RAFFLES INSTITUTION
H2 Mathematics (9758)
2017 Year 6

2017 H2 Math 9758 Preliminary Examination Paper 2 : Suggested Solutions

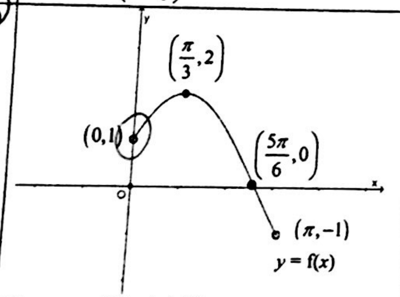
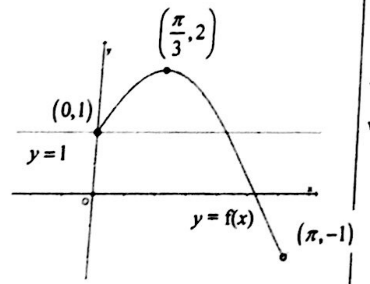
Qn 1		Comments
(i) [4]	$\overrightarrow{OA}=\begin{pmatrix}2\\3\\-1\end{pmatrix}$, $\overrightarrow{OB}=\begin{pmatrix}5\\-2\\3\end{pmatrix}$, $\overrightarrow{OC}=\begin{pmatrix}4\\1\\-2\end{pmatrix}$ Since M is the mid-point of AC, $\overrightarrow{OM}=\dfrac{\begin{pmatrix}2\\3\\-1\end{pmatrix}+\begin{pmatrix}4\\1\\-2\end{pmatrix}}{2}=\frac{1}{2}\begin{pmatrix}6\\4\\-3\end{pmatrix}$ $\overrightarrow{AB}=\begin{pmatrix}5\\-2\\3\end{pmatrix}-\begin{pmatrix}2\\3\\-1\end{pmatrix}=\begin{pmatrix}3\\-5\\4\end{pmatrix}$ $\overrightarrow{AM}=\frac{1}{2}\begin{pmatrix}6\\4\\-3\end{pmatrix}-\begin{pmatrix}2\\3\\-1\end{pmatrix}=\begin{pmatrix}1\\-1\\-\frac{1}{2}\end{pmatrix}$ Area of ΔABM $=\frac{1}{2}\left\lvert\overrightarrow{AB}\times\overrightarrow{AM}\right\rvert$ $=\frac{1}{2}\left\lvert\begin{pmatrix}3\\-5\\4\end{pmatrix}\times\begin{pmatrix}1\\-1\\-\frac{1}{2}\end{pmatrix}\right\rvert$ $=\frac{1}{4}\left\lvert\begin{pmatrix}3\\-5\\4\end{pmatrix}\times\begin{pmatrix}2\\-2\\-1\end{pmatrix}\right\rvert=\frac{1}{4}\left\lvert\begin{pmatrix}13\\11\\4\end{pmatrix}\right\rvert=\frac{\sqrt{306}}{4}=\frac{3\sqrt{34}}{4}$	Many candidates did this question correctly. Some candidates wrongly expressed $\overrightarrow{OM}$ as $\frac{1}{2}\overrightarrow{AC}$. A small minority did not realize that the question asked for "exact area", and left their answer as 4.37, rounded to 3 sf, which is not acceptable. For those who attempted to express the final answer as exact, do note that values like $\frac{\sqrt{76.5}}{2}$ can be simplified to $\frac{3\sqrt{34}}{4}$ by writing 76.5 as a fraction and rationalizing the denominator.

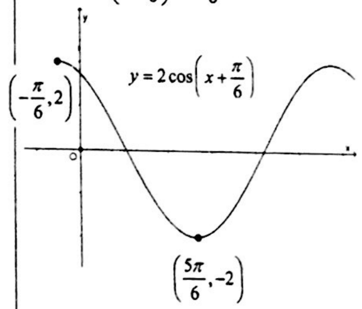
	Or: $\overrightarrow{AB}=\begin{pmatrix}5\\-2\\3\end{pmatrix}-\begin{pmatrix}2\\3\\-1\end{pmatrix}=\begin{pmatrix}3\\-5\\4\end{pmatrix}$, $\overrightarrow{AC}=\begin{pmatrix}4\\1\\-2\end{pmatrix}-\begin{pmatrix}2\\3\\-1\end{pmatrix}=\begin{pmatrix}2\\-2\\-1\end{pmatrix}$ Area of ΔABC $=\frac{1}{2}\left\lvert\overrightarrow{AB}\times\overrightarrow{AC}\right\rvert$ $=\frac{1}{2}\left\lvert\begin{pmatrix}3\\-5\\4\end{pmatrix}\times\begin{pmatrix}2\\-2\\-1\end{pmatrix}\right\rvert$ $=\frac{1}{2}\left\lvert\begin{pmatrix}13\\11\\4\end{pmatrix}\right\rvert=\frac{\sqrt{306}}{2}$ Area of $\Delta ABM=\frac{1}{2}$Area of $\Delta ABC=\frac{\sqrt{306}}{4}=\frac{3\sqrt{34}}{4}$	
(ii) [4]	l_{AB}: $\mathbf{r}=\begin{pmatrix}2\\3\\-1\end{pmatrix}+\lambda\begin{pmatrix}3\\-5\\4\end{pmatrix}$, $\lambda\in\mathbb{R}$ Since point N is on the line AB, $\overrightarrow{ON}=\begin{pmatrix}2\\3\\-1\end{pmatrix}+\lambda\begin{pmatrix}3\\-5\\4\end{pmatrix}$ for some λ $\overrightarrow{MN}=\begin{pmatrix}3\lambda-1\\1-5\lambda\\\frac{1}{2}+4\lambda\end{pmatrix}$ For $\overrightarrow{MN}$ to be perpendicular to $\overrightarrow{AB}$, $\overrightarrow{MN}.\overrightarrow{AB}=0$ $\begin{pmatrix}3\lambda-1\\1-5\lambda\\\frac{1}{2}+4\lambda\end{pmatrix}.\begin{pmatrix}3\\-5\\4\end{pmatrix}=0$ $\lambda=\frac{3}{25}$ $\overrightarrow{ON}=\begin{pmatrix}2\\3\\-1\end{pmatrix}+\frac{3}{25}\begin{pmatrix}3\\-5\\4\end{pmatrix}=\frac{1}{25}\begin{pmatrix}59\\60\\-13\end{pmatrix}$	Most candidates demonstrated understanding of the concept of finding $\overrightarrow{MN}$ and using it to find λ. A significant number of the candidates who got the wrong answer made careless mistakes in the following arithmetic operations i.e. $\overrightarrow{MN}=\overrightarrow{ON}-\overrightarrow{OM}$ $=\begin{pmatrix}2+3\lambda\\3-5\lambda\\-1+4\lambda\end{pmatrix}-\begin{pmatrix}3\\2\\-1.5\end{pmatrix}$ $=\begin{pmatrix}\vdots\\\vdots\\2.5+4\lambda\end{pmatrix}$ and $\begin{pmatrix}3\lambda-1\\1-5\lambda\\\frac{1}{2}+4\lambda\end{pmatrix}.\begin{pmatrix}3\\-5\\4\end{pmatrix}=0$ $\cdots\cdots\cdots+2+8\lambda=0$

Qn 2		Comments
(a)(i) [1]	$\frac{1}{r-1}-\frac{2}{r}+\frac{1}{r+1}=\frac{r(r+1)-2(r-1)(r+1)+r(r-1)}{r(r-1)(r+1)}$ $=\frac{r^2+r-2(r^2-1)+r^2-r}{r(r-1)(r+1)}$ $=\frac{2}{r(r-1)(r+1)}$	Almost all students managed to show this. Just a note that it might be easier to start from LHS and combine the fraction to arrive at RHS instead of trying to break up RHS into partial fractions.
(a)(ii) [4]	$\sum_{r=3}^{n}\frac{4}{r(r-1)(r+1)}=2\sum_{r=3}^{n}\frac{2}{r(r-1)(r+1)}$ $=2\sum_{r=3}^{n}\left(\frac{1}{r-1}-\frac{2}{r}+\frac{1}{r+1}\right)$ $=2\left[\frac{1}{2}-\frac{2}{3}+\frac{1}{4}\right.$ $+\frac{1}{3}-\frac{2}{4}+\frac{1}{5}$ $+\frac{1}{4}-\frac{2}{5}+\frac{1}{6}$ $+\frac{1}{5}-\frac{2}{6}+\frac{1}{7}$ $+\ldots$ $+\frac{1}{n-3}-\frac{2}{n-2}+\frac{1}{n-1}$ $+\frac{1}{n-2}-\frac{2}{n-1}+\frac{1}{n}$ $\left.+\frac{1}{n-1}-\frac{2}{n}+\frac{1}{n+1}\right]$ $=2\left(\frac{1}{6}-\frac{1}{n}+\frac{1}{n+1}\right)=\frac{1}{3}-\frac{2}{n}+\frac{2}{n+1}$	Most students were able to recognize that this involved MOD. However, some common mistakes were still prevalent: 1. Forgot about the factor 2, i.e. $\sum_{r=3}^{n}\frac{4}{r(r-1)(r+1)}$ $=\sum_{r=3}^{n}\left(\frac{1}{r-1}-\frac{2}{r}+\frac{1}{r+1}\right)$ 2. Did not write down the correct leftover terms after the cancellations, i.e either missed out $\frac{1}{3}$ or $\frac{1}{n}$ in the final expression. 3. Question mentioned that "There is no need to express answer as a single algebraic fraction" but that does not mean that liked terms need not be simplified. 4. Some students tried to split the sum up without realizing that r **cannot** start

		from 1, e.g. $\sum_{r=3}^{n}\left(\frac{1}{r-1}-\frac{2}{r}+\frac{1}{r+1}\right)$ $=\sum_{r=1}^{n}\left(\frac{1}{r-1}-\frac{2}{r}+\frac{1}{r+1}\right)$ $-\sum_{r=1}^{2}\left(\frac{1}{r-1}-\frac{2}{r}+\frac{1}{r+1}\right)$
(b)(i) [2]	Amount Amy saves in nth week $=25+(n-1)(4)=21+4n$ Amount Ben saves in nth week $=ar^{n-1}=2(1.22)^{n-1}$ When Ben saves more than Amy, $2(1.22)^{n-1}>21+4n$ From GC, in the 20th week, Amy saves \$101, Ben saves \$87.47 in the 21st week, Amy saves \$105, Ben saves \$106.72 Hence, Ben first saves more than Amy in the 21st week. Or: $2(1.22)^{n-1}>21+4n \Rightarrow 2(1.22)^{n-1}-21-4n>0$ When $n=20$, $2(1.22)^{n-1}-21-4n=-13.5<0$ When $n=21$, $2(1.22)^{n-1}-21-4n=1.72>0$	Students need to recognize the use of GC for these 2 parts based on the number of marks and also the nature of the inequality which is not easy to solve algebraically. There were a number of students who could not progress after forming the inequality. Some common mistakes made for both (i) and (ii): 1. Use the sum of AP and GP formulae for both parts. 2. Use only the nth term formulae for both parts. 3. Forgot either the sum of AP/GP formula or the nth term formula for AP/GP. 4. Take $r=0.22$.
(b)(ii) [2]	Total amount in Amy's account after nth week, $=\frac{n}{2}(2a+(n-1)d)=\frac{n}{2}(50+(n-1)(4))=\frac{n}{2}(46+4n)$ Total amount in Ben's account after nth week, $=\frac{a(r^n-1)}{r-1}=\frac{2(1.22^n-1)}{1.22-1}$ For their total saving to exceed \$2400, $\frac{n}{2}(46+4n)+\frac{2(1.22^n-1)}{1.22-1}>2400$ From GC, in the 22nd week, total savings= \$2186.89 < \$2400	

	in the 23rd week, total savings= \$2458.72 > \$2400 $\therefore$ least $n=23$. Hence, their total saving will exceed \$2400 after 23 complete weeks.	

Qn 3		Comments
(i) [2]	$f(x)=R\sin(x+\alpha)=R\sin x\cos\alpha+R\cos x\sin\alpha$ compare with $f(x)=\sqrt{3}\sin x+\cos x$ $\Rightarrow R\cos\alpha=\sqrt{3},\ R\sin\alpha=1$ $\Rightarrow R=\sqrt{1^2+\sqrt{3}^2}=2,\quad \alpha=\tan^{-1}\frac{1}{\sqrt{3}}=\frac{\pi}{6}$ $f(x)=2\sin\left(x+\frac{\pi}{6}\right)$	Most students are able to do this part correctly.
(ii) [3]	 The range of f is $(-1,2]$.	Students have to learn to read the question carefully. Many did not label the intercepts. Students also need to consider the domain of f and indicate the coordinates of the end points on the sketch. Quite a handful miss out the open circle on both the end points. A few did not write down the range although they got the graph correct, which is very wasteful.
(iii) [2]	 	The key word here is **"exactly"**. This means that students have to show algebraic working to get the correct value of $\frac{2\pi}{3}$. Answers without working will not get any credit.

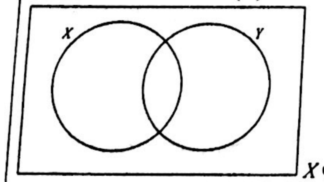
	$f(x)=2\sin(x+\frac{\pi}{6})=1$ $\Rightarrow x+\frac{\pi}{6}=\frac{\pi}{6}$ or $\pi-\frac{\pi}{6}$ $\Rightarrow x=0$ or $\frac{2\pi}{3}$ The set of values of x is $\left[\frac{2\pi}{3},\pi\right)$	
(iv) [1]	$g(x)=2\cos\left(x+\frac{\pi}{6}\right),\quad -\frac{\pi}{6}\le x\le b.$  For g^{-1} to exist, g has to be a 1-1 function. The largest exact value of b, is $\pi-\frac{\pi}{6}=\frac{5\pi}{6}$.	
(v) [3]	The domain of g^{-1} = the range of g = $[-2,2]$. Range of f $=(-1,2]\subseteq$ Domain of g^{-1} $=[-2,2]$, therefore $g^{-1}f$ exists. $g^{-1}f(x)=x,\quad 0<x<\pi$ $g(x)=f(x)$ $2\cos(x+\frac{\pi}{6})=2\sin(x+\frac{\pi}{6})$ $\tan(x+\frac{\pi}{6})=1$ $x+\frac{\pi}{6}=\frac{\pi}{4}$ $x=\frac{\pi}{12}$	Many students failed to see that the easiest way to solve this is to solve $g(x)=f(x)$. Many proceeded to find the function $g^{-1}f(x)$.

	(Note: $0 < x \le \frac{5\pi}{6}$ considering domain of f and g)	Again the key word is **"exactly"**. Marks will not be awarded for answers without algebraic working.

Qn 4		Comments
(i) [3]	$l_1 : \mathbf{r} = \begin{pmatrix}0\\0\\1\end{pmatrix} + \lambda\begin{pmatrix}-3\\12\\4\end{pmatrix},\ \lambda \in \mathbb{R}$ $l_2 : \mathbf{r} = \begin{pmatrix}1\\4\\1\end{pmatrix} + \mu\begin{pmatrix}-3\\1\\4\end{pmatrix},\ \mu \in \mathbb{R}$ Since $\begin{pmatrix}-3\\12\\4\end{pmatrix}$ and $\begin{pmatrix}-3\\1\\4\end{pmatrix}$ are not parallel, l_1 and l_2 are not parallel. If the two lines intersect, there will be a unique value of λ and μ for the system of equations $\begin{pmatrix}0\\0\\1\end{pmatrix} + \lambda\begin{pmatrix}-3\\12\\4\end{pmatrix} = \begin{pmatrix}1\\4\\1\end{pmatrix} + \mu\begin{pmatrix}-3\\1\\4\end{pmatrix}$ $-3\lambda + 3\mu = 1 \quad (1)$ $12\lambda - \mu = 4 \quad (2)$ $4\lambda - 4\mu = 0 \quad (3)$ Using GC, no solution of λ and μ exist. Hence, the lines do not intersect. Hence, l_1 and l_2 are skew lines.	Common mistakes: - Obtained $\begin{pmatrix}-3\\0\\4\end{pmatrix}$ as the direction vector for l_2 instead of $\begin{pmatrix}-3\\1\\4\end{pmatrix}$. - Concluded that l_1 and l_2 are skew lines without showing that they are not parallel.
(ii) [3]	A normal to $p = \begin{pmatrix}-3\\12\\4\end{pmatrix} \times \begin{pmatrix}-3\\1\\4\end{pmatrix} = \begin{pmatrix}44\\0\\33\end{pmatrix} = 11\begin{pmatrix}4\\0\\3\end{pmatrix}$	Most students did well for this part, although there were a few who did not give the final equation in cartesian form.

	Since (1, 4, 1) lies on l_2 which is on p, $\begin{pmatrix}1\\4\\1\end{pmatrix} \cdot \begin{pmatrix}4\\0\\3\end{pmatrix} = 7$ Hence a cartesian equation for p is $4x + 3z = 7$.	
(iii) [6]	(0, 0, 1) is a point on l_1. $\begin{pmatrix}0\\a\\1\end{pmatrix} - \begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}0\\a\\0\end{pmatrix}$ Distance from A to l_1 $= \frac{\left\lvert \begin{pmatrix}0\\a\\0\end{pmatrix} \times \begin{pmatrix}-3\\12\\4\end{pmatrix} \right\rvert}{\left\lvert \begin{pmatrix}-3\\12\\4\end{pmatrix} \right\rvert} = \frac{1}{13}\left\lvert \begin{pmatrix}4a\\0\\3a\end{pmatrix} \right\rvert = \frac{1}{13}\lvert a\rvert \left\lvert \begin{pmatrix}4\\0\\3\end{pmatrix} \right\rvert = \frac{5}{13}\lvert a\rvert$ (1, 4, 1) is a point on p. $\begin{pmatrix}0\\a\\1\end{pmatrix} - \begin{pmatrix}1\\4\\1\end{pmatrix} = \begin{pmatrix}-1\\a-4\\0\end{pmatrix}$ Distance from A to p $= \frac{\left\lvert \begin{pmatrix}-1\\a-4\\0\end{pmatrix} \cdot \begin{pmatrix}4\\0\\3\end{pmatrix} \right\rvert}{\left\lvert \begin{pmatrix}4\\0\\3\end{pmatrix} \right\rvert} = \frac{1}{5}\lvert -4\rvert = \frac{4}{5}$	Note that this method is much faster than finding foot of perpendicular, F from A to l_1 and then taking the length of $\overline{AF}$. Instead of factorising $\lvert a\rvert$ out from $\begin{pmatrix}4a\\0\\3a\end{pmatrix}$, most students simplify the expression using definition, i.e., $\left\lvert \begin{pmatrix}4a\\0\\3a\end{pmatrix} \right\rvert = \sqrt{(4a)^2 + 0^2 + (3a)^2} = \sqrt{25a^2}$, which many mistakenly simplify as $5a$ instead of $5\lvert a\rvert$, hence obtaining only one value of a as final answer in the last step. Similarly, this method is much faster than finding foot of perpendicular, N from A to p and then taking the length of $\overline{AN}$. Many of those who attempted to find $\overline{OF}$ and $\overline{ON}$ committed careless mistakes and lost marks for not getting the correct position vectors and distances.

	Since the point A is equidistant to p and l_1, $\frac{5}{13}\lvert a\rvert=\frac{4}{5}$ $a=\pm\frac{52}{25}$	

Qn 5		Comments
[3]	$P(X\mid Y)=\frac{1}{2}$ $\Rightarrow \frac{P(X\cap Y)}{P(Y)}=\frac{1}{2}$ $\Rightarrow P(X\cap Y)=\frac{1}{2}P(Y)$ $P(Y\mid X)=\frac{2}{3}$ $\Rightarrow \frac{P(X\cap Y)}{P(X)}=\frac{2}{3}$ $\Rightarrow P(X\cap Y)=\frac{2}{3}P(X)$ $P(X\cap Y)=\frac{1}{2}P(Y)=\frac{2}{3}P(X)$ $P(X\cup Y)=\frac{5}{6}$ $\Rightarrow P(X)+P(Y)-P(X\cap Y)=\frac{5}{6}$ $\Rightarrow P(X)+\frac{4}{3}P(X)-\frac{2}{3}P(X)=\frac{5}{6}$ $\Rightarrow P(X)=\frac{1}{2}$	Most students were able to do this part correctly. The most common method used was to apply standard results to obtain equations connecting P(X), P(Y) and P($X\cap Y$). Unfortunately, some wrong formula were seen, for example: $P(X\mid Y)=\frac{P(X\cap Y)}{P(X)}$ instead of $\frac{P(X\cap Y)}{P(Y)}$, $P(Y\mid X)=\frac{P(Y\cap X)}{P(Y)}$ instead of $\frac{P(Y\cap X)}{P(X)}$, and most frequently, $P(X\cup Y)=P(X)+P(Y)+P(X\cap Y)$ instead of $P(X)+P(Y)-P(X\cap Y)$. Interestingly, there were a few students who used their GC to solve the three equations, obtaining $P(X)=\frac{1}{2}$, $P(Y)=\frac{2}{3}$ and $P(X\cap Y)=\frac{1}{3}$ all at one go.
(ii) [2]	$P(X\cap Y)=\frac{2}{3}P(X)=\frac{2}{3}\left(\frac{1}{2}\right)=\frac{1}{3}$  $X\cup Y'$	Significant fewer number of students were able to handle this part well. The most common problem was the failure to understand $P(X\cup Y')$. A large number of students tried to simplify it through the result $P(X\cup Y')=P(X)+P(Y')-P(X\cap Y')$, but often end up with a complicated sum

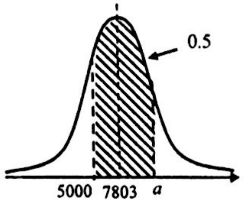
	$P(X\cup Y')=1-P(Y)+P(X\cap Y)$ $=1-\frac{4}{3}\left(\frac{1}{2}\right)+\frac{1}{3}$ $=\frac{2}{3}$	/ difference which eventually worked out to be P(Y') instead. Those students who drew a Venn diagram were generally quick and successful in identifying the correct region and thus were able to obtain the correct answer rather effortlessly.

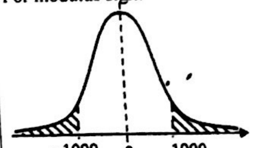
Qn 6		Comments
[6]	Let $X=W-100$. Then we have $\sum x=26$, $\sum x^2=273$ $\overline{w}=\frac{1}{50}\sum x+100=\frac{26}{50}+100=100.52$ $s_w^{\,2}=s_x^{\,2}=\frac{1}{n-1}\left[\sum x^2-\frac{\left(\sum x\right)^2}{n}\right]$ $=\frac{1}{49}\left(273-\frac{26^2}{50}\right)$ $=5.29551$ To test $H_0:\mu=100$ vs $H_1:\mu>100$ Perform a 1–tail test at 3% level of significance. Under H_0, $\overline{W}\sim N\left(\mu_0,\frac{s^2}{n}\right)$ **approximately** where $\mu_0=100$ and $n=50$ Using a z - test, p – value $=0.0550$ (3 s.f.) Since p – value $=0.0550>0.03$, we **do not reject** H_0 and conclude that there is **insufficient** evidence, at 3% significance level, that the salesman made an understatement on the average power consumption of the Effixion laptops.	Define your random variable if you are not using W. Do not confuse μ with $\overline{w}$. If we know the population mean, there would be no need to perform hypothesis testing. W is normally distributed, so do not quote CLT. However, there is still an approximation due to use of the unbiased **estimate** of population variance.
[4]	To test $H_0:\mu=100$ vs $H_1:\mu>100$ Perform a 1–tail test at 1% level of significance. Sample variance $=6.25$ $s^2=\frac{n}{n-1}$[sample variance]	Sample variance is **neither** population variance, **nor** its unbiased estimate.

	$= \frac{50}{49}(6.25) = 6.37755$ In order to reject H_0 at 1% level of significance, we need $P(\overline{W} > \overline{w}) \le 0.01$ where $\overline{W} \sim N\left(100, \frac{6.37755}{50}\right)$ approximately $\Rightarrow P(\overline{W} < \overline{w}) \ge 0.99$ $\Rightarrow \overline{w} \ge 100.8308$ Or $z-value \ge z_{0.99}$ $\frac{\overline{w}-100}{\sqrt{\frac{6.37755}{50}}} \ge 2.3263$ $\overline{w} \ge 100.8308$ (4 d.p.) Set of values of $\overline{w}$ is $\{\overline{w} \in \mathbb{R} : \overline{w} > 100.83\}$ or $\{\overline{w} \in \mathbb{R} : \overline{w} \ge 100.84\}$	Note the alternative hypothesis is right-tailed. Standardisation is not necessary, as parameters of $\overline{W}$ are assumed to be known in the test. Due to rounding off, 100.83 is not within the range, while 100.84 is.

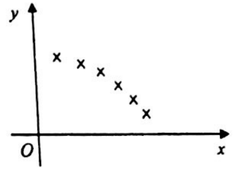
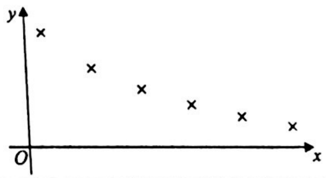
Qn 7		Comments
(i) [4]	| Die shows | 1 | 2 | 3 | | Probability | $\frac{1}{6}$ | $\frac{1}{3}$ | $\frac{1}{2}$ | P(Adrian pays Benny in a round) = P(total score is 2) + P(total score is 3) $= \frac{1}{6}\times\frac{1}{6} + \left(\frac{1}{6}\times\frac{1}{3}\right)2$ $= \frac{5}{36}$ Let Y be the number of rounds, out of 20, that Adrian pays Benny. $Y \sim B\left(20, \frac{5}{36}\right)$ $P(Y \ge 5)$ $= 1 - P(Y \le 4)$ $= 0.134$ (3 s.f.)	This is NOT the usual 6 sided die. Using a table of this form will help to determine the total score of 2 rolls easily. For P(total score is 3), there are 2 cases : 1st roll 1 & 2nd roll 2 or 1st roll 2 & 2nd roll 1. Define the random variable Y properly and state its distribution. Be careful when taking the complement as the GC can only compute $P(Y \le k)$ and not $P(Y < k)$ for Binomial Distributions.

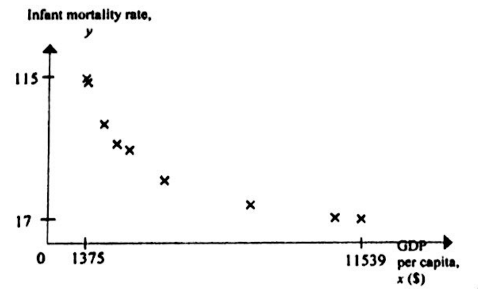
(ii) [5]	P(total score is 4) $= \left(\frac{1}{6}\times\frac{1}{2}\right)2 + \frac{1}{3}\times\frac{1}{3} = \frac{5}{18}$ Method 1 : P(total score is 5 or 6) $= 1 - \left(\frac{1}{36}+\frac{1}{9}+\frac{5}{18}\right) = \frac{7}{12}$ Method 2 : P(total score is 5 or 6) $= \left(\frac{1}{3}\times\frac{1}{2}\right)2 + \frac{1}{2}\times\frac{1}{2} = \frac{7}{12}$ Given : X represents Benny's winnings in each round | x | 6 | 3 | −2 | 0 | | $P(X=x)$ | $\frac{1}{36}$ | $\frac{1}{9}$ | $\frac{5}{18}$ | $\frac{7}{12}$ | $E(X) = -\frac{1}{18}$ Since $E(X) < 0$, Benny is expected to lose in the long run. Thus, Benny should not accept Adrian's invitation to play the game.	Method 1 is faster, but it does not "help to check" that you are on the right track. Use Method 2 (unless finding this probability is way too long/tedious) to help you check that $\sum P(X=x) = 1$. Many did not realize that x can be zero. Do check that $\sum P(X=x) = 1$. If the probabilities do not add up, check the individual probabilities and also re-evaluate if there were missing x values. $E(X)$ is the long-run average winnings of Benny in one round. Thus, $E(X) < 0$ does not imply that Benny is expected to lose every round.
(iii) [1]	For the game to be fair, $E(X) = 0$. $\frac{a}{36} + \frac{1}{3} - \frac{10}{18} = 0$ $a = 8$	

Qn 8		Comments
(ai) [1]	Let X be the random variable denoting the household income per capita in dollars of a randomly chosen family in Country S. Then $X \sim \text{N}(2601,\ 768^2)$. $\text{P}(X>1800) = 0.852$ (3s.f.)	A significant number of students misread the question and found the probability to be less than 1800 dollars instead.
(aii) [3]	Let Y be the random variable denoting the gross income in dollars of a randomly chosen family with 3 family members. $Y = 3X \sim N(3\times 2601, 9\times 768^2)$ $\Rightarrow Y \sim \text{N}(7803, 9\times 768^2)$ $\text{P}(5000 < Y < a) = 0.5$ $\Rightarrow \text{P}(Y<a) - \text{P}(Y<5000) = 0.5$ $\Rightarrow \text{P}(Y<a) = 0.5 + \text{P}(Y<5000) = 0.61188$ $\Rightarrow \text{P}(Y<a) = 0.61188$ $\Rightarrow a = 8458$ (to nearest dollars) Alternative: $\text{P}(5000 < Y < a) = 0.5$ $\text{P}\left(\frac{5000}{3} < X < \frac{a}{3}\right) = 0.5$ since $Y = 3X$ $\Rightarrow \text{P}\left(X < \frac{a}{3}\right) - \text{P}\left(X < \frac{5000}{3}\right) = 0.5$ $\Rightarrow \text{P}\left(X < \frac{a}{3}\right) = 0.5 + \text{P}\left(X < \frac{5000}{3}\right) = 0.61188$ $\Rightarrow \frac{a}{3} = 2819.28$ (2dp) $\Rightarrow a = 8458$ (to nearest dollar)	Many made the mistake of $Y = X_1 + X_2 + X_3$, which is the sum of the income per capita of three randomly chosen households. The correct relationship can be derived from the first line in the question as $X = \frac{Y}{3}$. A number of students assumed symmetry which is not true. Many did not get the correct relationship of the probabilities. A sketch of normal distribution curve may help. 
(bi) [2]	Let V be the random variable denoting the family's monthly expenditure in dollars. Then $V \sim \text{N}(\mu,\ 650^2)$ $\text{P}(V>5900) = 0.1$ $\Rightarrow \text{P}(V<5900) = 0.9$ $\Rightarrow \text{P}(Z < \frac{5900-\mu}{650}) = 0.9$ From GC: $\text{P}(Z < 1.28155) = 0.9$ $\Rightarrow \frac{5900-\mu}{650} = 1.28155$ $\Rightarrow \mu = 5067$ (to the nearest dollar)	Common mistakes: - Some did not find the probability on the left tail. Note: for invNorm on TI 84 Plus C SE, the area is shaded from the left, i.e. lower tail. - Some did the standardisation wrongly as $\frac{\mu-5900}{650}$ Those who arrived at the answer 5067 with both of the above mistakes were not awarded full credits.

(bii) [4]	Let W be the random variable denoting the family's monthly saving in dollars. $W = T + 1500 - V$, where T denotes Mr Tan's monthly income. Then $W \sim N(7500-5067, 1000^2 + 0 + 650^2)$ i.e. $W \sim N(2433, 1422500)$ and $W_1 - W_2 \sim N(0, 2845000)$ $\Rightarrow \text{P}(\lvert W_1 - W_2 \rvert > 1000)$ $= 2\text{P}(W_1 - W_2 < -1000)$ $= 0.553$ (3sf)	Some did not find the distribution of the DIFFERENCE between 2 months. For modulus sign: 
(biii) [1]	It is assumed that Mr Tan's income and the family's expenditure in a particular month are independent. Alternatively, Mr Tan's income and the family's expenditure in a month are independent of what he earned and how much the family spent in another month.	We need to assume INDEPENDENCE to be able to add and subtract normal distributions to obtain new normal distributions.

Qn 9		Comments

(i)
[2]
	For (A): $y = a + bx^2$, with $a > 0$ and $b < 0$.
	
	
	For (B): $y = c + d\ln x$, with $c > 0$ and $d < 0$.
	
	
		A handful of students plotted points on the axes though the questions asked for positive x and y values.
		
		A number of students also plotted more than 6 data points.
		
		Students should take note that the data points should not lie close to a straight line to clearly illustrate the relationship between x and y.

(ii) [2]		Students should ensure that the scatter plot is sketched to scale. Many scatter plots were out of proportion.
(iii) [2]	From GC, the product moment correlation coefficient is −0.898 (3s.f.). Since −0.898 is close to −1, it suggests a strong negative linear correlation between X and Y. However, it can be observed from the scatter plot that the values of y are decreasing at a decreasing rate with increasing values of x, which will not be the case if the data follows a linear model (the decrease in y should be approximately constant for a linear model).	Many students fail to elaborate on the implication of the value of r in relation to the linear relationship between the variables. They also did not mention the behavior of y as x increases. Many simply mentioned that the scatter plot does not exhibit a linear relationship.
(iv) [3]	(B) is the appropriate model. Using model (B), the product moment correlation coefficient is −0.978 (3s.f.). From GC, $y = 430.30 - 45.010(\ln x)$. $\therefore y = 430 - 45.0 \ln x$ (to 3s.f.)	Most students did well in this part of the question, except for those who had made errors in their data entry. Students are reminded to leave their final answers in 3 sf (unless specified otherwise). Thus, intermediate working should be at least 5 sf.
(v) [3]	At a GDP per capita of \$723, the infant mortality rate is estimated to be $y = 430.30 - 45.010(\ln 723) = 133.98$ (5s.f.) $= 134$ (3s.f.) Since $x = 723$ is outside the range of the data values, the estimation is not reliable.	Many students used 3 sf answer found in (iv) to obtain the estimated value of y. This affects their accuracy of answers.

Qn 10		Comments
(a) [3]	Complement Method: Probability $= 1 -$ P(22 people were all born on different days) $= 1 -$ P(21 people were all born on different days) $\times \frac{365-21}{365}$ $\approx 1 - 0.55631 \times \frac{344}{365}$ $= 0.476$ (3sf) Alternative method: P(at least 2 with same date of birth in 21 people) + P(21 people all born on different days and 22nd person shares same date of birth with someone else) $= (1 - 0.55631) + (0.55631) \times \frac{21}{365}$	It is inappropriate to describe the distribution of the number of people who shares the same date of birth as someone else in the group as following the Binomial distribution. There may be more than one common date of birth!
(bi) [2]	Number of ways $= {}^3C_1 \times {}^6C_4 \times {}^9C_4 \times {}^4C_2 = 34020$	
(ii) [3]	Complement Method: Number of ways = n(teams formed without restriction) − n(teams which include both twins) $= 34020 - {}^3C_1 \times {}^5C_3 \times {}^8C_3 \times {}^4C_2$ $= 34020 - 10080 = 23940$ Alternative method: Number of ways = n(teams which include the twin defender and not the twin midfielder) + n(teams which include the twin midfielder and not the twin defender) + n(teams which do not include both twins) $= {}^3C_1 \times {}^5C_4 \times {}^8C_4 \times {}^4C_2 + {}^3C_1 \times {}^5C_3 \times {}^8C_4 \times {}^4C_2 + {}^3C_1 \times {}^5C_4 \times {}^8C_3 \times {}^4C_2$ $= 6300 + 12600 + 5040$	1) *Read the qn carefully!* Qn asks for number of teams with **at most** one twin brother, *not both twin brothers* and *not exactly one twin brother*. 2) Complement method is more efficient since it only involves one case.
(iii) [4]	Let A denote the event that player 1 and player 11 are both in the team. Let B denote the event that player 4 and player 7 are both in the team. $n(A) = {}^6C_4 \times {}^8C_3 \times {}^4C_2 = 5040$ $n(B) = {}^3C_1 \times {}^5C_3 \times {}^8C_3 \times {}^4C_2 = 10080$ $n(A \cap B) = {}^5C_3 \times {}^7C_2 \times {}^4C_2 = 1260$ $\therefore n(A \cup B) = 5040 + 10080 - 1260 = 13860$ Hence required probability $= 1 - \frac{13860}{34020} = \frac{16}{27}$	1) Qn asks for probability, not number of ways. 2) There are many different methods available for this part, but no matter which one you apply, do always check whether your cases are mutually exclusive (ie whether there is "double" counting). There is a need to account for the duplication.