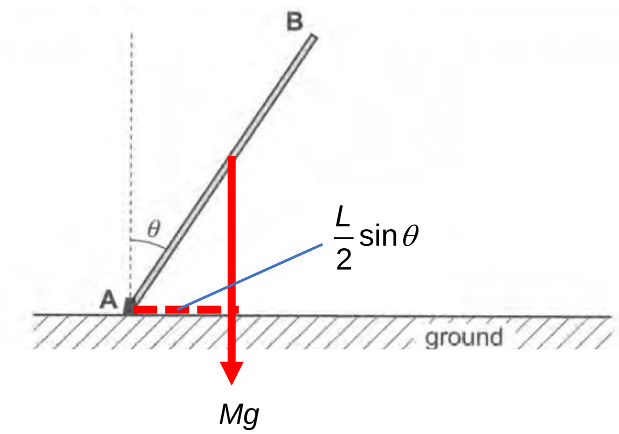
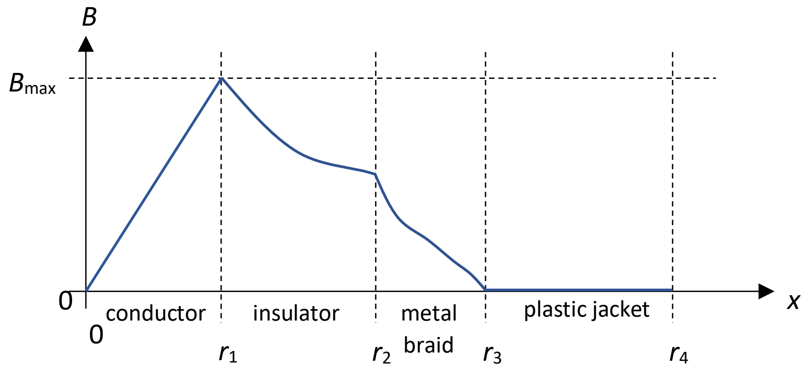
2022 H3 A Level Suggested Solutions

1	**(a)**	**(i)**	The net torque generated on rod due to its weight is given by, $$\tau = \int g\left(\frac{M}{L}\right)dl.x = \int g\left(\frac{M}{L}\right)dl.(lsin\theta) = \frac{gMsin\theta}{L}\left(\frac{L^2}{2}\right) = \frac{MgL\,sin\theta}{2}$$ (**Alternatively**, since the rod is uniform, you may obtain the torque as follows:  The torque due to the weight of the rod about A would be $Mg\left(\frac{L}{2}\sin\theta\right)$. As $\tau = I_A\alpha$, and $I_A = \frac{ML^2}{12} + M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{3}$ (The term $\frac{ML^2}{12}$ is the moment of inertia about centre of mass of the rod) We have, $$\frac{MgL\sin\theta}{2} = \frac{ML^2}{3}\alpha$$ $$\alpha = \frac{3g\sin\theta}{2L}$$
			Comments: Several students did not consider the factor of two in their answers.
		(ii)	Comparing with the given form, $k = 1.5$.

<table>
<tr><td></td><td>(b)</td><td colspan="2">As the total energy is conserved, the gravitational PE is converted to the rotational kinetic energy of the rod,
$$Mg\left(\frac{L}{2}\cos\theta\right)+0=0+\frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2$$
$$\omega=\sqrt{\frac{3g\cos\theta}{L}}$$</td></tr>
<tr><td></td><td></td><td colspan="2">Comments: Using the conservation of energy is the most straightforward method. Some students erroneously used SUVAT equations, however, the angular acceleration was not constant.</td></tr>
<tr><td>2</td><td>(a)</td><td colspan="2">$$\oint \vec{B}\cdot d\vec{\ell}=\mu_0 I_{enc}$$
Where $\oint \vec{B}\cdot d\vec{\ell}$ is the net magnetic flux along the loop, μ_0 is the magnetic permeability of free space, I_{enc} is the enclosed current and $d\ell$ is the small length on the circular path (loop).</td></tr>
<tr><td></td><td>(b)</td><td>(i)</td><td>Since $I_1=I_2$,
$$\int J_1\,dA=\int J_2\,dA$$
$$\int_0^{r_1} J_1\,(2\pi r\,dr)=\int_{r_2}^{r_3} J_2\,(2\pi r\,dr)$$
$$\pi r_1^2 J_1=\pi(r_3^2-r_2^2)J_2$$
$$J_2=\frac{r_1^2}{r_3^2-r_2^2}J_1$$</td></tr>
<tr><td></td><td></td><td>(ii)1</td><td>Draw a circular Amperian loop of radius r_1 centred on the axis of the wire,
$$\oint \vec{B}\cdot d\vec{\ell}=\mu_0 I_{enc}=\mu_0 I_{1,enc}=\mu_0(\pi r_1^2 J_1)$$
$$B(2\pi r_1)=\mu_0\pi r_1^2 J_1$$
$$B=\frac{1}{2}\mu_0 J_1 r_1$$</td></tr>
<tr><td></td><td></td><td>(ii)2</td><td>Draw a circular Amperian loop of radius r_2 centred on the axis of the wire,
$$\oint \vec{B}\cdot d\vec{\ell}=\mu_0 I_{enc}=\mu_0 I_1=\mu_0(\pi r_1^2 J_1)$$
$$B(2\pi r_2)=\mu_0\pi r_1^2 J_1$$
$$B=\frac{\mu_0 J_1 r_1^2}{2r_2}$$</td></tr>
</table>

		(iii)	

The shapes of the graph are:

- 0 to r_1: linear
- (Maximum at r_1)
- r_1 to r_2: curve
- r_2 to r_3: curve
- r_3 to r_4: zero

Calculations for each region (no need to show):

Repeating the working in 2(b)(ii)1 and 2 but considering a loop with a general radius r, we have:

$$B = \begin{cases} \frac{1}{2}\mu_0 J_1 r, & 0 < r \le r_1 \\ \frac{\mu_0 J_1 r_1^2}{2r}, & r_1 \le r \le r_2 \end{cases}$$

That is, $B \propto r$ for $0 < r \le r_1$ and $B \propto \frac{1}{r}$ for $r_1 \le r \le r_2$

For $r_2 \le r \le r_3$:

Draw a circular Amperian loop of radius r centred on the axis of the wire, where $r_2 \le r \le r_3$.

(Note that the current in the metal braid flows in the opposite direction, so the net current is $I_1 - I_{2,enc}$.)

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc} = \mu_0\left(I_1 - I_{2,enc}\right) = \mu_0(\pi r_1^2 J_1 - \pi(r^2 - r_2^2)J_2)$$

$$B(2\pi r) = \mu_0\left(\pi r_1^2 J_1 - \pi(r^2 - r_2^2)\frac{r_1^2}{r_3^2 - r_2^2}J_1\right)$$

$$B(2\pi r) = \mu_0 \pi J_1 r_1^2\left(1 - \frac{r^2}{r_3^2 - r_2^2} + \frac{r_2^2}{r_3^2 - r_2^2}\right)$$

$$B(2\pi r) = \frac{\mu_0 \pi J_1 r_1^2}{r_3^2 - r_2^2}(r_3^2 - r^2)$$

$$B = \frac{\mu_0 J_1 r_1^2}{2(r_3^2 - r_2^2)}\left(\frac{r_3^2}{r} - r\right)$$

If you expand the bracket you will get two terms:

			- The first term is proportional to $1/r$, and initially this dominates. However it gets smaller as r increases. - The second term is proportional to r. This gets larger as r increases Therefore, you get a curve with decreasing gradient, that eventually becomes a line with negative gradient. (Basically, a hyperbola that approaches an asymptote of $B = -kr$ for some constant k). A kind of curve that is decreasing would suffice. <u>For $r_3 \leq r \leq r_4$:</u> Intuitively, this must be zero because $I_{enc} = I_1 - I_2 = 0$ (two equal currents flowing in opposite directions). However, if you want to check, you could perform this calculation: Draw a circular Amperian loop of radius r centred on the axis of the wire, where $r_3 \leq r \leq r_4$ $$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc} = \mu_0 (I_1 - I_2)$$ Since $I_1 = I_2 = I$, $$B(2\pi r) = 0$$ $$\therefore B = 0$$
	(c)	The magnetic field outside a standard transmission cable, unlike that of a coaxial cable, is *not* zero when a current is flowing through it. A high frequency signal will mean that energy will be dissipated in metal components in the surroundings.	
		Comments: Several students did not talk about the effects outside of the cable.	

3	**(a)**	**(i)**	$R = u\cos\theta\left(t_{flight}\right)$ $v_y = u_y + gt_{flight}$ $\left(\uparrow +\right)$ $-u\sin\theta = u\sin\theta + (-g)t_{flight}$ $t_{flight} = \dfrac{2u\sin\theta}{g}$ $R = u\cos\theta\left(\dfrac{2u\sin\theta}{g}\right) = \dfrac{u^2 2\sin\theta\cos\theta}{g} = \dfrac{u^2\sin 2\theta}{g}$
		(ii)	For maximum range R, $\sin\left(2\theta\right) = 1 \rightarrow \theta = 45°$
		(iii)	$R = \dfrac{u^2\sin 2\theta}{g} = \dfrac{150^2\sin(2\times 45°)}{9.81} = 2290\text{ m} = 2.29\text{ km}$
	(b)	**(i)**	$R = u\cos\theta\left(t_{flight}\right)$ $t_{flight} = \dfrac{R}{u\cos\theta}$
		(ii)	$\tan\phi = \dfrac{h}{R} \rightarrow h = R\tan\phi$
		(iii)	$s = s_0 + u_y t_{flight} + \dfrac{1}{2}a_y t_{flight}^2$ $\left(\uparrow +\right)$ $0 = h + u\sin\theta\left(t_{flight}\right) + \dfrac{1}{2}\left(-g\right)t_{flight}^2$ $h = -u\sin\theta\left(t_{flight}\right) + \dfrac{1}{2}\left(g\right)t_{flight}^2$ $R\tan\phi = -u\sin\theta\left(\dfrac{R}{u\cos\theta}\right) + \dfrac{1}{2}\left(g\right)\left(\dfrac{R}{u\cos\theta}\right)^2$ $R\tan\phi = -R\tan\theta + \dfrac{R^2 g}{2u^2\cos^2\theta}$ $R = \dfrac{2u^2\cos^2\theta}{g}\left(\tan\phi + \tan\theta\right)$ $R = \dfrac{u^2}{g}\left[2\cos^2\theta\tan\phi + \dfrac{2\sin\theta\cos^2\theta}{\cos\theta}\right] = \dfrac{u^2}{g}\left[\left(1+\cos 2\theta\right)\tan\phi + \sin 2\theta\right]$
			Comments: Many could handle the rigorous algebra here.

		(iv)	$R=\frac{u^2}{g}\left[\left(1+\cos 2\theta\right)\tan\phi+\sin 2\theta\right]$ $\phi\to 0, \tan\phi\to 0$ $R\to\frac{u^2\sin 2\theta}{g}$
		(v)	$R=\frac{u^2}{g}\left[\left(1+\cos 2\theta\right)\tan\phi+\sin 2\theta\right]$ $\frac{dR}{d\theta}=0\to 2\cos 2\theta+2(-\sin 2\theta)\tan\phi=0$ $\cos 2\theta=\sin 2\theta\tan\phi$ $\tan 2\theta=\frac{1}{\tan\phi}=\tan\left(\frac{\pi}{2}-\phi\right)$ $2\theta=\frac{\pi}{2}-\phi$ $\theta=\frac{\pi}{4}-\frac{\phi}{2}$
			Comments: Many did not realize that ϕ was a constant.

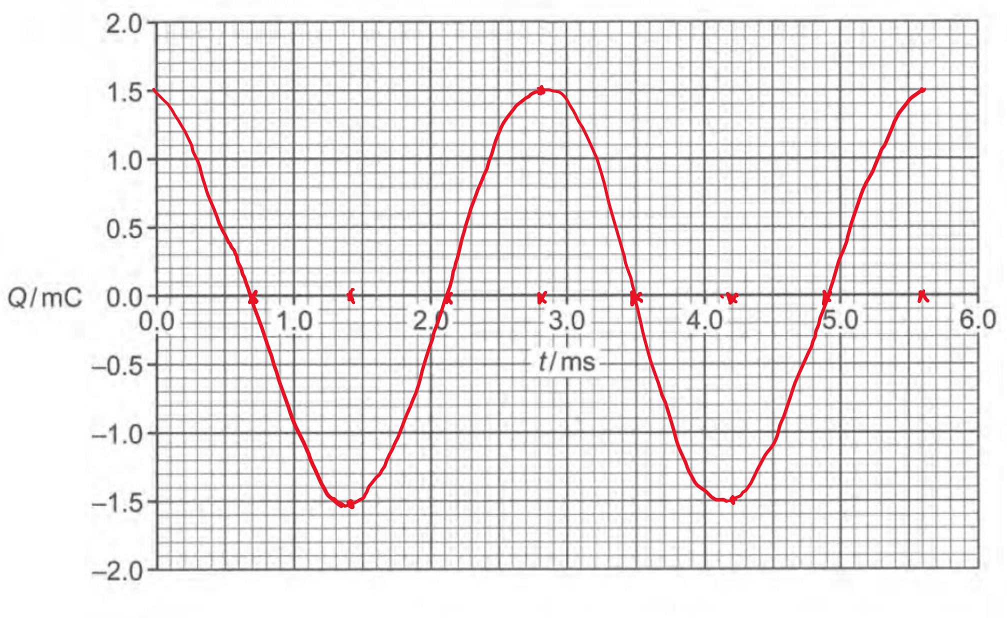
4	**(a)**	**(i)**	In Fig. 4.1, at the equilibrium, $Mg = k\left(\frac{L}{2} - l_0\right)$, where $\left(\frac{L}{2} - l_0\right)$ is the extension. In Fig. 4.2, $Mg \mp F_{hand} = k\left(\frac{L}{2} - l_0\right)_{left} + k\left(\frac{L}{2} - l_0\right)_{right}$ When it is released, $F_{hand} \to 0$, $Mg < k\left(\frac{L}{2} - l_0\right)_{left} + k\left(\frac{L}{2} - l_0\right)_{right}$. The upwards force is greater than *Mg.* Hence, when the system is released, it moves upwards.
		(ii)	Fig. 4.2 must eventually be in equilibrium. $\frac{Mg}{2} = k(e)$, where e is the extension for the spring either on left or right side in Fig. 4.2. The load is halved on either side. Hence, $e = \frac{Mg}{2k}$ The total length on either side is $L_1 = \frac{L}{2} + l_0 + e$, where $\frac{L}{2}$ is the length of string, l_0 is the unstretched length of spring and e is the extension. From Fig. 4.1, $Mg = k\left(\frac{L}{2} - l_0\right) \to \frac{L}{2} = \frac{Mg}{k} + l_0$ Hence, $L_1 = \frac{L}{2} + e = \frac{Mg}{k} + l_0 + l_0 + \frac{Mg}{2k}$ $L_1 = 2l_0 + \frac{3Mg}{2k}$
			Comments: Very challenging. Many were confused with extensions in Fig. 4.1 and Fig. 4.2.

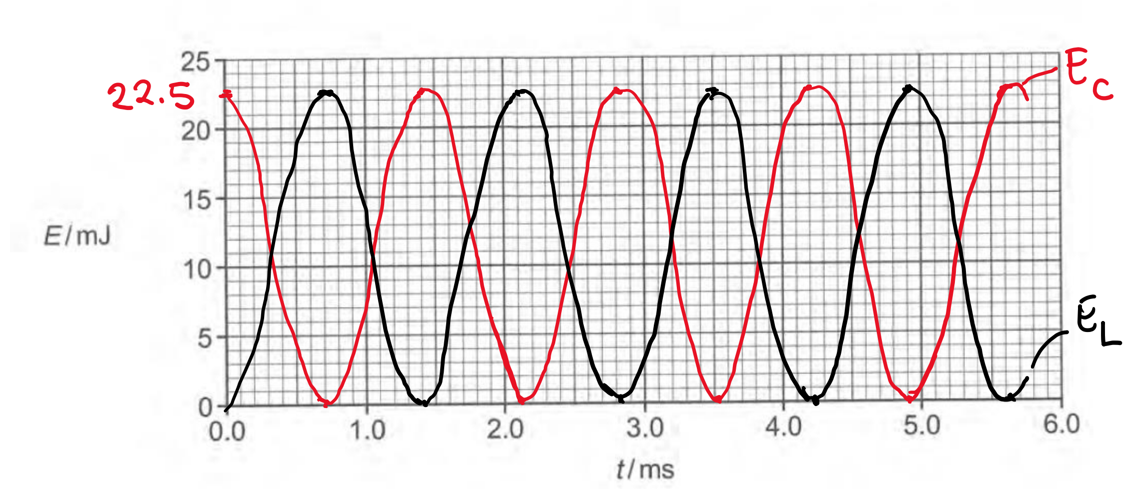
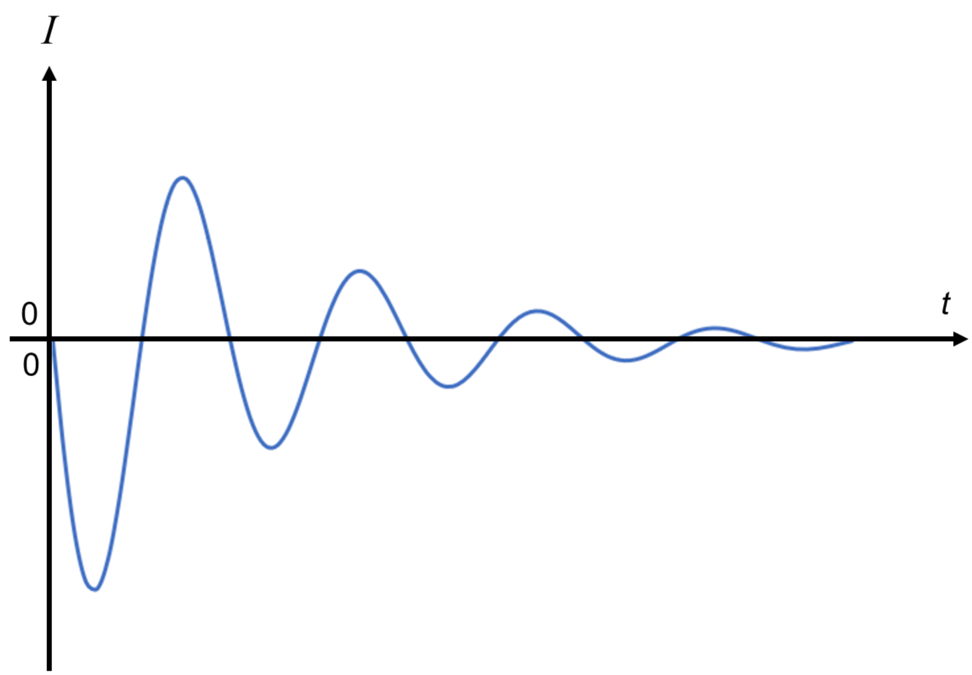
	(b)	**(i)**	On either side, $\frac{1}{2}\frac{5}{2}Mg = k(e)_{new} \rightarrow (e)_{new} = \frac{5}{4}\frac{Mg}{k}$ The new total length on either side is $L_2 = \frac{L}{2} + l_0 + e_{new}$, where $\frac{L}{2}$ is the length of string, l_0 is the unstretched length of spring and e_{new} is the new extension. $L_2 = \frac{L}{2} + l_0 + e_{new}$ $\frac{17}{14}L_1 = \left(\frac{Mg}{k} + l_0\right) + l_0 + \frac{5}{4}\frac{Mg}{k}$ $\frac{17}{14}\left(2l_0 + \frac{3}{2}\frac{Mg}{k}\right) = \left(\frac{Mg}{k} + l_0\right) + l_0 + \frac{5}{4}\frac{Mg}{k}$ $\frac{34}{14}l_0 - 2l_0 = \frac{Mg}{k}\left(1 + \frac{5}{4} - \frac{17}{14}\frac{3}{2}\right) \rightarrow \frac{6}{14}l_0 = \frac{Mg}{k}\left(\frac{12}{28}\right)$ $k = \frac{Mg}{l_0}$
			Comments: Very challenging for many.
		(ii)	$L_1 = 2l_0 + \frac{3Mg}{2k} = 2\left(\frac{Mg}{k}\right) + \frac{3Mg}{2k} = \frac{7Mg}{2k}$ From the condition in Fig. 4.1, $\frac{L}{2} = \frac{Mg}{k} + l_0 = \frac{Mg}{k} + \frac{Mg}{k} = 2\frac{Mg}{k}$ $\rightarrow \frac{Mg}{k} = \frac{L}{4}$ $L_1 = \frac{7}{2}\left(\frac{Mg}{k}\right) = \frac{7}{2}\frac{L}{4} = \frac{7L}{8}$

5	**(a)**	**(i)**	The rod is light, finding the moment of inertia of spheres about the axis of rotation, $I = 2m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{2}$ $\tau = I\alpha$ $= \frac{mL^2}{2}\left(\frac{a}{L}\right) = \frac{mL^2}{2}\left(\frac{\omega^2 L\theta}{L}\right) = \frac{mL^2}{2}\omega^2\theta$ $\tau = \kappa\theta = \frac{mL^2}{2}\omega^2\theta \rightarrow \kappa = \frac{mL^2}{2}\frac{4\pi^2}{T^2} = \frac{2\pi^2 mL^2}{T^2}$
			Comments: Remember to include omega ω as the motion is modelled as SHM and relate the torque to the moment of inertia.
		(ii)	15.0 cm + 2.5 cm + 0.1 cm = 17.6 cm
			Comments: Incorrect answers used diameters, not radii.
		(iii)	For small angle θ_1, $s = \frac{L}{2}\theta_1 \rightarrow \theta_1 = \frac{2s}{L} = \frac{2\left(4.1\times10^{-3}\right)}{1.80} = 45.6\times10^{-4}$ rad $\frac{\Delta\theta_1}{\theta_1} = \frac{\Delta s}{s} + \frac{\Delta L}{L} = \frac{0.1}{4.1} + \frac{0.01}{1.80}$ $\Delta\theta_1 = \theta_1\left(\frac{0.1}{4.1} + \frac{0.01}{1.80}\right) = 45.6\times10^{-4}\left(\frac{0.1}{4.1} + \frac{0.01}{1.80}\right) = 0.0001\text{ rad}$ $\theta_1 = 0.0046 \mp 0.0001\text{ rad}$
			Comments: Some used the full length of the rod, rather than the distance to the centre.
		(iv)	The gravitational force provides the torque on the system, $\tau = F_g L = \kappa\theta_1$ $\frac{GMm}{r^2}L = \kappa\theta_1$ $G = \frac{\kappa\theta_1 r^2}{MmL} = \frac{2\pi^2 mL^2}{T^2}\frac{\theta_1 r^2}{Mm} = \frac{2\pi^2 L}{MT^2}r^2\theta_1$
			Comments: The couple provides the torque.

	(b)	**(i)**	The deflection of the laser beam on the screen is caused by the angle between the incident beam and the reflected beam. $\tan\theta = \dfrac{15.6\text{ cm}}{12.00\text{ m}} \rightarrow \theta = 0.7448°$ $\theta_2 = \dfrac{\theta}{2} = 0.372°$
			Comments: A factor of two was missing. Some did not express their answers in degrees.
		(ii)	10 minutes 22 seconds is 3*T*/4. Hence, the period $T = \dfrac{622}{3/4} = 829.3\text{ s}$ $G = \dfrac{2\pi^2 L}{MT^2} r^2 \theta_2 = \dfrac{2\pi^2 (1.80\text{ m})}{(158\text{ kg})(829.3\text{ s})^2}(0.176\text{ m})^2(0.00649\text{ rad})$ $G = 6.58 \times 10^{-11}\text{ m}^3\text{ kg}^{-1}\text{ s}^{-2}$
			Comments: Several students did not realize that the duration given in the question was 3*T*/4. Some forgot to convert the degrees to radians in their calculation.
		(iii)	13 hours 48 minutes 18 seconds = 49 698 seconds $\dfrac{49698}{829.3} = 59.93$ oscillations Rounding this to 60 oscillations and adding the additional *T*/4 oscillation observed in the morning, makes 60.25 oscillations. $49698 / 60.25 = 825\text{ s}$
			Comments: Some did not consider the additional quarter period of oscillation.
		(iv)	It was assumed that the rod was light. This must have **mainly** led to the difference in the value of *G*.
			Comments: Several students talked about damping or energy losses due to air resistance or modelling the spheres as point-masses.

6	**(a)**	Faraday's law: The magnitude of the induced e.m.f. ε is directly proportional to the rate of change of magnetic flux linkage or rate of cutting of magnetic flux. $$\varepsilon \propto \frac{d\phi}{dt}$$ Lenz's law: The polarity of the induced e.m.f. is such that it tends to produce a current that creates a magnetic field so as to oppose the change in magnetic flux.		
		Comments: Some were not clear about the direction or polarity of induced current/e.m.f. when they explained Lenz's law.		
	(b)	**(i)**	Total energy is constant. $U=\frac{Q^2}{2C}+\frac{1}{2}LI^2$ $\frac{dU}{dt}=0=\frac{d}{dt}\left(\frac{Q^2}{2C}+\frac{1}{2}LI^2\right)=\frac{2Q}{2C}\frac{dQ}{dt}+\frac{2LI}{2}\frac{dI}{dt}$ $=\frac{Q}{C}\frac{dQ}{dt}+LI\frac{dI}{dt}=\frac{Q}{C}I+LI\frac{dI}{dt}$ $=\frac{Q}{C}+L\frac{d}{dt}\left(\frac{dQ}{dt}\right)$ $\rightarrow\frac{Q}{C}+L\frac{d^2Q}{dt^2}=0$	
			Comments: Those who expressed the stored energy in a capacitor in terms of voltage had difficulties to make progress. Some did not follow the instruction of using energy consideration and started with Kirchoff's voltage.	
		(ii)	$Q=Q_0\cos(\omega t+\phi)$ at $t=0$, $Q=Q_0$ and $I=0$ $\frac{dQ}{dt}=-\omega Q_0\sin(\omega t+\phi)=0\rightarrow\phi=0$ Hence, $Q=Q_0\cos(\omega t)$	
	(c)	**(i)**	At $t=0$, $Q=Q_0$. The capacitor will be next fully charged half of the period later. $\omega=\frac{2\pi}{T}=\sqrt{\frac{1}{LC}}\rightarrow T=2\pi\sqrt{LC}=2\pi\sqrt{(4.0\times10^{-3})(50\times10^{-6})}=2.8\times10^{-3}$ s $\frac{T}{2}=1.4\times10^{-3}$ s	
			Comments: Several students forgot about the factor of two.	

		(ii)	 **Fig. 6.2**
		(iii)	This happens when E_C is equal to E_L which is equal to half of the total energy. $\frac{Q^2}{2C} = \frac{1}{2}\frac{Q_o^2}{2C} \rightarrow \frac{Q_0^2\cos^2(\omega t)}{2C} = \frac{Q_o^2}{4C}$ $\rightarrow \cos^2(\omega t) = \frac{1}{2} \rightarrow \cos(\omega t) = \frac{1}{\sqrt{2}}$ $\omega t = \frac{\pi}{4} \rightarrow \frac{2\pi}{T}t = \frac{\pi}{4} \rightarrow t = \frac{T}{8} = \frac{2.8\times10^{-3}}{8}$ $t = 3.5\times10^{-4}$ s
			Comments: This part was difficult for significant number of students.

		(iv)	$U=\dfrac{Q_0^2}{2C}=\dfrac{\left(1.5\times10^{-3}\right)^2}{2\left(50\times10^{-6}\right)}=0.0225\text{ J}=22.5\text{ mJ}$  **Fig. 6.3**
		(v)	1. 
			2. $\omega=\sqrt{\dfrac{1}{LC}-\dfrac{R^2}{4L^2}}=\dfrac{2\pi}{T}\rightarrow T=\dfrac{2\pi}{\sqrt{\dfrac{1}{LC}-\dfrac{R^2}{4L^2}}}$ Angular frequency becomes smaller, the period increases.

7	**(a)**	**(i)**	Consider a small element as a horizontal strip of width dc, $dI_x = r^2 dm = r^2 \sigma dA$, where σ is area density. $dI_x = c^2 \sigma 2R(dc)$ $I_x = \sigma 2R \int_{-c/2}^{c/2} c^2 dc = \sigma 2R \frac{2c^3}{24} = \frac{M}{c(2R)} 2R \frac{2c^3}{24} = \frac{1}{12} Mc^2$
		(ii)	Consider a small element as a vertical strip of width dR, $dI_y = r^2 dm = r^2 \sigma dA$, where σ is area density. $dI_x = R^2 \sigma c(dR)$ $I_x = \sigma c \int_{-R}^{R} R^2 dR = \sigma c \frac{2R^3}{3} = \frac{M}{c(2R)} c \frac{2R^3}{3} = \frac{1}{3} MR^2$
		(iii)	$I_z = \frac{Mc^2}{12} + \frac{MR^2}{3} = \frac{1}{12} M\left(c^2 + 4R^2\right)$
		(iv)	$M = \rho V = (1470)(2 \times 3.84)(0.20)(0.03) = 67.7$ kg Hence $I_z = \frac{1}{12} M\left(c^2 + 4R^2\right) = \frac{1}{12} 67.7(0.20^2 + 4 \times 3.84^2) = 333$ kg m^2
			Comments: Several students forgot about the factor of two.
		(v)	For constant angular acceleration, $\omega = \omega_0 + \alpha t \rightarrow \alpha = \frac{\omega - \omega_0}{t} = \frac{2\pi\left(\frac{320}{60} - \frac{20}{60}\right)}{9.0} = 3.49$ rad s^{-2} $\tau = I_z \alpha = (333)(3.49) = 1162$ kg m^2 s^{-2}
	(b)	**(i)**	Since the equation should be homogeneous, $C_L = \frac{2dL}{\rho (r\omega)^2 cdr}$ Unit of C_L is $\frac{\text{kg m s}^{-2}}{(\text{kg m}^{-3})(\text{m}^2)(\text{s}^{-2})(\text{m})(\text{m})} = 1$
			Comments: Several confused the lift force (L) with length.

		(ii)	$\int dL = \frac{1}{2} C_L \rho \omega^2 c \int_{-R}^{R} r^2 dr = \frac{1}{2} C_L \rho \omega^2 c \frac{2R^3}{3} = \frac{C_L \rho \omega^2 c R^3}{3}$
	(c)	**(i)**	As the lift on the part of the rotor blade increases with distance from center, this creates a torque that causes the blades to bend increasingly upwards until tension in the blades increases sufficiently to provide an opposing torque.
		(ii)	For the helicopter to hover, the net force on it must be zero, hence, $mg = L = \frac{C_L \rho \omega^2 c R^3}{3} \cos(10.0°)$ $\omega^2 = \frac{3mg}{C_L \rho \omega^2 c R^3 \cos(10.0°)} = \frac{3(2500)(9.81)}{(0.400)(1.25)(0.20)(3.84)^3 \cos(10.0°)}$ $\omega = 115$ rad s^{-1} $\frac{115}{2\pi} \times 60 = 1097$ revolutions per minute
			Comments: Several students made arithmetic errors.
	(d)	In order for a stable flight, the net moment on the helicopter about the vertical axis should be zero. As the main rotor blades generate a clockwise moment, the tail rotor is needed to generate an anticlockwise moment so that the net moment is zero.	
	(e)	As Ingenuity has two pairs of rotor blades, each can be set to rotate in the opposite direction. This will ensure that net moment about the vertical axis is zero.	

8	**(a)**	**(i)**	For **WXY branch**, $C = \frac{q}{V_{WX}}$ Same charge must be stored on 3*C* as they (*C* and 3*C*) are in series. $3C = \frac{q}{V_{XY}} \rightarrow V_{XY} = \frac{q}{3C} = \frac{CV_{WX}}{3C} = \frac{V_{WX}}{3}$ Hence, the p.d. across *C* is 3 times larger than the p.d. across 3*C*. For **WZY branch**, $2C = \frac{q_{WZY}}{V_{WZ}} \rightarrow q_{WZY} = 2C\left(V_{WZ}\right)$ $4C = \frac{q_{WZY}}{V_{ZY}} \rightarrow V_{ZY} = \frac{q_{WZY}}{4C} = \frac{2C\left(V_{WZ}\right)}{4C} = \frac{\left(V_{WZ}\right)}{2}$ Hence, the p.d. across 2*C* is 2 times larger than the p.d. across 4*C*. The total p.d. across the capacitor network is 24 V because half of the full current is registered at the ammeter at time *T*. Therefore, the p.d. across *C* is 18 V and the p.d. across 3*C* is 6 V. Likewise, the p.d. across 2*C* is 16 V and the p.d. across 4*C* is 8 V.
		(ii)	$\left(\frac{1}{C} + \frac{1}{3C}\right)^{-1} = \frac{3C}{4}$ $\left(\frac{1}{2C} + \frac{1}{4C}\right)^{-1} = \frac{4C}{3}$ $C_{eq} = \left(\frac{3C}{4} + \frac{4C}{3}\right) = 2.08C$
		(iii)	At $t = 0$ when the capacitors are uncharged, the current is $I_0 = \frac{48}{R}$ When the capacitors are charged, no current passes through the resistor, hence $48 = \frac{Q_0}{C_{eq}}$. Considering the potential differences in the circuit at time *t*,

			$48\text{ V} = IR + \frac{q}{C_{eq}} = \frac{dq}{dt}R + \frac{q}{C_{eq}}$ $\frac{Q_0}{C_{eq}} = \frac{dq}{dt}R + \frac{q}{C_{eq}} \rightarrow \frac{dq}{dt}R = \frac{Q_0}{C_{eq}} - \frac{q}{C_{eq}}$ $\frac{dq}{q-Q_0} = -\frac{dt}{RC_{eq}}$ Integrating the charge from 0 to Q as time increases from 0 to *t*, $\int_0^Q \frac{dq}{q-Q_0} = -\frac{1}{RC_{eq}}\int_0^t dt$ $\ln\left(q-Q_0\right)_0^Q = -\frac{t}{RC_{eq}} \rightarrow \frac{Q-Q_0}{-Q_0} = e^{-\frac{t}{RC_{eq}}}$ $Q = Q_0\left(1-e^{-\frac{t}{RC_{eq}}}\right)$
			Comments: Several students struggled at using correct limits of integration.
		(iv)	Differentiating the total charge with respect to time, we obtain the current $\frac{dQ}{dt} = I = \frac{Q_0}{-RC_{eq}}\left(-e^{-\frac{t}{RC_{eq}}}\right) = \frac{48\text{ V}}{R}e^{-\frac{t}{RC_{eq}}} = I_0 e^{-\frac{t}{RC_{eq}}}$ At $t = T$, the current is $\frac{I_0}{2}$. $\frac{I_0}{2} = I_0 e^{-\frac{T}{RC_{eq}}} \rightarrow T = \ln(2)\left(130\times10^3\right)\left(\frac{500\times10^{-6}}{4}\times2.0833\right) = 23.4\text{ s}$
			Comments: Several students could not produce a correct expression for current as a function of time.
		(v)	$C = \frac{500\times10^{-6}}{4} = 125\times10^{-6}\text{ F}$ $C = \frac{q}{V} = \frac{q}{18} \rightarrow q = 125\times10^{-6}\times18 = 2.25\times10^{-3}\text{ C}$
			Comments: Many did not use the correct p.d.

		(vi)	Energy stored in the capacitor network would be $E = \frac{1}{2} C_{eq} V_{WY}^2 = \frac{1}{2}\left(\frac{500 \times 10^{-6}}{4} 2.0833\right) 24^2 = 0.075 \text{ J}$
			Comments: Several students struggled and did not use correct values.
	(b)	**(i)**	E D B or F H C
			Comments: Many labelled the edges for the square rather than the corners.
		(ii)	There are altogether 12 capacitors. By joining the points with the same potentials, We have (3 capacitors in parallel: AE, AB, AD) in series to (6 capacitors in parallel: EF, EH, BF, BC, DC, DH) in series to (3 capacitors in parallel: FG, HG, CG) $\frac{1}{C_{eq}} = \frac{1}{3C} + \frac{1}{6C} + \frac{1}{3C}$ $C_{eq} = 1.2C = 1.2 \times 25 \mu F = 30 \mu F$
			Comments: Many could not obtain the total capacitance.