $k=b$	

(ii)	$\dfrac{\mathrm{d}Q}{\mathrm{d}t}=k\left(\dfrac{16-Q^2}{Q}\right)$ $\int\left(\dfrac{Q}{16-Q^2}\right)\mathrm{d}Q=\int k\ \mathrm{d}t$ $-\dfrac{1}{2}\int\left(\dfrac{-2Q}{16-Q^2}\right)\mathrm{d}Q=kt+C$ where C is an arbitrary constant. $\dfrac{1}{2}\ln\left\|16-Q^2\right\|=-kt-C$ $\ln\left\|16-Q^2\right\|=-2kt-2C$ $16-Q^2=\pm\mathrm{e}^{-2kt-2C}$ $D=\pm\mathrm{e}^{-2C}$ $Q^2=16-D\mathrm{e}^{-2kt}$ When $t=0, Q=7$ $7^2=16-D$ $D=-33$ When $t=15, Q=6.8$ $(6.8)^2=16+33\mathrm{e}^{-30k}$ $0.91636=\mathrm{e}^{-30k}$ $-30k=-0.087342$ $k=0.0029114$ $Q^2=16+33\mathrm{e}^{-2(0.0029114)t}$ $A=33$ $B=-2k=-0.0058228\approx-0.00582$	
(iii)	When $t=60$, $Q^2=16+33\mathrm{e}^{-0.0058228(60)}$ Using GC, $Q=6.267\approx6.27$	
(iv)	Since the glucose level after 1 hr is 6.27 mmol/L which is not within the normal range, hence the clinical trial is not as effective as it claims.	
(v)	When $t\to\infty$, $Q^2\to16$. Hence $Q\to4$. Hence the amount of glucose in the patient's bloodstream approaches 4 mmol/L in the long run. The model might not be feasible in the long run as there may be other external factors such as consumption of food, that might affect the glucose level in the patient's bloodstream.	

11(i)	$x = a(\cos\theta - e)$ $\quad y = b\sin\theta$ $\frac{x}{a} + e = \cos\theta$ $\quad \sin\theta = \frac{y}{b}$ $\cos\theta = \frac{x+ea}{a}$ $\sin^2\theta + \cos^2\theta = 1$ $\left(\frac{x+ea}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$ $\frac{(x+ea)^2}{a^2} + \frac{y^2}{b^2} = 1$	
(ii)	 $A = \frac{1}{2}\left(a\left(\cos\theta_1 - \frac{1}{2}\right)b\sin\theta_1\right) + \int_{x=a\left(\cos\theta_1-\frac{1}{2}\right)}^{x=\frac{a}{2}} y\,\mathrm{d}x$ $= \frac{ab}{2}\sin\theta_1\cos\theta_1 - \frac{ab}{4}\sin\theta_1 + \int_{\theta=\theta_1}^{\theta=0} y\frac{\mathrm{d}x}{\mathrm{d}\theta}\mathrm{d}\theta$ $= \frac{ab}{4}2\sin\theta_1\cos\theta_1 - \frac{ab}{4}\sin\theta_1 + \int_{\theta=\theta_1}^{\theta=0} b\sin\theta(-a\sin\theta)\mathrm{d}\theta$ $= \frac{ab}{4}\sin 2\theta_1 - \frac{ab}{4}\sin\theta_1 + ab\int_{\theta=0}^{\theta=\theta_1}\sin^2\theta\,\mathrm{d}\theta$	

	$A=\frac{ab}{4}\sin 2\theta_1-\frac{ab}{4}\sin\theta_1+ab\int_{\theta=0}^{\theta=\theta_1}\sin^2\theta\,\mathrm{d}\theta$ $=\frac{ab}{4}\sin 2\theta_1-\frac{ab}{4}\sin\theta_1+\frac{ab}{2}\int_{\theta=0}^{\theta=\theta_1}(1-\cos 2\theta)\mathrm{d}\theta$ $=\frac{ab}{4}\sin 2\theta_1-\frac{ab}{4}\sin\theta_1+\frac{ab}{2}\left[\theta-\frac{\sin 2\theta}{2}\right]_0^{\theta_1}$ $=\frac{ab}{4}\sin 2\theta_1-\frac{ab}{4}\sin\theta_1+\frac{ab}{2}\left[\left(\theta_1-\frac{\sin 2\theta_1}{2}\right)-\left(0-\frac{\sin 0}{2}\right)\right]$ $=\frac{ab}{4}\sin 2\theta_1-\frac{ab}{4}\sin\theta_1+\frac{ab}{2}\theta_1-\frac{ab}{4}\sin 2\theta_1$ $=\frac{ab}{2}\theta_1-\frac{ab}{4}\sin\theta_1$	
(iii)	Area swept out by the planet between P_1 and P_2, $=\frac{ab}{2}\left(\frac{\pi}{4}\right)-\frac{ab}{4}\sin\left(\frac{\pi}{4}\right)=\frac{ab}{4}\left(\frac{\pi}{2}-\frac{1}{\sqrt{2}}\right)=\frac{ab}{8}\left(\pi-\sqrt{2}\right)$	
(iv)	 Let A_ϕ be the area swept out by the planet from P_0 to P_ϕ. $A_\pi=\frac{ab}{2}\pi-\frac{ab}{4}\sin(0)=\frac{ab}{2}\pi$ $A_{\theta_3}=\frac{ab}{2}\theta_3-\frac{ab}{4}\sin\theta_3$ Hence, Area swept out by the planet from P_3 to P_4, $A=A_\pi-A_{\theta_3}$ $=\frac{ab}{2}\pi-\left(\frac{ab}{2}\theta_3-\frac{ab}{4}\sin\theta_3\right)$ $=\frac{ab}{2}(\pi-\theta_3)+\frac{ab}{4}\sin\theta_3$	

	Since the areas swept out must be equal in the same amount of time (as given by Kepler's Second Law) $\frac{ab}{4}\left(\frac{\pi}{2}-\frac{1}{\sqrt{2}}\right)=\frac{ab}{4}\left(2\pi-2\theta_3+\sin\theta_3\right)$ $2\pi+\sin\theta_3-2\theta_3=\frac{\pi}{2}-\frac{1}{\sqrt{2}}$ $\sin\theta_3-2\theta_3=-\frac{3\pi}{2}-\frac{1}{\sqrt{2}}$ From GC, $\theta_3=2.8523=2.85$	