ANGLO-CHINESE JUNIOR COLLEGE
JC1 PROMOTIONAL EXAMINATION

Higher 2

MATHEMATICS 9758/01

Paper 1 **27 September 2024**
QUESTION PAPER **3 hr**

Additional Materials: Printed Answer Booklet
List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

1 Solve exactly the inequality

$$\frac{3x-2}{x-1} \le \frac{4}{x+2}.$$ [3]

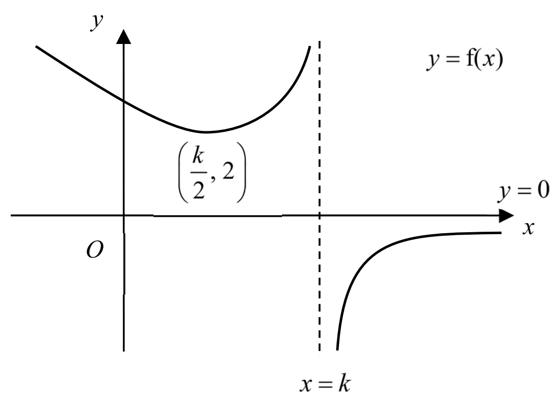
Hence solve $\dfrac{3|x|-2}{|x|-1} \le \dfrac{4}{|x|+2}$. [2]

2 It is given that $y = \sqrt{1+\tan x}$.

(i) Show that $2y\dfrac{\mathrm{d}y}{\mathrm{d}x} = 1+\left(y^2-1\right)^2$ and $y\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = 2y\left(y^2-1\right)\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)$. [3]

(ii) Hence find the first three terms in the Maclaurin expansion of y. [2]

3 The diagram below shows the graph of the curve $y = \mathrm{f}(x)$, with turning point at $\left(\dfrac{k}{2}, 2\right)$ and asymptotes with equations $x = k$ and $y = 0$.



The curve undergoes the following sequence of transformations in succession

A: Translate k units in the negative x-direction

B: Reflection about the y-axis

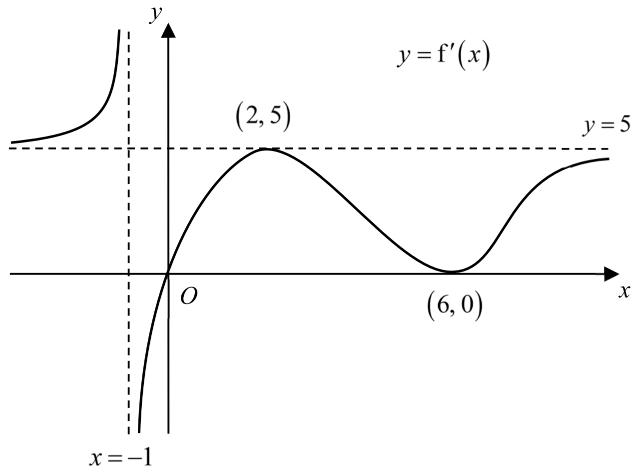
C: Translate k units in the positive x-direction

(i) Sketch the curve after the transformations, indicating clearly the equations of the asymptotes and the coordinates of the turning point. [2]

(ii) Find the equation of the curve after the transformations in the form $y = \mathrm{f}(ax+b)$ where a and b are constants to be determined, in terms of k where appropriate. [1]

(iii) Another curve $y = \mathrm{g}(x)$ is such that $\mathrm{g}(x) = \mathrm{g}(4-x)$ for all x. State the equation of a line of symmetry of this curve. [1]

4 The diagram shows the graph of $y = \mathrm{f}'(x)$, which passes through the origin and has turning points at $(2, 5)$ and $(6, 0)$. The graph of $y = \mathrm{f}'(x)$ has a vertical asymptote $x = -1$ and a horizontal asymptote $y = 5$.



(i) Sketch the graph of $y = \mathrm{f}''(x)$, stating the equation of asymptote(s) and the coordinates of any intersection(s) with the axes where possible. [3]

(ii) Write down the x-coordinate(s) of the stationary point(s) of $y = \mathrm{f}(x)$ and determine their nature. [2]

(iii) If the tangent to the graph of $y = \mathrm{f}(x)$ at $x = 2$ passes through the origin, find the equation of the normal to the graph of $y = \mathrm{f}(x)$ at $x = 2$. [2]

5 The functions f and g are defined as follows

$$\mathrm{f} : x \mapsto \frac{\ln(3x-2)}{x}, x > \frac{2}{3}$$

$$\mathrm{g} : x \mapsto 3x^2 - 12x + 13, x \in \mathbb{R}$$

(i) Sketch the graph of $y = \mathrm{f}(x)$, indicating the equations of asymptotes, coordinates of intersections with the axes and turning points, if any. [3]

(ii) Show that fg exists and find its range. [2]

The function h is defined as

$$\mathrm{h} : x \mapsto \mathrm{g}(x), x \le k$$

such that h^{-1} exists.

(iii) State the largest value of k. For this value of k, find h^{-1} in a similar form. [4]

6 Relative to the origin O, points A and B have position vectors **a** and **b** respectively, where **a** and **b** are non-zero and non-parallel vectors. The points M is the midpoint of OA and the point N lies on OB such that $ON : NB$ is $1:2$. The point C with position vector **c** is the point of intersection of lines AN and BM.

(i) Show that $\mathbf{c} = \frac{2}{5}\mathbf{a} + \frac{1}{5}\mathbf{b}$. [3]

(ii) Give the geometrical meaning of $\left|\mathbf{a}\cdot\hat{\mathbf{c}}\right|$, where $\hat{\mathbf{c}}$ is the unit vector in the direction of **c**. [1]

The points O, A and C lies on a circle where OA is the diameter of the circle. It is further given that $\mathbf{a}\cdot\mathbf{b} = \frac{1}{2}\left|\mathbf{a}\right|^2$.

(iii) Find the ratio of $\left|\mathbf{a}\right| : \left|\mathbf{c}\right|$. [3]

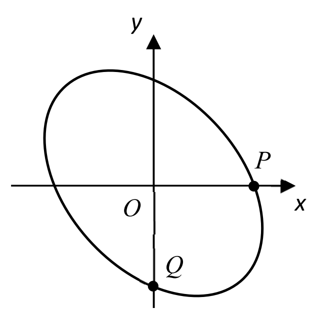
(iv) Hence find the area of triangle OAC, giving your answer in terms of $\left|\mathbf{a}\right|$. [2]

7 The parametric equations of a curve C are

$$x = 2a\cos\left(t + \frac{\pi}{6}\right),\ y = a\sin t,$$

where $0 \le t < 2\pi$ and a is a positive constant.

A sketch of curve C is shown below, with C intersecting the positive x-axis at P and the negative y-axis at Q.



(i) Show that the normal to a point on the curve C with parameter θ has gradient $1 + \sqrt{3}\tan\theta$. [3]

(ii) Show that $t = \frac{4\pi}{3}$ at the point Q. [1]

(iii) Hence, find the equation of the normal to the curve C at Q. [2]

The normal to the curve C at Q intersects the x-axis at T.

(iv) Show that the ratio $\frac{OT}{OP}$ is independent of a and find its value exactly. [3]

8 **(a)** It is given that $\sum_{r=1}^{n} \frac{1}{4r^2-1} = \frac{n}{2n+1}$.

(i) Explain why the series $\sum_{r=1}^{\infty} \frac{1}{4r^2-1}$ converges and write down the value to which it converges. [2]

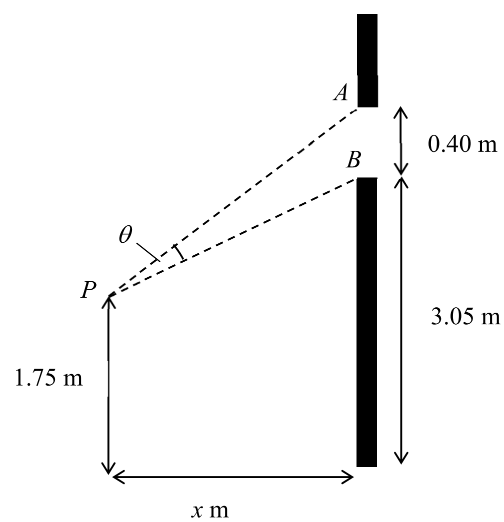
(ii) Find $\sum_{r=6}^{N} \frac{1}{(2r+1)(2r+3)}$ in terms of N, express your answer in a single fraction. [3]

(b) The sequence $u_1, u_2, u_3, \ldots$ is defined by $u_1 = 2$, $u_{n+1} = 1 - \frac{1}{u_n}$, $n \geq 1$.

Find the value of u_2, u_3 and u_4. Hence find the value of $\sum_{r=1}^{50} u_r$. [4]

9 As part of a game, a person stands x m away from a wall and tries to throw an object through a 0.40 m wide hole in the wall.

The point B denotes the bottom of the hole, which is located 3.05 m above the ground. The object is thrown from point P at a height of 1.75 m above the ground. The angle APB is denoted by θ in radians.



(i) Show that $\theta = \tan^{-1}\left(\frac{1.7}{x}\right) - \tan^{-1}\left(\frac{1.3}{x}\right)$. [2]

(ii) Using differentiation, find the value of x which gives the stationary value of θ, giving your answer to four significant figures. [3]

(iii) Show that the value of x in **(ii)** corresponds to a maximum value of θ, and suggest why knowing this value of x may be useful in this situation. [2]

(iv) The person walks away from the wall at a constant rate of 0.1 m/s. Find the rate of change of θ at the instant when the person is 1 m away from the wall. [2]

10 The curve C has equation $y=\dfrac{ax^2-2ax+a-2}{x-2}$, $a\in\mathbb{R}, a\neq 0$.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(ii) Find the range of values of a for which C has no stationary points. [3]

For $a=1$,

(iii) sketch the curve C, indicating the equations of asymptotes, coordinates of intersections with the axes and turning points, if any. [3]

(iv) By sketching a suitable curve on the diagram in **(iii)**, state the number of positive and negative roots of the equation $x^3-2x^2-x=k\left(x-2\right)$ where $k>0$. [2]

11 The line l_1 has the vector equation $\mathbf{r}=\begin{pmatrix}a\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}0\\1\\b\end{pmatrix}$, $\lambda\in\mathbb{R}$, where a and b are real constants.

The plane p_1 has cartesian equation $x+2y+4z=4$.

The line l_1 is perpendicular to the line l_2 with cartesian equation $\dfrac{1-y}{2}=z-3$, $x=5$.

The line l_1 intersects the plane p_1 at the point with coordinates $\left(a,1,0\right)$.

(i) By first finding the vector equation of line l_2, show that $a=b=2$. [3]

(ii) Verify that the point A with coordinates $\left(2,2,2\right)$ lies on l_1. [1]

(iii) Find the coordinates of the foot of perpendicular, F, from point A to the plane p_1. Hence find the shortest distance from point A to the plane p_1. [5]

(iv) The plane p_2 contains the line l_1 and the point F. Find the vector equation of the plane p_2, giving your answer in the form $\mathbf{r}\cdot\mathbf{n}=d$, where $\mathbf{n}$ is a vector and d is a real constant to be determined. [2]

12 The school's step challenge is an initiative by the PE department to encourage students to lead more active lifestyles. The table below shows the corresponding daily step counts and the number of health points that can be earned.

Steps Category	
Steps Count (Daily)	**Health Points (Daily)**
5000-7499	5
7500-9999	15
10000 and more	25 (Max)

Student A walks 4500 steps on the first day. For each subsequent day, he increases his step count by 350 steps from the previous day.

(a) **(i)** On which day will Student A first earn 15 Health Points in one day? [2]

(ii) How many Health Points would Student A earn by the end of the 20th day? [3]

Student B walks 7500 steps on the first day. For each subsequent day, he increases his step count by x % from the previous day.

(b) **(i)** Student B would like to complete a total of at least 261,500 steps in the first 20 days. Find the minimum value of x, giving your answer correct to 2 decimal points. [3]

(ii) Given that $x = 3.80$, find the total number of health points that Student B can earn in 20 days. [3]

(c) A student earns a \$2 coupon for completing at least 10,000 steps on Day 1. For each subsequent day, the student will earn a new coupon and the value of each new coupon decreases by 9% from the value of the previous day's coupon. If the student consistently completes 10,000 steps every day, show that the total coupon value accumulated over time will not exceed \$23. [2]

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Summary of Areas for Improvement			
Knowledge (K)	**Careless Mistakes (C)**	**Read/Interpret Qn wrongly (R)**	**Presentation (P)**