1. Quantities & Measurement

Content Page

Learning Objectives

1.1 Physical quantities and SI units

(a) Recall and use the following SI base quantities and their units: mass (kg), length (m), time (s), current (A), temperature (K), amount of substance (mol).

(b) Recall and use the following prefixes and their symbols to indicate decimal sub-multiples or multiples of both base and derived units: pico (p), nano (n), micro (μ), milli (m), centi (c), deci (d), kilo (k), mega (M), giga (G), tera (T).

(c) Express derived units as products or quotients of the base units and use the named units listed in 'Summary of Key Quantities, Symbols and Units' as appropriate.

(d) Use SI base units to check the homogeneity of physical equations.

(e) Make reasonable estimates of physical quantities included within the syllabus.

1.2 Errors and uncertainties

(f) Show an understanding of the distinction between random and systematic errors (including zero error), which limit precision and accuracy.

(g) Assess the uncertainty in derived quantities by adding absolute or relative (i.e. fractional or percentage) uncertainties or by numerical substitution (rigorous statistical treatment not required).

1.3 Scalars and vectors

(h) Distinguish between scalar and vector quantities, and give examples of each.

(i) Add and subtract coplanar vectors.

(j) Represent a vector as two perpendicular components.

All videos in this lecture notes can be found in this playlist:

1.1 PHYSICAL QUANTITIES AND SI UNITS

1.1.1 Importance of SI Base Units

The International System of Units (SI)[1] is the globally accepted standard for measurement. It ensures that scientists and engineers around the world can communicate their findings accurately and consistently. Here are some key reasons why SI units are crucial:

1. **Consistency**: SI units provide a consistent framework for measurements, which is essential for comparing results from different experiments and studies.
2. **Accuracy**: Using standardized units reduces the risk of errors and misunderstandings in scientific communication.
3. **Global Collaboration**: SI units facilitate international collaboration in science, engineering, and industry, as everyone uses the same measurement system.
4. **Education and Training**: Teaching and learning are simplified when a single, coherent system of units is used.

Case Study: Mars Climate Orbiter[2]

In 1999, NASA's Mars Climate Orbiter mission failed because of a mix-up between metric and imperial units. The spacecraft was lost because one team used imperial units (pounds) while another used metric units (newtons) for a crucial calculation. This incident highlights the importance of using a standardized system of units to avoid costly mistakes.

Self-study resources		
A SLS lesson on Physical Quantities and SI units	https://for.edu.sg/01si	

[1] The International System of Units (SI) **provides definitions of units of measurement** that are widely accepted in science and technology. The International Bureau of Weights and Measures (BIPM) located in Sèvres near Paris, France has the task of ensuring world-wide uniformity of measurements and their traceability to the SI. A*STAR's National Metrology Centre (NMC) is the national measurement institute of Singapore and it establishes and maintains measurement standards at the highest level of accuracy locally.

[2] Read more about the importance of indicating physical units in the found in Page 38 below

1.1.2 Physical Quantities: Base Quantities and Derived Quantities

In Physics, quantities that can be measured are known as **physical quantities**. Each quantity consists of a **numerical magnitude** and a **unit**. (For vectors, there are **directions** as well.)

Quantities can be classified as **base quantities** or **derived quantities**.

Base quantities are the 7 physical quantities of the *S.I. system* by which all other physical quantities are defined. They are arbitrarily chosen by scientists so that they:

a) form the smallest set of physical quantities that will lead to a complete description of physics in the simplest terms.

b) are based on international agreement by scientists.

Derived quantities are obtained from one or more of the base quantities through a **defining equation.**

Base Units

- There are 7 base units, one for each of the base quantities.
- **Base units** are the 7 base units of the *S.I. system*, related to a base quantity, whose magnitude is defined without referring to any other units. The units used in measurement are the International System of Units (SI).

Base Quantity	Base Unit	Name
Length	m	metre
Mass	kg	kilogram
Time	s	second
Electric current	A	ampere
Temperature	K	kelvin
Amount of substance	mol	mole
Luminous Intensity[3]	cd	candela

- "Kilometres", "Centimetres", "Grams", etc are multiples or sub-divisions of the respective SI Base units.

[3] * *Not in A level syllabus. Luminous intensity is "perceived brightness" and it is different from intensity because the sensitivity of the human eyes varies across different wavelengths. Hence a very intense x-ray source will have zero perceived brightness as seen by the human eye.*

S.I. Derived Units

- A **derived unit** can be expressed in terms of base units by using the defining equation of the quantity.
- It is obtained from the base units by multiplication and/or division; without including any numerical factors.
- There are 22 SI derived units that are given special names (scan QR on right). You do not need to memorise these special names.

https://for.edu.sg/non-si-derived-units

- Some examples of SI derived units are shown below:

DERIVED QUANTITIES	**Defining Equation**	**SI Derived Units**	**Special Name**
Volume	$Vol = l^3$	m^3	-
Velocity	$v = s/t$	$m\ s^{-1}$	-
Frequency	$f = 1/T$	s^{-1}	Hz
Force	$F = ma$	$kg\ m\ s^{-2}$	N
Work or Energy	$W = Fs$	$kg\ m^2\ s^{-2}$	J
Pressure	$P = F/A$	$kg\ m^{-1}\ s^{-2}$	Pa
Charge	$Q = It$	A s	C

- There are also some non-S.I. Units that are accepted for use together with S.I. units
- You may refer to the QR code on the right for some of the non-SI units that are accepted for use with SI units. You do not need to memorise any of them.

https://for.edu.sg/si-derived-unit-poster

- SI units are presented in index form i.e. m s^{-1} instead of m/s

A summary of Key Quantities, Symbols and Units can be found at the back of this lecture notes under Appendix.

Example 1.1

The energy E of a photon of frequency f is given by $E = hf$, where h is the Planck constant. What is the unit of h in base units?

Example 1.2

The drag coefficient C_D of a car moving with speed v through air of density ρ is given by $C_D = \frac{F}{2pv^2A}$ where F is the drag force exerted on the car and A is the maximum cross-sectional area of the car perpendicular to the direction of travel. Determine the units of the constant C_D.

A quick note about what "dimensionless" means

In older textbooks, the 7 base quantities are also called the 7 fundamental dimensions. Hence, you may encounter the term "dimensionless quantity" in your A level exam.

A dimensionless quantity is a quantity without any physical units and thus a pure number, for example the constant C_D in example 1.2 is a dimensionless quantity. However, there is one exception. Angles in physics is dimensionless but has a unit of "radian".

1.1.3 Prefixes

In physics, it is common to encounter quantities that are very large or minute in magnitude.

For example, Earth's mean radius is estimated to be 6 400 000 m. Radius of a hydrogen nucleus is approximately 0.000 000 000 000 001 3 m.

To include all the zeros in all computation or steps will be undesirable and hence scientists adopted any of the 2 methods: use of scientific notation (standard form) or **prefixes**.

Prefix	Symbol	Sub-multiple	Prefix	Symbol	Sub-multiple
Pico	p	10^{-12}	Kilo	k	10^{3}
Nano	n	10^{-9}	Mega	M	10^{6}
Micro	μ	10^{-6}	Giga	G	10^{9}
Milli	m	10^{-3}	Tera	T	10^{12}
Centi	c	10^{-2}			
Deci	d	10^{-1}			

There are two new prefixes (highlighted in blue) that you need to know in the A level Physics syllabus. The rest of the prefixes were introduced in the O level Physics syllabus.

Example 1.3

Convert the following measurements to their equivalent values using the appropriate prefixes.

5,000 m = ______ km

0.002 g = ______ mg

3,000,000 W = 3 ___

0.000001 s = 1 _____

2,000,000,000 Hz = ______ GHz

1.1.4 Homogeneity of Physical Equations

Dimensional Consistency

An equation must always be dimensionally consistent. Only physical quantities with the same dimension (e.g., length, time, mass) can be directly compared to each other.

Example: 1 cm is less than 1 inch (1 inch = 2.54 cm).

If physical quantities have different dimensions (e.g., mass vs. time), they cannot be compared.

Example: It is meaningless to ask whether a kilogram is greater than, equal to, or less than an hour.

This also means that any two quantities can only be equated, added, or subtracted if they have the same dimensions (i.e., they can be made to have the same units with appropriate conversions).

- **Example**:
  - 4 N + 2 kg: Dimensionally inconsistent (cannot be evaluated).
  - 4 cm – 2 mm: Dimensionally consistent.
    - Converting all terms to meters: 0.04 m – 0.002 m = 0.038 m.

Homogeneous equations

An equation where each term has the same base units (dimensionally consistent) is known as a homogeneous equation.

Rules for Homogeneous Equations

1. **Addition/Subtraction**: Only terms with the same units can be added or subtracted.

   e.g. $C = A + B$ implies that *A* and *B* must have the same units.

2. **Equality**: Units on both sides of the equation must be the same.

   e.g. $A = B$ implies that A and B must have the same units.

3. **Exponents**: The exponent of a term must not have any unit.

   e.g. $e^{\frac{-t}{RC}}$ implies that units of RC combined must have the same unit as time, t.

Important Note

A homogenous equation **may not be correct, but an equation that is not homogeneous is definitely incorrect**. For example, the equation: $E_k = mv^2$ is homogenous but incorrect because of a missing dimensionless constant term. The correct version is $E_k = \frac{1}{2}mv^2$.

Example 1.4

Which of the following is the correct expression for the period T of a simple pendulum of length l? g is the acceleration due to gravity.

(A) $T = 2\pi\sqrt{gl}$ **(B)** $T = \frac{2\pi}{\sqrt{gl}}$ **(C)** $T = 2\pi\sqrt{\frac{l}{g}}$ **(D)** $T = 2\pi\sqrt{\frac{g}{l}}$

Example 1.5

One of the equations of motion for constant acceleration is $s = ut + \frac{1}{2}at^2$ where s represents displacement, u is the initial velocity, t is time and a is acceleration.

Show that this equation is homogenous.

Reasonable estimations

"Reasonable estimates" refer to order of magnitude estimates, which represent values typically expressed to the nearest power of ten. These estimates are usually written to 1 significant figure (or at most 2 significant figures) and not more.

Some Examples (No need to memorise)

Length

Diameter of an atom	1×10^{-10} m (1 angstrom)
Diameter of nucleus	1×10^{-15} m
Wavelength of visible light	4 to 7×10^{-7} m

Density

Density of air	1 kgm^{-3}
Density of water	1000 kgm^{-3}
Density of metals	10000 kgm^{-3}

Mass

Mass of an apple	0.1 kg
Mass of person	60 kg
Mass of a car	1000 kg
Mass of a bus	10000 kg

Speed

Speed of running person	10 kmh^{-1} or 3 ms^{-1}
Speed of car	60 kmh^{-1} or 20 ms^{-1}
Speed of air molecules	400 ms^{-1} (at room temp)
Speed of light	3×10^{8} ms^{-1}
Speed of sound	300 ms^{-1}

Others

Atmospheric pressure	1×10^{5} Pa
Earth's magnetic field	5×10^{-5} T

1.2 ERRORS AND UNCERTAINTIES

Error refers to the difference between a measured quantity and its true value. Errors are unavoidable when taking measurements during an experiment. In experimental physics, the challenge is to invent better and more precise instruments or methodologies to determine fundamental physical constants (such as the speed of light, Planck constant, and rest masses of fundamental particles) with greater precision.

Any changes in these constants over time will significantly impact our understanding of the universe and its evolution.

Errors can be classified into two categories: Systematic Errors and Random Errors.

1.2.1 Systematic Errors

A systematic error is one that leads to readings that are consistently more or consistently less than the true value.

Note that the phrase is "consistently more or consistently less" NOT "consistently more or less."

- Systematic errors have the same magnitude and sign when measurements are repeated
- **Cause:** It is a reproducible error caused by imperfect equipment, wrongly calibrated instruments, experimental techniques, and/or environmental conditions.
- **Elimination**: It can be eliminated if the source is known. For example, zero error of a vernier caliper.
- **Detection**: To detect systematic errors, you need to:
    - Make measurements under different experimental conditions.
    - Use another technique to perform the experiment.
    - Use a second instrument to do the measurements and compare the readings or results. A consistently different set of results reveals the presence of systematic errors.

Some common Systematic Errors

1. **Zero Error**: Occurs when the pointer of an instrument does not exactly coincide with the zero mark when it is supposed to. Always check for zero errors before using instruments such as a micrometer screw gauge or vernier calipers.
2. **End Error**: For example, in some rulers, the 0-cm mark starts right at the edge of the ruler. End error occurs when there is wear and tear at the ends of such a ruler after being used for many years, such that the 0-cm mark is no longer present.

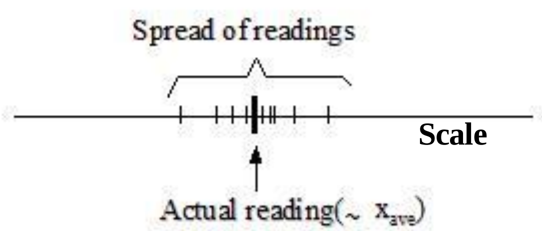
In experiments, systematic errors that can be accounted for or rectified should be corrected. They should not be included when discussing errors in your practical experiments.

1.2.2 Random Errors

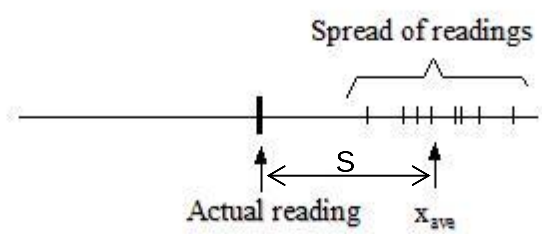
Even if every step of an experiment is done properly and all systematic errors are accounted for, repeated measurements of a quantity may not give identical values. The variation in the values of repeated measurements is known as random error.

- Random error have different magnitudes and signs when measurements are repeated.
- **Nature**: A random error gives rise to a **scatter of readings around a mean value**. Random errors cannot be eliminated, even if the source is known, because it is impossible to reproduce exactly the same conditions in each measurement.

Pure Random Error



Random with Systematic Error



S : Systematic Error

Types of Random Errors

- **Pure Random Error**: Variations purely due to random factors.
- **Random with Systematic Error**: Variations that include both random and systematic errors.

Reducing Random Errors

Random errors can be reduced by using an appropriate method of averaging. (See Example 1.6 below)

Some common Sources of Random Errors

1. **Fluctuating Environmental Conditions**: Such as temperature, pressure, vibrations, etc.
2. **Errors of Judgment**: For example, the observer's estimate of a fraction of the smallest division may vary from time to time.

Example: "Faster-than-Light" Neutrinos (more on page 38).

In the OPERA experiment, two sources of systematic error were not accounted for, resulting in the calculated speed of the neutrinos being faster than the speed of light in a vacuum. This highlights the importance of accounting for all potential sources of error to ensure accurate and reliable measurements.

1.2.3 Distinction between Systematic Errors and Random Errors

	Systematic Errors	Random Errors
Effect on readings	It causes the readings to be consistently more or consistently less than the actual value. This error has **a constant value**.	It causes a scatter of readings about a mean value.
Detection	It can be detected by making measurements under a **different experimental conditions** or by using **another technique** to perform the experiment. A consistently different set of results may reveal the presence of systematic errors. It can also be detected by comparing the trend of points in a plotted graph and the equation that relates the quantities.	It can be detected by plotting a graph and drawing a best-fit line to the points; the presence of random errors is reflected by the **scattering of points about the best-fit line.**
Minimization/ Elimination	Systematic errors **can be eliminated** if the cause is known and rectified.	**Random errors cannot be eliminated,** but **can be minimized by averaging**.

1.2.4 Precision and Accuracy

Accuracy is a measure of <u>how close</u> a measurement or <u>result is to the true value</u>. It depends on how well systematic errors can be controlled or compensated for.

Accuracy depends on the equipment used, the skill of the experimenter, and the techniques employed.

Precision is a measure of <u>how close</u> the repeated measured values are <u>to each other</u> regardless of true value. (We can also say precision is a measure of how small the spread of values are) It depends on how well random errors can be minimized.

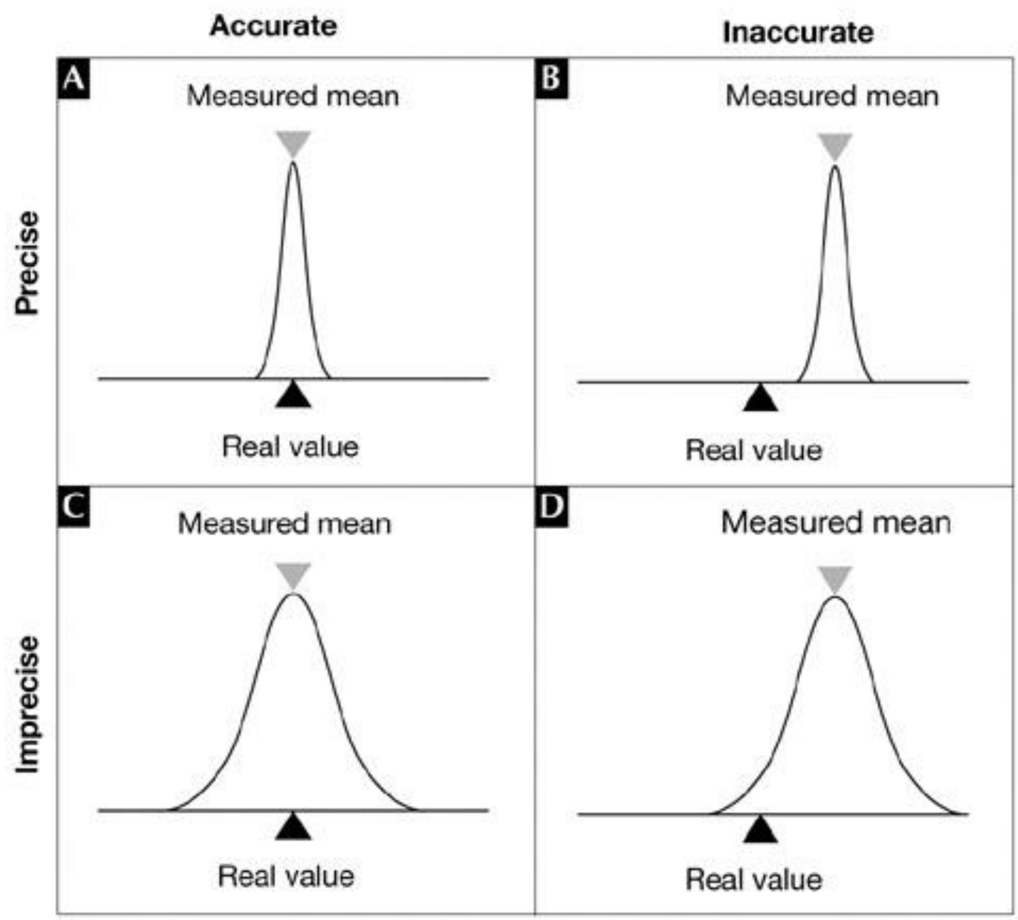
Precision is within the control of the experimenter. The experimenter may choose different measuring instruments and use them with varying levels of skill, affecting the precision of measurement.

Examples of representations of Precision vs Accuracy in Experimental measurements

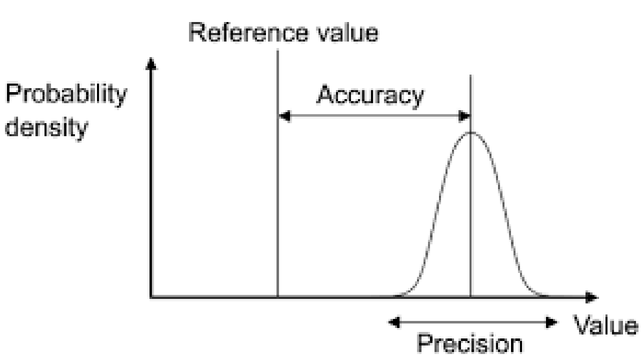
<u>Graphical representation</u>

y-axis: number of measurements or probability density

x-axis: value of measurement



In general:



When the peak (measured mean) coincides with the real value, the measurements are accurate.

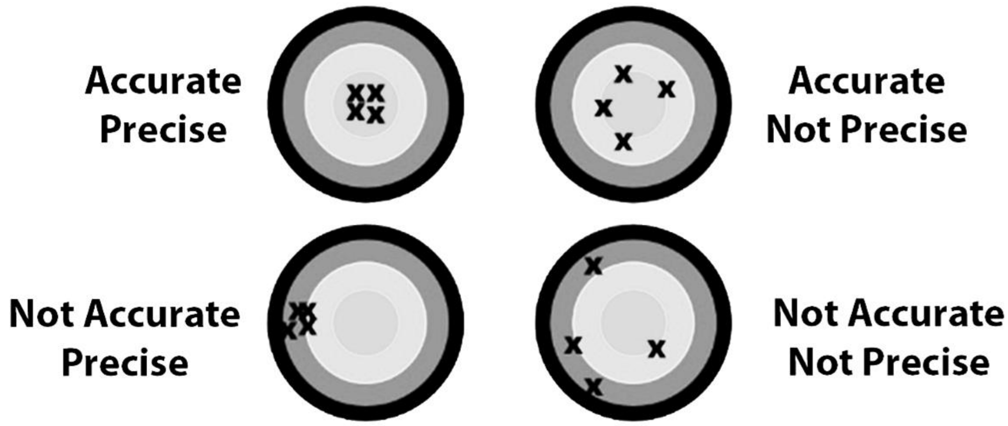
(inaccurate: peak is shifted)

When the peak is sharp (spread of readings are small), the measurements are precise.

(imprecise: peak is broad/flat)

Another representation – Target board:

In this target board, the bullseye (the center) represents the true value of what you're measuring. Each measurement is marked with a cross on the target board.



- If your crosses are spread out evenly around the bullseye, the average position of these crosses will be close to the center. This means your measurements are accurate.
- If your crosses are tightly clustered together, your measurements are precise.

Checkpoint!

Consider the following two groups of measurements of *x* which has a true value of 20.3 m s^{-1}.

	1st reading	2nd reading	3rd reading	4th reading	5th reading	average
Group 1	20.1	20.3	20.2	20.1	20.5	20.2
Group 2	20.1	20.1	20.2	20.2	20.1	20.1

Group _____ is more accurate because ____.

Group _____ is more precise because ____.

1.2.5 Experimental uncertainties

Self-study resources		
A SLS lesson on uncertainties	https://for.edu.sg/uncertainties	

Random errors and unknown systematic errors give rise to uncertainty in a measured value or in a calculated value of some physical quantity.

In Physics, every measurement comes with uncertainty.

Expressing measurements with its uncertainty

The uncertainty of any measurement is always rounded to 1 s.f. as it is usually an estimate.

The number of significant figures in a measurement indicates its precision. The measured value should be stated to the same precision as the uncertainty. For example, if ΔL = 0.5 cm, then the value of L (in cm) should be rounded off to one decimal place,

e.g.:

L = (16.9 ± 0.5) cm ✓

L = (16.93 ± 0.5) cm ✗

L = (17 ± 0.5) cm ✗

In the example above, the uncertainty is in the first decimal place. Hence the length measurement could vary from 16.4 cm to 17.4 cm.

Any additional decimal places beyond the first are not significant and do not improve the precision of the measurement of L.

Example 1.6

Express the following values in an appropriate form, corresponding to the units given.

L	ΔL	Correct form
37.3333 cm	0.0388 cm	
10.1667 mm	0.99 mm	
20793 m	1003 m	

In many cases, it may be necessary to express the value in standard form or using prefixes,

e.g.:

L = (0.0000237 ± 0.0000002) m	←	L = (2.37 ± 0.02) × 10^{-5} m
L = (17.0 ± 0.5) cm	←	L = (17.0 ± 0.5) × 10^{-2} m
L = (2400 ± 100) m	←	L = (2.4 ± 0.1) × 10^{3} m = (2.4 ± 0.1) km

Fractional and Percentage Uncertainties

Suppose a value R is written with its uncertainty: $R \pm \Delta R$

- actual or absolute uncertainty of $R = \pm \Delta R$
- fractional uncertainty of $R = \frac{\Delta R}{R}$. Fractional uncertainty has no unit.
- percentage uncertainty of $R = \frac{\Delta R}{R} \times 100\%$

Both fractional and percentage uncertainty is usually written to 2 significant figures.

Fractional or percentage uncertainty tells you how large the uncertainty is relative to the average/ measured value.

For example, it is only possible to judge if an actual uncertainty of ΔL = 0.5 cm is significant, if we know how large it is compared to the measured quantity:

Measurement and absolute uncertainty: $L \pm \Delta L$	Percentage uncertainty in $L = \frac{\Delta L}{L} \times 100\%$
$L = (17.0 \pm 0.5)\ cm$	$\frac{0.5}{17.0} \times 100\% = 2.9\%$
$L = (1.7 \pm 0.5)\ cm$	$\frac{0.5}{1.7} \times 100\% = 29\%$ (relatively large)
$L = (170.0 \pm 0.5)\ cm$	$\frac{0.5}{170} \times 100\% = 0.29\%$ (relatively small)

If the fractional or percentage uncertainty is very large, it means that the experiment is poorly conducted (i.e. poor technique, inappropriate procedure and/or wrong choice of instrument).

We can reduce the percentage uncertainty in a measurement by either:

(i) reducing absolute uncertainty by using a more precise instrument or
(ii) increasing the measured value (see example below).

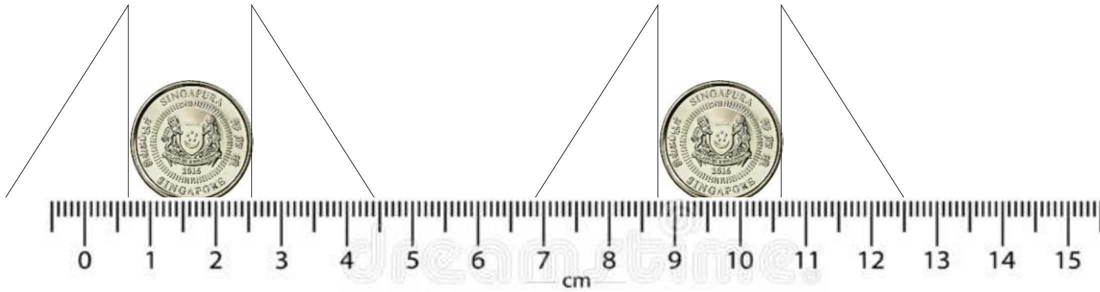
Example 1.7

A student wishes to determine the diameter of a 10-cent coin. He only has a 15-cm ruler with him and eight 10-cent coins. Which of the following two methods of averaging should the student use to reduce percentage error and why?

Method A:

Line up the eight coins in a straight line (as shown below), measure the total length from one end to another and then divide the length by 8 to get the average diameter of one coin.

Method B: Measure each of the 8 coins separately to get 8 readings and then take the average of the 8 readings to get the average diameter of one coin.



1.2.6 Consequential Uncertainty

Certain quantities, like speed, are sometimes derived indirectly from other measurements (e.g. distance and time). Estimating the consequential uncertainty of derived values is essential.

The table below shows the two most commonly used formulae in calculating uncertainties in a derived quantity. The symbols k, m, n and p represent constants (e.g. 4, 2.7 and 6π) while A, B and C represents independent variables.

You can choose to memorise the two formulae or you can use first principles, for example: $\Delta Z = Z_{max} - Z_{average}$ to calculate the uncertainty.

	Equation	Uncertainty	Remarks
1	$Y = mA + nB$ or $Y = mA - nB$ Pure addition and/or subtraction	$DY = mDA + nDB$	Note that for both addition and subtraction, we <u>add</u> the uncertainties.
2	$Z = k\left(\frac{A^n B^m}{C^p}\right)$ Pure multiplication and/or division (including exponential)	$\frac{\Delta Z}{Z} = \lvert n\rvert\left(\frac{\Delta A}{A}\right) + \lvert m\rvert\left(\frac{\Delta B}{B}\right) + \lvert p\rvert\left(\frac{\Delta C}{C}\right)$	In this case, it is easier to calculate the fractional uncertainty first before multiplying the fractional uncertainty with average Z to get ΔZ.

Note that the absolute values of n, m and p are always used when calculating the uncertainty.

First principle method (max – best method):

1) Determine Z_{max}

2) Determine $Z_{average}$

3) Calculate $\Delta Z = Z_{max} - Z_{average}$

Example 1.8

Given that $Y = 3A - 2B$, prove from first principles, that the uncertainty in Y is given by $\Delta Y = 3\Delta A + 2\Delta B$.

Example 1.9

In an experiment to determine the resistivity of a piece of metallic wire, the following measurements were obtained:

Resistance of wire: $R = 0.800 \pm 0.002\ \Omega$

Length of wire: $l = 1.000 \pm 0.002$ m

Diameter of wire: $d = 0.50 \pm 0.01$ mm

The resistivity, ρ of the wire is given by $\rho = \frac{\pi}{4}\left(\frac{Rd^2}{l}\right)$.

Determine the value of ρ together with its uncertainty using

(a) $\Delta\rho = \rho_{max} - \rho$

(b) $\frac{\Delta\rho}{\rho} = \frac{\Delta R}{R} + \frac{\Delta l}{l} + 2\frac{\Delta d}{d}$

Example 1.10

The period T of a simple pendulum of length L is given by the following equation

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where g is the acceleration of free fall.

In an experiment to measure the acceleration g, the length L is measured as (6.25 ± 0.05) cm.

What is the maximum percentage uncertainty in T such that g can be determined with a maximum uncertainty of $\pm$ 2% ?

A 0.6 % **B** 1.2 % **C** 1.4 % **D** 2.8 %

Example 1.11

The period of a pendulum was measured to be T = (2.46 $\pm$ 0.04) s.
The frequency of the pendulum is given as $f = \frac{1}{T}$. Determine f and its associated uncertainty.

Method 1: First principle, Max – average method

Method 2: Formula method

Asymmetric Uncertainty

Example 1.12

Given that $\theta = (25 \pm 2)°$. Determine the value of $\cos\theta$ together with its associated uncertainty.

θ /°	$\cos(\theta/°)$
23	0.9205
25	0.9063
27	0.8910

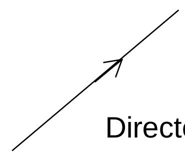
1.3 SCALARS AND VECTORS

Scalar quantities are physical quantities that can be represented by a **magnitude** only.

They do not have a direction associated with them. Some scalar quantities, like electric charge, magnetic flux, and potential energy, may have a positive or negative sign, which does not indicate direction.

Vector quantities are physical quantities that possess a **magnitude** and a **direction** in space.

They are represented by a directed line segment (which is basically an "arrow"). The length of the "arrow" represents the magnitude of the vector while the direction of the "arrow" indicates direction



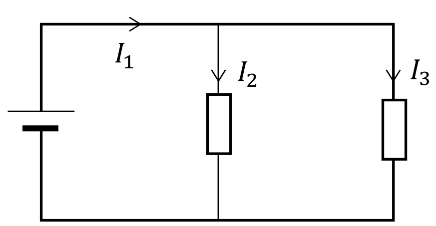
Directed line segment

For A level Physics, vector quantities are common, so proficiency in vector operations is essential.

Example 1.13

Is electric current a scalar or vector quantity?

Solution



Example 1.14

Categorise the following quantities into Scalar vs Vector Quantity

Distance, displacement, speed, temperature, velocity, acceleration, energy, force, power, mass, density, momentum, weight, moment / torque

Scalar Quantity	Vector Quantity

1.3.1 Vector addition

The resultant of two or more vectors is found by

- ➢ Method 1(A): Scale drawing
- ➢ Method 1(B): Sketch the vectors and apply the sine rule, cosine rule, or geometry
- ➢ Method 2 : Resolve each vector into perpendicular components, then add these components separately and use pythagora's theorem to determine resultant.

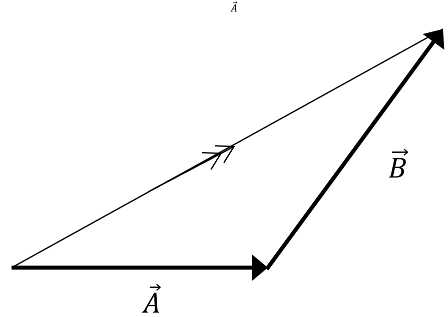
The resultant vector is usually represented by a double arrow in a sketched diagram.

Note: For A level Physics, scale drawing is only used if explicitly requested in the question.

Sketching Techniques for Vector addition of vectors $\vec{A}$ and $\vec{B}$:

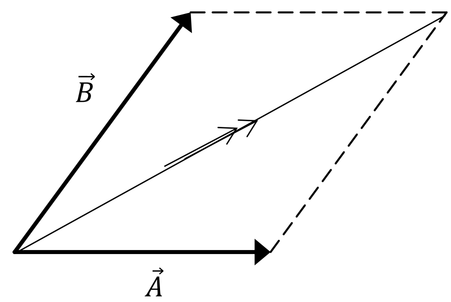
Triangle method

1) Draw the first vector $\vec{A}$
2) Draw the second vector $\vec{\boldsymbol{B}}$ with its tail at the head of the first vector $\vec{A}$
3) The resultant vector joins the tail of the first to the head of the second.

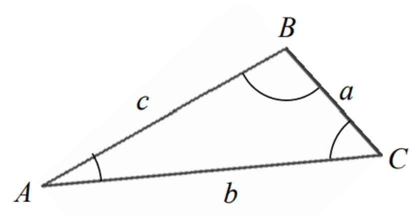


Parallelogram method

1) Draw both vectors $\vec{A}$ and $\vec{\boldsymbol{B}}$ from the same starting point, forming adjacent sides of a parallelogram.
2) Complete the parallelogram with dotted lines.
3) The resultant vector is the diagonal from the common starting point to the point the dotted lines intersect.



Useful Formulae:

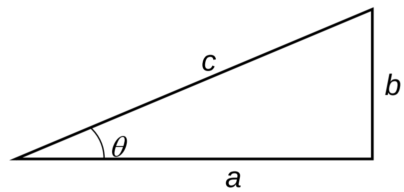


Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Only for right-angled triangle:

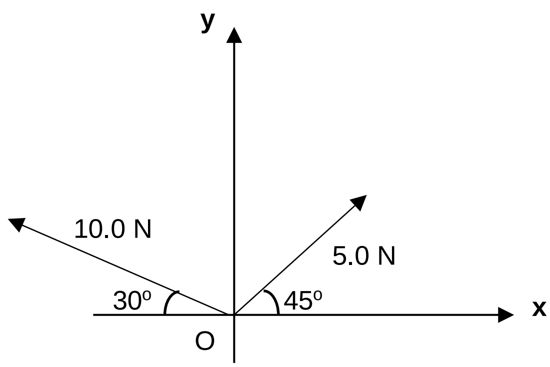
Pythagoras theorem: $a^2 + b^2 = c^2$



Example 1.15

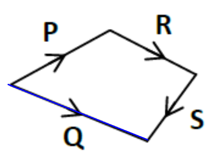
The figure shows two forces acting at a point O. Determine the magnitude and direction of the resultant force.

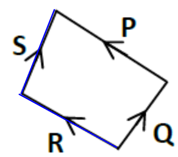
Solution:

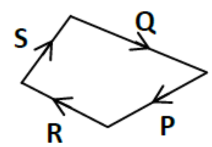


Example 1.16

(a) Write down the equation relating vectors **P**, **Q**, **R** and **S** for each of the diagrams given below:







(b) The three vector diagrams shown above are drawn to scale. If all 4 vectors in each diagram are added together, which diagram has the largest resultant?

Solution:

Example 1.17

A man starts from point O, walks 4.0 km due east to point A, followed by 3.6 km in a direction 56.3° north of east to point P. Determine the displacement of P relative to O.

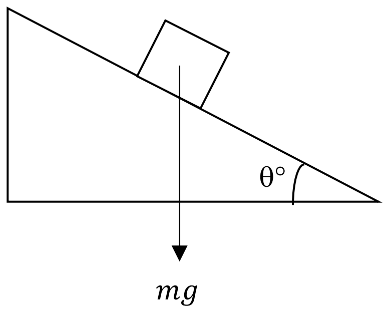
Solution

Method 2: By Resolving Vectors perpendicularly

Self-study resources		
A SLS lesson on 2-D Vector Addition by Resolving into Perpendicular Components	https://for.edu.sg/2dresolve	

Example 1.18

A box of weight mg rests on a slope that is inclined at $\theta°$ to the horizontal. Draw and label the components of the weight of the object parallel to and perpendicular to the slope in terms of mg and θ.



It is easier to see the relation between the angles if we model the box as a point mass

Points to note from Example 1.18:

(1) When resolving components of vectors, axes don't have to be vertical and horizontal. In this example, it is more convenient to choose the axes to be parallel to the slope and perpendicular to the slope because the box is expected to move along the slope (no movement in the direction normal to slope).

(2) The "side" or component touching or adjacent to θ is multiplied by "cosθ" while the side opposite θ is multiplied by "sinθ". The original vector to be resolved is **always the hypotenuse** of the right angle triangle.

(3) When we resolve a vector into its perpendicular components, the vector effectively replaced by its two components. We should avoid double counting the original vector. Normally you may leave the original vector visible, but draw the two components in dotted lines to show the breakdown.

1.3.2 Scalar multiplication of a vector

When you multiply a vector by a scalar, the direction of the vector is not affected.

- If the scalar is greater than 1, the magnitude of the vector increases
- If the scalar is less than 1, the magnitude of the vector decreases
- If the scalar is 1, then there is no change to the vector.
- If the scalar is -1, then the vector reverses direction while preserving its original magnitude.

1.3.3 Vector subtraction

Change in a vector

In many physics context, it is common to consider a 'change in vector' e.g. change in velocity, change in displacement

Consider an example where we are asked to subtract vectors to determine a change in velocity, Δv.

Since $\Delta v = v_f - v_i$ where v_f and v_i denote the final and initial velocity of some object respectively.

We can rewrite the above equation as $\Delta v = v_f + (-v_i)$

Since multiplying the vector v_i by -1 just reverses its direction and addition of vectors is the same as finding resultant of the vectors, $\Delta \boldsymbol{v}$ is just the resultant of $\boldsymbol{v_f}$ and $(-\boldsymbol{v_i})$.

Example 1.19

A car is initially travelling 12 ms^{-1} due east. Calculate the change in velocity of the car if its final velocity is

(a) 16 ms^{-1} due east
(b) 9 ms^{-1} due east
(c) 16 ms^{-1} due west
(d) 12 ms^{-1} at 60° north of east

Solve using Method 1B (Sketch, Cosine rule, sine rule) and Method 2 (resolving vectors). You need to know both methods.

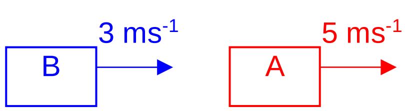
Solution

Relative velocity

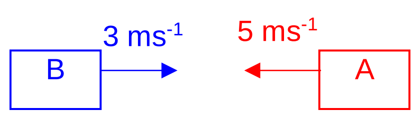
Vector subtraction will also be required when finding the relative velocity of an object. If car A is travelling at velocity $\overrightarrow{v_A}$ on a road and car B is travelling at velocity $\overrightarrow{v_B}$ on the same road, then the following applies:

Velocity of A relative to B, $\vec{v}_{\text{A relative to B}} = \overrightarrow{v_A} - \overrightarrow{v_B}$

You can use "common sense" to understand the above formula (taking rightwards as positive):



Velocity of A relative to B = $5 - 3$ = 2 ms^{-1}



Velocity of A relative to B = $-5 - 3$ = – 8 ms^{-1}

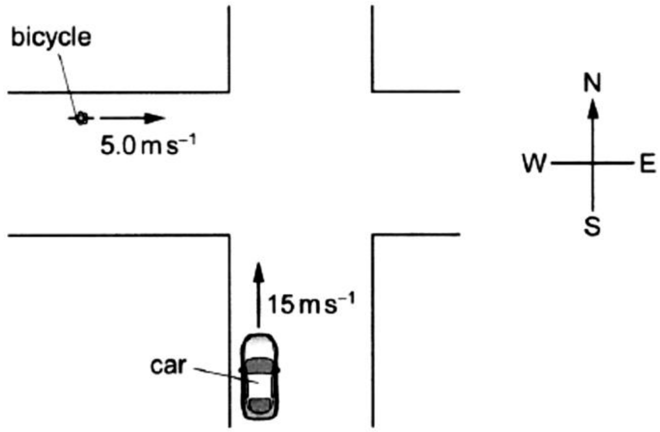
Velocity is a vector, so need to assign rightwards positive. Since A is moving leftwards, its velocity is -5 ms^{-1}.

Relative velocity of car B with respect to car A is looking at how car B is moving from the perspective of car A. In Mathematical symbol, we can denote it as $\boldsymbol{V}_{BA}$.

Example 1.20 (modified from 2020 A Level P1Q5)

A car and a bicycle are equal distances from a crossroads. The car is travelling north with a speed of 15 m s^{-1}. The bicycle is travelling east with a speed of 5.0 m s^{-1}.

At this instant, what is the velocity of the bicycle relative to the car?



<u>Solution</u>

1.3.4 Scalar Product of two vectors (dot product)

Certain quantities such as work done (w) by a force involves the scalar product of two vectors (force and displacement). The scalar product of two vectors, for example, force $\vec{F}$ and displacement $\vec{s}$ is defined by:

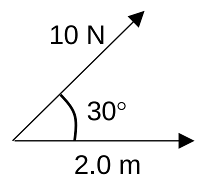
$$w = \vec{F} \cdot \vec{s} = |\vec{F}||\vec{s}| \cos \theta$$ where θ is the angle between $\vec{F}$ and $\vec{s}$

The dot is the symbol used to denote the scalar product.

It is called a "scalar" product because the final quantity, in this case w, is a scalar quantity.

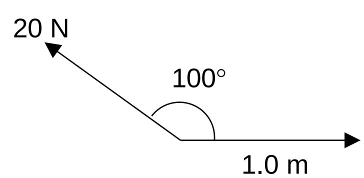
Example 2.9

(a) Calculate the work done by the 10 N force if the displacement is 2.0 m in the direction shown below:



Solution

(b) Calculate the work done by the 20 N force if the displacement is 1.0 m in the direction shown below:



ADDITIONAL READING MATERIALS

1. The Importance of Indicating Physical Units in Data

Mars Climate Orbiter Incident

In 1998, NASA launched the Mars Climate Orbiter to study the Martian climate. However, the mission failed because of a mix-up between metric and imperial units:

- **NASA's Navigation Team**: Used metric units (millimeters and meters).
- **Lockheed Martin**: Provided data in imperial units (inches, feet, and pounds).

https://for.edu.sg/nasa-mco-article

This mismatch caused the spacecraft to enter Mars' atmosphere incorrectly, leading to its destruction[1].

Gimli Glider Incident

In 1983, an Air Canada flight ran out of fuel mid-flight due to a similar unit conversion error:

- **Canada's Metrication**: Canada switched from imperial to metric units in the 1970s.
- **Fuel Calculation Error**: The crew calculated fuel in pounds instead of kilograms, resulting in only half the needed fuel being loaded.

https://for.edu.sg/gimli-accident

The plane, known as the Gimli Glider, had to glide to a safe landing thanks to the pilots' skill[2].

Why This Matters

These incidents show why it's crucial to always indicate physical units in data:

- **Prevents Misunderstandings**: Ensures everyone is on the same page, avoiding dangerous mistakes.
- **Maintains Accuracy**: Accurate data is essential for safety and success in scientific and engineering projects.
- **Promotes Consistency**: Using standard units helps maintain consistency across different teams and systems.

Understanding and correctly using physical units is vital for the reliability and safety of scientific and engineering endeavors.

2. The importance of accounting for experimental uncertainty.

In 2011, scientists working on the OPERA experiment thought they had observed neutrinos traveling faster than light. This was surprising because, according to Einstein's theory of special relativity, nothing can travel faster than light.

What Happened?

- **Initial Observation**: The OPERA team announced in September 2011 that neutrinos seemed to travel faster than light. This result was unexpected and caused a lot of excitement and skepticism in the scientific community.
- **Errors Found**: By early 2012, the team discovered two major errors in their experiment:
  - A fiber optic cable was not properly connected, which made the neutrinos appear to travel faster.
  - A clock oscillator was ticking too fast, further contributing to the error.
- **Corrections and Confirmation**: After fixing these issues, the OPERA team, along with other experiments like ICARUS, found that neutrinos travel at speeds consistent with the speed of light.

Why is This Important?

This incident highlights the importance of carefully accounting for experimental uncertainties. Even small mistakes in the setup or measurement can lead to incorrect conclusions. Scientists must always double-check their work and be open to scrutiny from others to ensure their findings are accurate.

Understanding and correcting these uncertainties is crucial for the progress of science, as it helps maintain the reliability and credibility of scientific discoveries.[4][5][6]

[4] https://en.wikipedia.org/wiki/2011_OPERA_faster-than-light_neutrino_anomaly
[5] https://thereader.mitpress.mit.edu/when-science-fails-opera-neutrinos
[6] https://www.sciencedaily.com/releases/2011/09/110923084425.htm

APPENDIX

SUMMARY OF KEY QUANTITIES, SYMBOLS AND UNITS

The following list illustrates the symbols and units that will be used in question papers.

Quantity	***Usual symbols***	***Usual unit***
Base Quantities		
mass	m	kg
length	l	m
time	t	s
electric current	I	A
thermodynamic temperature	T	K
amount of substance	n	mol
Other Quantities		
distance	d	m
displacement	s, x	m
area	A	m^2
volume	V, v	m^3
density	ρ	$kg\,m^{-3}$
speed	u, v, w, c	$m\,s^{-1}$
velocity	u, v, w, c	$m\,s^{-1}$
acceleration	a	$m\,s^{-2}$
acceleration of free fall	g	$m\,s^{-2}$
force	F	N
weight	W	N
momentum	p	N s
work	w, W	J
energy	E, U, W	J
potential energy	E_p	J
kinetic energy	E_k	J
heating	Q	J
change of internal energy	ΔU	J
power	P	W
pressure	p	Pa
torque	T, τ	N m
gravitational constant	G	$N\,kg^{-2}\,m^2$
gravitational field strength	g	$N\,kg^{-1}$
gravitational potential	ϕ	$J\,kg^{-1}$
angle	θ	°, rad
angular displacement	θ	°, rad
angular speed	ω	$rad\,s^{-1}$
angular velocity	ω	$rad\,s^{-1}$

Quantity	***Usual symbols***	***Usual unit***
period	T	s
frequency	f	Hz
angular frequency	ω	rad s^{-1}
wavelength	λ	m
speed of electromagnetic waves	c	m s^{-1}
electric charge	Q	C
elementary charge	e	C
electric potential	V	V
electric potential difference	V	V
electromotive force	E	V
resistance	R	Ω
resistivity	ρ	Ω m
capacitance	C	F
electric field strength	E	N C^{-1}, V m^{-1}
permittivity of free space	ε_0	F m^{-1}
magnetic flux	Φ	Wb
magnetic flux density	B	T
permeability of free space	μ_0	H m^{-1}
force constant	k	N m^{-1}
Celsius temperature	θ, T	°C
specific heat capacity	c	J K^{-1} kg^{-1}
molar gas constant	R	J K^{-1} mol^{-1}
Boltzmann constant	k	J K^{-1}
Avogadro constant	N_A	mol^{-1}
number	N, n, m	
number density (number per unit volume)	n	m^{-3}
Planck constant	h	J s
work function energy	Φ	J
activity of radioactive source	A	Bq
decay constant	λ	s^{-1}
half-life	$t_{\frac{1}{2}}$	s
relative atomic mass	A_r	
relative molecular mass	M_r	
atomic mass	m_a	kg, u
electron mass	m_e	kg, u
neutron mass	m_n	kg, u
proton mass	m_p	kg, u
molar mass	M	kg mol^{-1}
proton number	Z	
nucleon number	A	
neutron number	N	

DATA AND FORMULAE

Data

speed of light in free space	$c = 3.00 \times 10^{8}$ m s^{-1}
permeability of free space	$\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}
permittivity of free space	$\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1}
	$(\frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^{9}$ m F$^{-1})$
elementary charge	$e = 1.60 \times 10^{-19}$ C
Planck constant	$h = 6.63 \times 10^{-34}$ J s
unified atomic mass constant	$u = 1.66 \times 10^{-27}$ kg
rest mass of electron	$m_e = 9.11 \times 10^{-31}$ kg
rest mass of proton	$m_p = 1.67 \times 10^{-27}$ kg
molar gas constant	$R = 8.31$ J K^{-1} mol^{-1}
Avogadro constant	$N_A = 6.02 \times 10^{23}$ mol^{-1}
Boltzmann constant	$k = 1.38 \times 10^{-23}$ J K^{-1}
gravitational constant	$G = 6.67 \times 10^{-11}$ N m^{2} kg^{-2}
acceleration of free fall	$g = 9.81$ m s^{-2}

Formulae

uniformly accelerated motion	$s = ut + \frac{1}{2}at^2$
	$v^2 = u^2 + 2as$
work done on / by a gas	$W = p\Delta V$
pressure	$p = \frac{F}{A}$
gravitational potential	$\phi = -\frac{GM}{r}$
temperature	$T/\text{K} = T/{}^\circ\text{C} + 273.15$
pressure of an ideal gas	$p = \frac{1}{3}\frac{Nm}{V}\langle c^2 \rangle$
mean translational kinetic energy of an ideal gas particle	$E = \frac{3}{2}kT$

9478 PHYSICS GCE ADVANCED LEVEL H2 SYLLABUS

displacement of particle in s.h.m.	$x = x_0 \sin \omega t$
velocity of particle in s.h.m.	$v = v_0 \cos \omega t = \pm \omega \sqrt{(x_0^2 - x^2)}$
electric current	$I = nAvq$
resistors in series	$R = R_1 + R_2 + \ldots$
resistors in parallel	$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$
capacitors in series	$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$
capacitors in parallel	$C = C_1 + C_2 + \ldots$
energy in a capacitor	$U = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$
charging a capacitor	$Q = Q_0\left[1 - e^{\frac{t}{\tau}}\right]$
discharging a capacitor	$Q = Q_0 e^{\frac{t}{\tau}}$
RC time constant	$\tau = RC$
electric potential	$V = \frac{Q}{4\pi\varepsilon_0 r}$
alternating current / voltage	$x = x_0 \sin \omega t$
magnetic flux density due to a long straight wire	$B = \frac{\mu_0 I}{2\pi d}$
magnetic flux density due to a flat circular coil	$B = \frac{\mu_0 N I}{2r}$
magnetic flux density due to a long solenoid	$B = \mu_0 n I$
energy states for quantum particle in a box	$E_n = \frac{h^2}{8mL^2} n^2$
radioactive decay	$x = x_0 e^{-\lambda t}$
radioactive decay constant	$\lambda = \frac{\ln 2}{t_{\frac{1}{2}}}$