Chapter 4 FORCES

Content

- Types of force
- Centre of gravity
- Turning effects of forces
- Equilibrium of forces
- Upthrust

Learning Outcomes

Candidates should be able to:

(a)	recall and apply Hooke's law ($F = kx$, where k is the force constant) to new situations or to solve related problems		
(b)	describe the forces on a mass, charge and current-carrying conductor in		
(c)	show a qualitative understanding of normal contact forces, frictional forces and viscous forces including air resistance (no treatment of the coefficients of friction and viscous to required)		
(d)	show an understanding that the weight of a body may be taken as acting at a single point known as its centre of gravity		
(e)	define and apply the moment of a force and the torque of a couple		
(f)	show an understanding that a couple is a pair of forces which tends to produce rotation only		
(g)	apply the principle of moments to new situations or to solve related problems		
(h)	show an understanding that, when there is no resultant force and no resultant torque, a system is in equilibrium		
(i)	use a vector triangle to represent forces in equilibrium		
(j)	derive, from the definitions of pressure and density, the equation $p = \rho g h$		
(k)	solve problems using the equation $p = \rho g h$		
(I)	show an understanding of the origin of the upthrust acting on a body in a fluid		
(m)	state that upthrust is equal to the weight of the fluid displaced by a submerged or floating object		
(n)	calculate the upthrust in terms of the weight of the displaced fluid		
(0)	recall and apply the principle that, for an object floating in equilibrium, the upthrust is equal to the weight of the object to new situations or to solve related problems.		

Magnetic Force

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Types of Forces

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In general, forces can be categorized as either non-contact forces or contact forces.

Non-contact Forces Gravitational Force





Gravitational force is an attractive force exerted by one mass on another,

$$F = \frac{GMm}{r^2}$$

Gravitational force is a force on a mass in a gravitational field, in the direction of the field, F = mg

The weight (a.k.a. gravitational force exerted by earth on the object) of an object placed in the gravitational field of the Earth is the gravitational force exerted by the Earth on the object.

weight = mass × gravitational field strength

W = mg

The centre of gravity of a body is the point at which the entire weight (of body) appears to act.

Electric Force



Electric force is a force exerted by one charge on another.

Like charges repel, unlike charges attract.

$$F = \frac{Qq}{4\pi\varepsilon_0 r^2}$$



Electric force is a force on a charge in an electric field, in the direction of the field on a positive charge, opposite the direction of the field on a negative charge,

$$F = qE$$

Magnetic force is a force on a moving charge in a magnetic field.

$$F = Bqv$$

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Contact Forces

Normal Contact Force (a.k.a. normal reaction)

The normal contact force is the force that the one body exerts on another body that are physically in contact. It is always perpendicular (normal) to the surfaces in contact.

Frictional Force

A frictional force arises when the surfaces in contact are rough to resist motion or tendency of motion. The direction of the frictional force on a body is always in the direction opposite to its motion or impending motion, parallel to the rough surface.

Note:

- 1. Static friction is the frictional force when there is no relative motion between the two surfaces. The magnitude of static friction is self-adjusting such that it is just sufficient to prevent motion, but only up to a maximum value or the limiting static friction.
- 2. Kinetic friction is the frictional force when the two surfaces slide against each other. When the bodies just begin to move against each other, the kinetic friction between the surfaces is smaller than the static friction just before they begin to move.
- 3. Frictional force depends on the
 - (a) nature of the surface
 - (b) magnitude of the normal contact force.

Viscous Force

Viscous (drag) force is a resistive force that opposes relative motion. It is present when a body moves through a fluid (i.e. liquid or gas). Air resistance is a common example of viscous force.

Note:

- 1. The direction of viscous force is always opposite to the direction of motion of the body relative to the fluid.
- 2. Viscous force depends on the
 - (a) speed of the body moving through the fluid
 - (b) the fluid
 - (c) the shape and size of the body.





Lift

Lift is the force that acts on a body such as an airplane wing or a helicopter rotor. Lift acts perpendicular to airplane wings or helicopter rotor.

When an airplane is climbing, descending or banking in a turn, the lift is tilted with respect to the vertical.

Thrust

Thrust occurs when a system expels or accelerates mass in one direction. Thrust is the reaction force exerted by the expelled or accelerated mass on that system.

When a jet engine of an airplane expels hot gas, the jet engine exerts a force on the hot gas during the expulsion. By Newton's Third Law of Motion, the hot gas exerts a force that is equal in magnitude but opposite in direction called thrust on the jet engine, propelling the airplane forward.

 $T \longrightarrow F$ expelled hot gas



Hooke's Law

Hooke's Law states that extension (or compression) in a material is directly proportional to the applied force, provided that the limit of proportionality is not exceeded.

Mathematically,



where F is the magnitude of the applied force, x is the extension (or compression) produced, and k is a constant known as the force constant or spring constant.

Example 1

A spring obeying Hooke's Law has an unstretched (natural) length of 50 mm and a spring constant of 400 N m⁻¹. Determine the tension in the spring when its overall length is 70 mm.

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F = kx
= 400[(70-50)×10<sup>-3</sup>]
= 8.0 N
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Elastic Potential Energy

When an elastic material is stretched (or compressed), the energy used to strech the material is stored as the elastic potential energy. This elastic potential energy (strain energy) is given by the area under the force-extension graph.





Non-linear force-extension graph

For a body that **obeys Hooke's Law**, the force-extension graph will be linear and the elastic potential energy stored in the body, *E*,

E = Area under of F - x graph

$$= \frac{1}{2}Fx$$
$$= \frac{1}{2}(kx)x \quad (\because F = kx)$$
$$= \frac{1}{2}kx^{2}$$

Example 2

A sample is placed in a tensile testing machine. It is extended by known amounts and the tension is measured, as shown in the figure below.



Determine the work done on the sample when it is given a total extension of 9 mm. E_1 = Area under tension-extension graph E_2 = Area under tension-extension graph

$$= \frac{1}{2} (5.0 \times 10^{-3}) (60)$$

= 0.15 J
 $E = E_1 + E_2$
= 0.15 + 0.28
= 0.43 J

 $= \frac{1}{2} \left[(9.0 - 5.0) \times 10^{-3} \right] (60 + 80)$

$$= \frac{1}{2} \left[(9.0 - 5.0) \times 10^{-3} \right] (60 + 80)$$
$$= 0.28 \text{ J}$$

Example 3

The graph below shows the variation with extension x of the load F on a certain spring.



A load of 6.0 N is placed on the spring. Determine the additional elastic potential energy that will be stored in the spring if it is then extended further by 0.010 m.

 ΔE = area under the graph from x = 0.030 m to x = 0.040 m

 $=\frac{1}{2}(6.0+8.0)(0.040-0.030)$ =0.070 J

Fluid Pressure

Pressure is the normal force per unit area.

It is a scalar quantity and its SI unit is pascal (Pa).

Consider a point at a depth h below the surface of a fluid, the pressure due to the fluid at that point is due to the weight of the column of fluid above it.

For a plane surface of area A, the weight of the fluid column above it is given by W = mg



where ρ is the density of fluid.

Hence the fluid pressure, *p*, at this point is given by

 $\rho = \frac{F}{A}$ $= \frac{W}{A}$ $= \frac{Ah\rho g}{A}$ $= h\rho g$

The absolute pressure at this depth includes the pressure due to other fluids above it, such as that of the air i.e. atmospheric pressure p_0 ,

$$p_{absolute} = p_0 + h\rho g$$

Note:

- 1. Pressure acts in all directions, and does not depend on the area *A*.
- 2. All points at the same depth *h* within the fluid are at the same pressure.

Example 4

A long narrow tube is filled with water of density 1020 kg m⁻³ to a depth of 1.00 m. The tube is then inclined at 30° to the horizontal as shown below. If atmospheric pressure is 100 kPa, determine the pressure at point X inside the tube.

 $p_{\rm X} = p_0 + h\rho g$ = 100 × 10³ + (1.00 sin 30°)(1020)(9.81) = 1.05 × 10⁵ Pa





Lecture Notes

Example 5

A sealed U-tube contains nitrogen in one arm and helium at pressure P in the other arm. The gases are separated by mercury of density ρ with dimensions as shown in the diagram. The acceleration of free fall is g. Determine the pressure of the nitrogen.



Pressure at the same horizontal level is the same.

$$p_{N_2} = p_{He} + h_{Hg}\rho_{Hg}g$$
$$= P + x\rho g$$

Upthrust

Upthrust is the upward force exerted by a fluid on a submerged or floating object. It arises due to the difference in pressure between the upper and lower surfaces of the object.

Consider a cylindrical object of cross-sectional area A and height *h* submerged in a fluid of density ρ_r

Pressure at the upper surface of object, $p_1 = h_1 \rho_f g$

Force exerted by fluid on upper surface of object, $F_{1,}$

$$F_1 = p_1 A$$
$$= (h_1 \rho_f g) A$$

Pressure at the lower surface of object, $p_2 = h_2 \rho_f g$

Force exerted by fluid on lower surface of object, F_2 ,

$$F_2 = p_2 A$$
$$= (h_2 \rho_f g) A$$

As $h_2 > h_1$, the upward force, F_2 , is greater than the downward force, F_1 . This net upward force is called upthrust.

Upthrust exerted by fluid on object, *U*,



$$U = F_2 - F_1$$

= $h_2 \rho_f g A - h_1 \rho_f g A$
= $(h_2 - h_1) \rho_f g A$
= $h \rho_f g A$
= $V \rho_f g$ (:: $V = Ah$)

where V is the volume of object submerged in the fluid which is also equal to the volume of fluid displaced by the object, V_r . Therefore,

$$U = V_f \rho_f g$$
$$= m_f g \quad \left(\because \rho = \frac{m}{V}\right)$$

i.e. the magnitude of the upthrust exerted by a fluid on a submerged or floating object is equal to the weight of fluid displaced by the object.

Archimedes' Principle states that for an object immersed in a fluid, the magnitude of the upthrust is equal to the weight of the fluid displaced.

Note:

- 1. Archimedes' Principle can be applied to objects of any shape.
- 2. Upthrust is also known as buoyant force.
- 3. Upthrust acts at the centre of mass of the displaced fluid, known as the centre of buoyancy (the centre of buoyancy need not coincide with the centre of mass of the object).
- 4. Upthrust is not viscous force. Viscous force will only be exerted by the fluid on an object if the object moves through the fluid. On the contrary, upthrust is always present as long as the object is submerged or floating in the fluid (even when there is no relative motion between them).

For an object floating in equilibrium, the net force on the object is zero.



Principle of Flotation states that when an object floats in equilibrium, upthrust is equal to the weight of the object.

Example 6

A water droplet in a cloud is falling through air and is in equilibrium. Three forces act on it, its weight W, upthrust U and air resistance R. Which diagram, showing these three forces to scale, is correct?



As the water droplet is in equilibrium,

 $F_{net} = 0$ U + R = W $U = V \rho_{oir} q$

$$W = V \rho_{H_2O} g \qquad \left(\because V_{air} = V_{H_2O} \right)$$

Since $\rho_{air} \approx 1.2$ kg m⁻³ and $\rho_{H_2O} \approx 1000$ kg m⁻³, $W \approx 800U$ Answer: A

Example 7

A diver watches an air bubble as it rises to the surface in water. The weight of the air in the bubble is negligible. The volume and speed of ascent of the bubble increase as it rises. Two forces, upthrust and viscous drag, act on the bubble.

What changes occur in these forces as the bubble rises?

	upthrust	viscous drag
Α	decreases	decreases
В	increases	no change
С	increases	increases
D	no change	decreases

Using $U = V_{H_2O}\rho_{H_2O}g$, as volume increases while density of water, ρ_{H_2O} , and free fall acceleration, g,

remain constant, upthrust increases as the bubble rises.

Since viscous drag increases with speed, as speed and area increase, viscous drag increases. Answer: C

Example 8

The weight indicated on a balance is X when a beaker of water is placed on it. A solid object has weight Y in air and displaces water of weight Z when completely immersed. The diagram shows the object suspended from a light string and completely immersed in the beaker of water. Determine the balance reading in this arrangement.



From Archimedes' Principle, upthrust experienced by object is equal to the weight of water displaced i.e. U = Z

By N3L, the force exerted by object on water is equal and opposite to the force exerted by water on the object (upthrust).

Considering beaker of water,



Since beaker of water is in equilibrium,

 $F_{net} = 0$

N = X + Z

Example 9

A cuboid with dimensions 25 cm \times 20 cm \times 15 cm has mass 4.0 kg and is floating in water of density 1000 kg m⁻³ so that its largest faces are horizontal as shown in the figure below.



Calculate

- (i) the upthrust on the cuboid, As the cuboid is floating in equilibrium
 - $\boldsymbol{U} = \boldsymbol{W}$

= 39 N

= 4.0(9.81)



(ii) the fraction of the cuboid which is beneath the water surface,

Using
$$U = V_{f}\rho_{f}g$$
,
 $U = (Ah)\rho_{f}g$
 $h = \frac{U}{A\rho_{f}g}$
 $= \frac{39}{(25 \times 10^{-2})(20 \times 10^{-2})(1000)(9.81)}$
 $= 0.080 \text{ m}$

Fraction of cuboid beneath water surface,

 $=\frac{0.080}{15\times10^{-2}}$

- = 0.53
- (iii) the position of the centre of buoyancy. The centre of buoyancy would be situated at a position vertically halfway along the submerged section. It will be at the centre of the horizontal plane at a depth of 0.040 m, from the bottom of the cuboid.

Moment of a Force

The turning effect of a force is called its moment.

The moment of a force is the product of the force and the perpendicular distance of force from pivot.

Moment of a force is a vector, and its unit is N m.

Note: The unit for moment cannot be written as J as J is the unit for energy.





There are two ways of computing moment of a force:

Method 1: Finding perpendicular distance $M = Fd_{\perp}$ $= F(d\sin\theta)$

Method 2: Resolving force $M = F_{\perp}d$ $= (F \sin \theta) d$



Note: The component $F\cos\theta$ has no moment about pivot, since its line of action passes through the pivot.

Example 10

A uniform metre rule of weight 2.0 N is uniform metre rule

pivoted at the 60 cm mark. A 4.0 N weight is suspended from one end, causing the rule to rotate about the pivot. Determine the resultant turning

₹4.0 N

moment about the pivot, at the instant when the rule is horizontal.

$$M = 4.0 \left[(100 - 60) \times 10^{-2} \right] - 2.0 \left[(60 - 50) \times 10^{-2} \right]$$

=1.4 N m

Resultant moment is 1.4 N m, clockwise.

Example 11

A desk lamp is illustrated in Fig. A.



The lamp must be constructed so that it does not topple over when fully extended as shown in Fig. B. The base of the lamp is circular and has a radius of 10 cm. Other dimensions are shown on the above figures. The total weight of the light bulb and shade is 6.0 N and each of the two uniform arms has weight 2.0 N.

- (a) On Fig. B, draw an arrow to represent the weight of the base.
- (b) The lamp will rotate about a point if the base is not heavy enough. On Fig. B, mark this point and label it P.
- (c) Calculate the following moments about P.
 - 1. Moment of first arm

 $= 2.0 \left[(15 - 10) \times 10^{-2} \right]$ = 0.10 N m

- 2. Moment of second arm = $2.0 [(30+15-10) \times 10^{-2}]$ = 0.70 N m
- 3. Moment of light bulb and shade = $6.0 [(30+30-10) \times 10^{-2}]$ = 3.0 N m

Example 12

In which diagram is the moment of force about point P the greatest?



Torque of a Couple

A couple is a pair of equal in magnitude but opposite in direction forces whose lines of action do not coincide.

A couple produces only a turning effect and no translational effect. Diagram below shows a couple.



The torque of a couple is the product of one of the forces and the perpendicular distance between the forces.

Torque of the couple in the diagram above is Fd.

Example 13

Which is not true of the two forces that give rise to a couple?

- **A** They act in opposite directions.
- **B** They both act at the same point.
- **C** They both act on the same body.
- **D** They both have the same magnitude.

Answer: B

Example 14

A ruler of length 0.30 m is pivoted at its centre. Equal and opposite forces of magnitude 2.0 N are applied to the ends of the ruler, creating a couple as shown. Calculate the magnitude of the torque of the couple on the ruler when it is in the position shown.

$$au = Fd_{\perp}$$

 $= 2.0(0.30 \sin 50^{\circ})$

= 0.46 N m

Equilibrium

A body is in equilibrium if the

- 1. net force is zero
- 2. net moment about any point is zero.

Condition 1 ensures that the body is in **translational equilibrium**. If there are more than two forces acting on a body in equilibrium, the vector diagram will be a closed polygon.



Forces F_1 , F_2 , and F_3 act on a body, the three forces must form a closed triangle.

Condition 2 for equilibrium ensures that the body is in **rotational equilibrium**. This is also known as the Principle of Moments.

Principle of moments states that for a body in equilibrium, the sum of clockwise moments about any point is equal to the sum of anti-clockwise moments about the same point.

Note: For an object in equilibrium acted on by three non-parallel coplanar forces, the forces must pass through a common point.



2. Apply $\sum \tau = 0$ about a point in which the largest number of unknown forces passes through.

Example 15

A light rod is acted upon by three forces, P, Q and R. Which diagram shows the position and direction of each of the forces when the rod is in equilibrium?





As the rod is in equilibrium, the three forces must meet at a point. In option C, there is a net force to the left. Answer: B

Example 16

A ladder of weight W rests against a vertical wall. Friction between the ladder and the ground, and also between the ladder and the wall, prevents the ladder from slipping. Which diagram shows the directions of the forces on the ladder?



Answer: C

Example 17

An aircraft in level flight is moving with constant velocity relative to the ground. The resultant force acting on the aircraft is equal to

- Α the weight of the aircraft.
- В the resultant of the air resistance and the thrust of the engines.
- С the resultant of the air resistance and the weight of the aircraft.
- D zero.

Answer: D

Example 18

A body is acted on by two forces, P and Q. A frictional force F holds the body in equilibrium. Which vector triangle could represent the relationship between these forces?



As the body is in equilibrium, the vector diagram must a closed polygon (triangle) Answer: C

Example 19

A small ball of weight W is suspended by a light thread. When a strong wind blows horizontally, exerting a constant force F on the ball, the thread makes an angle θ to the vertical as shown.



Dividing (1) by (2), $\tan \theta = \frac{F}{W}$ Answer: C

Example 20

A student pulls a book by its top corner from a shelf, holding it between its forefinger and thumb. The book has weight W and dimensions x and y as shown.



What moment must the student provide at the corner of the book to stop it from rotating as it leaves the shelf?

Α	$\frac{vy}{2}$, anti-clockwise	В	$\frac{w_x}{2}$, anti-clockwise
С	$\frac{W(y-x)}{2}$, clockwise	D	Wx, anti-clockwise

As the book is still in equilibrium,

sum of clockwise moments about A = sum of anti-clockwise moments about A

$$M = W\left(\frac{x}{2}\right)$$
$$= \frac{Wx}{2}$$
Moment the student must provide is $\frac{Wx}{2}$, anti-clockwise.

Answer: B

Example 21

A uniform beam in a roof structure has a weight of 180 N. It is supported in two places X and Y, a distance 3.0 m apart. A load is placed on the beam a distance of 0.80 m from X. The support provided by Y is 220 N.



Determine the value of the load.

Taking moments about X,

 $F(0.80) + 180\left(\frac{3.0}{2}\right) = 220(3.0)$ F = 490 N

Example 22

A uniform cube (5.0 cm by 5.0 cm by 5.0 cm) of mass 1.0 kg rests on a rough surface. The maximum static friction between the cube and surface is 6.0 N and the kinetic friction is 4.5 N. An increasing horizontal force F (steadily increased from 0 N), is applied to the cube at a height of 4.0 cm above the surface. Determine whether the cube will topple or slide first.

When F is 6.0 N, taking moment of force about A,

$$M_F = Fd$$

$$= 6.0(4.0 \times 10^{-2})$$

= 0.24 N m



Moment of weight about A,

$$M_{W} = W\left(\frac{x}{2}\right)$$
$$= (1.0 \times 9.81) \left(\frac{5.0 \times 10^{-2}}{2}\right)$$

= 0.24525 N m

Since $M_W > M_{P}$ block does not topple. Hence, the cube will slide first.

APPENDIX: FLIPPING ICEBERGS -- Capsizing icebergs may release as much energy as a bomb

Source: https://www.sciencenewsforstudents.org/article/flipping-icebergs



Icebergs look like towering, frozen mountains that drift through water. Their peaks may soar hundreds of feet above the surface and large ones cover as much area as major cities. When one of these blocks of ice flips over, it causes a great splash. In recent experiments at the University of Chicago, scientists have calculated that an overturning iceberg may release as much energy as some of the most destructive events on the planet.

"It's easily as much energy as an atomic bomb," says physicist Justin Burton, who

designed and carried out the experiments. He says an iceberg takes about three or four minutes to flip, and afterward it may send out large waves called tsunamis. Such a frozen flip may even trigger an earthquake. Burton and his colleagues published their results in the January 20 issue of the Journal of Geophysical Research.

In especially cold areas, like Greenland or Antarctica, glaciers may flow over the land and into the ocean. Where the edge of the glacier floats on water, it forms an ice shelf. An iceberg forms when part of the ice shelf cracks and breaks off. That's when icebergs are most likely to capsize.

"Large icebergs break off glaciers and then they flip," says Burton. If an iceberg flips close enough to the glacier or some other solid surface, it may shake the ground hard enough to be detected as an earthquake. The force of gravity makes an iceberg flip. When an iceberg forms and plunges into the water, the block of ice may be unstable, or prone to move. A dropped ball is unstable and falls toward the ground; once it stops moving, it becomes stable. A balloon submerged in a pool of



water is unstable and quickly floats to the surface. A person swishing down a waterslide is unstable and doesn't stop moving until she reaches the bottom. In each of these cases, gravity causes an object to shift from instability to stability.

Burton and his colleagues built a model of an iceberg in their laboratory with a water tank that measured about 8 feet (244 centimeters) long, 11.8 inches (30 cm) wide and 11.8 inches tall. Burton says they initially wanted to use real ice to build their floating 'bergs, but the ice melted too quickly. Instead, they used a type of plastic that had the same density as the ice in icebergs. Density is a measure of the mass within a certain amount of space. Burton's team floated their plastic icebergs in the water tank, flipped them over, and then measured the waves.

Physicists already knew how to measure the energy released when gravity causes an unstable object to become stable. Burton and his colleagues used those same ideas to calculate the energy released by a flipping iceberg. Some of that energy is used to make the iceberg turn, but about 85 percent is simply released into the water.

Further Reading (with mathematical derivation) from Physics Magazine available in Drum: Physics Review, September 2016, Vol. 26, Number 1, Marshall, R., If Icebergs Capsize