Name:		Index Number:		Class:	

DUNMAN HIGH SCHOOL
Quiz 1
Year 5

MATHEMATICS (Higher 2) 9758

22 August 2024

40 minutes

Additional Material: Printed Answer Booklet

READ THESE INSTRUCTIONS FIRST

Answer **all** the questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

1 The first three terms of an infinite series are 16, x, 9.

(i) Find the value(s) of x, if the series is

(a) geometric,

(b) arithmetic. [3]

(ii) If all the terms in part **(i)(a)** are positive, calculate its sum to infinity. [2]

(iii) Given that the sum of the first n terms in part **(i)(b)** is less than -64, find the least value of n. [3]

Qn	Suggested Solution	Comments / Common Mistakes
1(i)	**(a)** $\frac{x}{16}=\frac{9}{x}$ $x^2=144$ $x=\pm 12$ OR $a=16$ $ar=x$ $ar^2=9$ $x=\pm\sqrt{16\times 9}=\pm 12$ **(b)** $x-16=9-x$ $x=12.5$	• Formulate equation using common ratio or consecutive terms of GP • Formulate equation using common difference of AP • Students should recognise that information in part **(ii)** may not necessarily apply to part **(i)**. For part **(i)(a)**, students may make the wrong assumption that x or r (common ratio) must be positive hence missed out on one solution. • Note that the question asked for values of x and not d or r.
(ii)	$S_\infty=\frac{16}{1-\frac{3}{4}}=64$	• Students should apply $S_\infty=\frac{\text{first term}}{1-\text{ratio}}$ formula for GP sum • Students may mistook $U_1=9,\ U_2=x,\ U_3=12,$ hence taking the common ratio $r=\frac{4}{3}$ instead (then sum to infinity won't exist)

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<td>(iii)</td>
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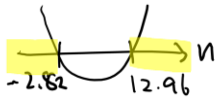
Let the number of terms be n.
$\frac{n}{2}\left[2(16)+(n-1)(-3.5)\right]<-64$
$n[32-3.5n+3.5]<-128$
$3.5n^2-35.5n-128>0$


$n<-2.82$ or $n>12.96$ OR
<table>
<tr><td>n</td><td>$S_n + 64$</td><td></td></tr>
<tr><td>12</td><td>25</td><td>>0</td></tr>
<tr><td>13</td><td>-1</td><td><0</td></tr>
</table>
From GC, least $n = 13$
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Students may apply wrong AP sum formula or wrong common difference
Note exact answers are not required hence students can use GC to solve the inequality graphically or by table
Students are reminded that the least value of $n = 13$ is obtained from the solution to the inequality $n > 12.96$, and not just derived by rounding up $n = 12.96$ to the nearest integer.

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2 **(i)** Find $\sum_{r=1}^{n}\left(\frac{1}{2}\right)^{r}$ in terms of n. [2]

A sequence is such that $u_0 = 3$ and $u_r - u_{r-1} = \left(\frac{1}{2}\right)^{r}$ for $r \geq 1$.

(ii) Show that $\sum_{r=1}^{n}\left(\frac{1}{2}\right)^{r} = u_n - u_0$. Hence find u_n in terms of n. [2]

(iii) Show that $S = 4n - 2 + 2\left(\frac{1}{2}\right)^{n}$, where S denotes the sum of the first n terms of the sequence. [2]

(iv) Determine, with reason, whether

(a) u_n converges, [1]

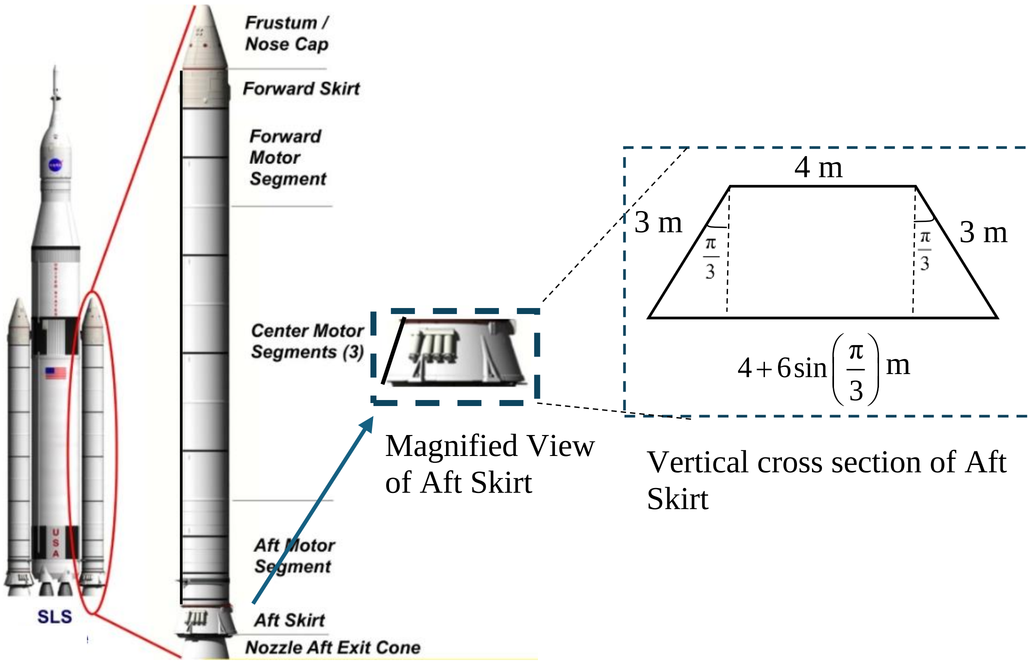
(b) S converges. [1]

Qn	Suggested Solution	Comments / Common Mistakes
2(i)	$\sum_{r=1}^{n}\left(\frac{1}{2}\right)^{r} = \left(\frac{1}{2}\right)^{1} + \left(\frac{1}{2}\right)^{2} + \left(\frac{1}{2}\right)^{3} + \ldots + \left(\frac{1}{2}\right)^{n}$ $= \frac{\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^{n}\right)}{1-\frac{1}{2}}$ $= 1-\left(\frac{1}{2}\right)^{n}$	Concepts used: 1. Sum of GP = $\frac{a\left(1-r^{n}\right)}{1-r}$

(ii)	$\sum_{r=1}^{n}\left(\frac{1}{2}\right)^{r}=\sum_{r=1}^{n}\left(u_r-u_{r-1}\right)$ $=\sum_{r=1}^{n}u_r-\sum_{r=1}^{n}u_{r-1}$ $=\left(u_1+u_2+\ldots+u_n\right)-\left(u_0+u_1+\ldots+u_{n-1}\right)$ $=u_n-u_0$ From (i) and $u_0=3$ $1-\left(\frac{1}{2}\right)^{n}=u_n-3$ $\therefore u_n=4-\left(\frac{1}{2}\right)^{n}$	
(ii)	**Alternative:** Using $u_r-u_{r-1}=\left(\frac{1}{2}\right)^{r}$ When $r=1,\ \cancel{u_1}\boxed{-u_0}=\left(\frac{1}{2}\right)^{1}$ When $r=2,\ \cancel{u_2}-\cancel{u_1}=\left(\frac{1}{2}\right)^{2}$ When $r=3,\ \cancel{u_3}-\cancel{u_2}=\left(\frac{1}{2}\right)^{3}$ … When $r=n-1,\ \cancel{u_{n-1}}-\cancel{u_{n-2}}=\left(\frac{1}{2}\right)^{n-1}$ When $r=n,\ \boxed{u_n}-\cancel{u_{n-1}}=\left(\frac{1}{2}\right)^{n}$ Sum up these equations, $u_n-u_0=\sum_{r=1}^{n}\left(\frac{1}{2}\right)^{r}$ (Shown) $\therefore u_n=3+1-\left(\frac{1}{2}\right)^{n}=4-\left(\frac{1}{2}\right)^{n}$ from (i) and $u_0=3$	Concepts used: 1. Observe pattern to perform method of differences.

(iii)	$S=\sum_{r=0}^{n-1}u_r=\sum_{r=0}^{n-1}\left(4-\left(\frac{1}{2}\right)^r\right)$ $=4n-\sum_{r=0}^{n-1}\left(\frac{1}{2}\right)^r$ $=4n-\frac{1\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\left(\frac{1}{2}\right)}$ $=4n-2+2\left(\frac{1}{2}\right)^n$ (shown) $\sum_{r=0}^{n-1}u_r\neq\sum_{r=0}^{n-1}u_n$ $u_0+u_1+...+u_{n-1}\neq u_n+u_n+...+u_n$	Concepts used: 1. Sum of GP = $\frac{a\left(1-r^n\right)}{1-r}$ 2. Understand that the first n terms of this sequence starts from u_0 to u_{n-1}.
(iv)	When $n\to\infty$, $\left(\frac{1}{2}\right)^n\to 0$. (a) $u_n\to 4$ (constant), u_n converges. (b) $S\to\infty$, S does not converge.	
		Total marks : 8

3 A famous entrepreneur, Elon Tusk, has designed a new rocket booster for his company SpaceY. A rocket booster consists of the following parts as shown in the following diagram.



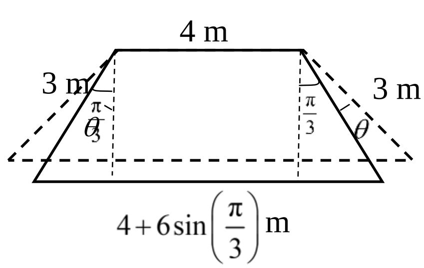
Magnified View of Aft Skirt

Vertical cross section of Aft Skirt

Rocket Booster

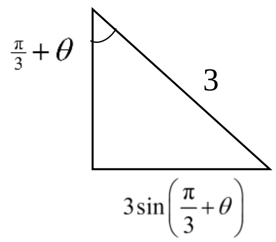
The design of the Aft Skirt can be viewed using a vertical cross sectional view with the stated dimensions (see dotted box above).

Due to the amount of heat and pressure generated from the thrust of the rocket during lift off, the sides of the Aft Skirt will tilt **outward** by a **very small angle**, θ, while keeping the slant length at 3 m (see diagram below).



Before the lift off, the diameter of the circular base of the Aft Skirt is given by $\left(4+6\sin\left(\frac{\pi}{3}\right)\right)$ m. In order for the booster to function properly during the lift off, the diameter at the bottom of the Aft Skirt must be less than 9.197 m.

Given that θ is a sufficiently small angle, show that θ satisfies the inequality $a\theta^2+b\theta+c<0$ where a, b and c are constants to be determined. Hence find the range of values for θ, giving your answers correct to 6 decimal places. [4]

Qn	Suggested Solution	Comments / Common Mistakes
3	Diameter at the bottom of the Aft Skirt $=4+2\times 3\sin\left(\frac{\pi}{3}+\theta\right)$ $4+6\sin\left(\frac{\pi}{3}+\theta\right)<9.197$ $\sin\left(\frac{\pi}{3}+\theta\right)<0.86617$ $\frac{\sqrt{3}}{2}\cos\theta+\frac{1}{2}\sin\theta<0.86617$ For small θ , $\frac{\sqrt{3}}{2}\left(1-\frac{\theta^2}{2}\right)+\frac{1}{2}\theta<0.86617$ $-\frac{\sqrt{3}}{4}\theta^2+\frac{1}{2}\theta+\frac{\sqrt{3}}{2}-0.86617<0$ Using GC, $\theta<0.000289$ or $\theta>1.154411$ (6dp) Since $\theta>0$ and it is small, $0<\theta<0.000289$ (6d.p)	• Common for students to misinterpret the meaning of "diameter" in this question. Diameter refers to the whole length of the base of the skirt. • Note that only the slant length remains unchanged at 3 m when the skirt opens.  • Apply the compound angle formula from MF27 and considered small θ to apply the formulas: $\sin\theta\approx\theta$ and $\cos\theta\approx 1-\frac{\theta^2}{2}$. • Since the skirt opens 1. by a small angle, rej. $\theta>1.554411$ 2. outward, $\theta>0$.
		Total marks : 4