

**TEMASEK JUNIOR COLLEGE** 

**JC2 Preliminary Examination** 



9649/02

Higher 2

FURTHER MATHEMATICS

Paper 2

15 September 2023 3 hours

Additional Materials: Answer Booklet List of Formulae (MF26)

### READ THESE INSTRUCTIONS FIRST

An answer booklet will be provided with this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Write your Civics Group and name on all the work that you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

#### Section A: Pure Mathematics [50 Marks]

**1** A particular solution to the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = (y-1)^2,$$

has y = -1.6 when x = 0.

- (a) Use the Euler method with step size 0.5 to estimate y at x = 1. [2]
- (b) Explain with the aid of a diagram why the Euler method with the step size chosen in (b) does not provide a good approximation to the exact solution. [2]
- (c) Use the improved Euler method with step size 0.5 to estimate y at x = 1. Suggest how the accuracy of the estimate can be improved. [3]
- 2 The population of a certain virus is studied in a laboratory. The research team proposes the following recurrence relation:

$$u_{n+1}\sqrt{n} = au_n\sqrt{n+1} - b\sqrt{n^2 + n}$$

where *a* and *b* are positive constants and  $u_n$  represents the size of population *n* days after the start of the project. Using the substitution of  $v_n = \frac{u_n}{\sqrt{n}}$ , show that the recurrence relation can be simplified to the form  $v_{n+1} = av_n - b$ . [1]

- (a) In the first instance, a is assigned the value 2 and  $u_1 = 10000$ .
  - (i) Find the solution for  $u_n$  in terms of b. [4]
  - (ii) Determine the range of values of *b* for which this model predicts that population of virus will eventually be zero. For the case when |10000-b| is small, describe the behaviour of  $u_n$  before this happens. [2]
- (b) If instead both *a* and *b* are assigned the value 2 and  $u_1 = 10000$ , find the number of weeks such that the population exceeds 100 times of  $u_1$ . [3]

- 3 Let **M** denote the matrix  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ , where *a*, *b*, *c* and *d* are real numbers such that a + c = 1, b + d = 1 and  $\mathbf{M} \neq \mathbf{I}$  (**I** is the 2×2 identity matrix).
  - (a) Show that the eigenvalues of **M** are 1 and a + d 1. [5]
  - (b) Find the corresponding eigenvectors of M, giving your answers where appropriate, in terms of a and d only. [4]
- 4 A toy company produces egg toy capsules by joining two half ellipses along their minor axes as shown in the diagram below, where a, b and c are positive constants such that a < b and a < c and  $b \neq c$ .



The toy egg capsule is obtained by rotating the region bounded by the 2 ellipses about the *y*-axis which is its axis of symmetry. All dimensions are in centimetres.

- (a) A particular type of egg capsule produced by the company is 6 cm long and 4 cm wide. Show that the volume contained by this egg capsule is  $16\pi$  cubic centimetres. [4]
- (b) Part of the ellipse from point (0,-c) to (a, 0) is rotated about the y-axis to form the curve surface of the lower part of the capsule. This lower part of the capsule is to be coated in paint. Find the curved surface area of the capsule that is coated in paint when c = 2a. Leave your answer in terms of a.

5 (a) Given that  $y = \frac{u}{x}$ , show that

$$\frac{d^2 y}{dx^2} = \frac{1}{x} \frac{d^2 u}{dx^2} - \frac{2}{x^2} \frac{du}{dx} + \frac{2u}{x^3}.$$

Hence find the general solution of the differential equation

$$\frac{d^2 y}{dx^2} + \frac{2}{x}\frac{dy}{dx} - 25y = 0, \quad x > 0.$$
 [5]

(b) Find the values of the constants p and q for which  $y = px \sin 2x + qx \cos 2x$  is a particular integral of the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 4y = \sin 2x \,.$$

Find the general solution of this differential equation.

For the case where  $x = n\pi$  and *n* is a large positive integer, show that whatever the initial conditions,  $\frac{y}{n}$  is approximately a constant which is to be stated. [7]

### Section B: Probability and Statistics [50 Marks]

6 At a launch event of a new Jpad pro tablet, 150 randomly selected customers were asked to indicate their colour preference, black, white, or grey of this new model. The results, classified by gender of customers, are shown in the table below.

		Colour preference			
		White	Black	Grey	
Gender	Male	10	1+k	43 - k	
	Female	20	59 - k	17 + <i>k</i>	

- (a) Given that there is no association at 5% level between gender of customers and colour preference of Jpad pro tablet. Using a chi-square test, determine the possible values of k.
- (b) Given that k = 30, calculate the *p*-value. Discuss what the *p*-value indicates about the association between the gender of customers and colour preference of Jpad pro tablet. [2]

7 The stimulant Vitalin has been shown to increase the attention span and improve mathematical performance in children. A researcher selects a random sample of 10 children, conducted a suitable test on them and collected the data before and after taking the stimulant. The results are as follows:

Child	1	2	3	4	5	6	7	8	9	10
Before	15	15	13	10	8	8	6	4	5	4
After	19	12	13	11	8	13	16	17	14	2

- (a) Conduct a Wilcoxon Signed Rank Test at 5% significance level to ascertain if the median score is higher after taking Vitalin.
  [6]
- (b) Show that the conclusion is reversed, if a sign test is conducted instead. Explain which conclusion should one take. [2]
- 8 In a computer game, a robot is placed on an infinite grid with starting position at point (0,0). The robot moves in steps and each step is either 1 unit in the positive *x*-direction with probability p or 1 unit in the positive *y*-direction with probability 1-p. The robot stops when it reaches a particular *y*-coordinate.
  - (a) If the robot stops at point (*X*, 1), find

(i) 
$$P(X=r)$$
 in terms of  $p$ , [1]

- (ii) P(X < r) in terms of p, [2]
- (iii) the value of p given that E(X) = 9. [2]
- (b) If the robot stops at point (X, 2), find in terms of p,
  - $(i) \quad P(X=r), \tag{1}$
  - (ii) P(X > 3). [3]
- 9 The continuous random variable *X* has probability density function

$$f(x) = \begin{cases} \frac{k}{x^n} & x \ge 1, \\ 0 & \text{otherwise} \end{cases}$$

where n and k are constants and n is an integer greater than 3.

- (a) Find k in terms of n. [3]
- (b) Given that  $E(X) = \frac{n-1}{n-2}$ , find Var(X) in terms of *n*, and hence prove that Var(X) < E(X).

(c) (i) When n = 4, find the cumulative distribution function of *X*. [2]

(ii) Hence find P(X > 7 | X > 5) when n = 4. [3]

[Turn over

[4]

10 To study whether the "CGarlic diet" (Consuming 2 cloves of Cooked Garlic every day for a period of 12 weeks) can reduce the seated systolic blood pressure in working adults, a random sample of 10 adults from a large company were selected to undergo the "CGarlic diet". The results in millimeters of mercury (mm Hg) are summarised by

$$\sum(x-100) = 149, \sum(x-100)^2 = 3277, \sum(y-100) = 99, \sum(y-100)^2 = 1527$$

where *x* mm Hg and *y* mm Hg are their seated systolic blood pressure taken before the "CGarlic diet" and after the "CGarlic diet" respectively.

(a) State the null and alternative hypotheses and carry out a 2-sample *t*-test at 5% level of significance.

[7]

State all assumptions required for the test to be valid.

The data for the decrease in seated systolic blood pressure after the "CGarlic diet" from the same sample of 10 adults are given in the table below.

Decrease in seated systolic blood pressure						
7	-4	18	17	-3		
-5	-2	1	10	11		

- (b) Construct a 95% confidence interval for the mean decrease in seated systolic blood pressure. State a necessary assumption for your method to be valid. [4]
- (c) Hence without doing further calculation, test whether there is sufficient evidence at the 5% level that there is a change in the seated systolic blood pressure before and after the "CGarlic diet".

# **BLANK PAGE**

## **BLANK PAGE**