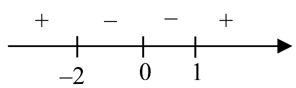
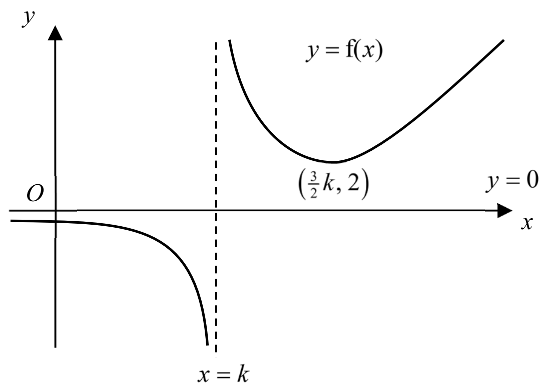
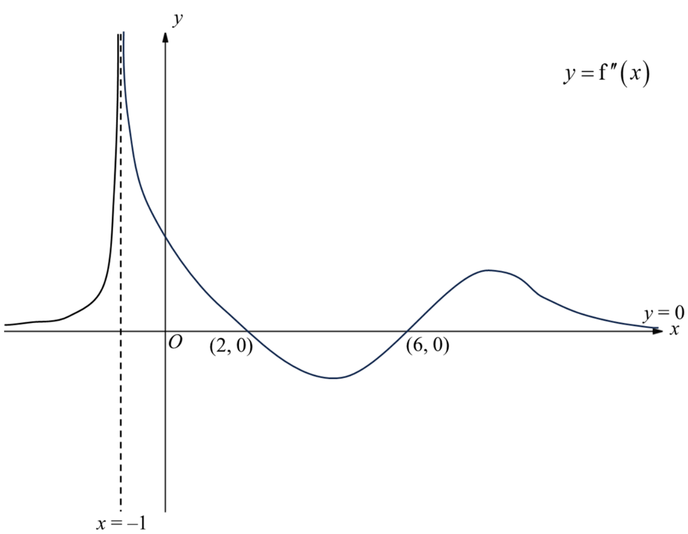
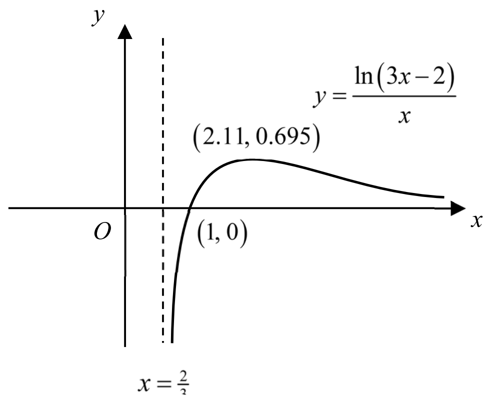
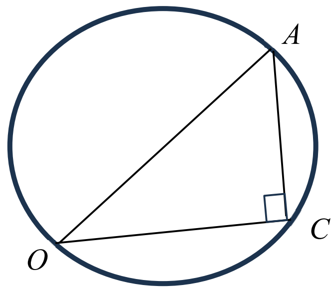
2024 ACJC H2 Math Promo Marking Scheme

Qn	Solution	
1	$\frac{3x-2}{x-1} \le \frac{4}{x+2} \Rightarrow \frac{3x-2}{x-1} - \frac{4}{x+2} \le 0$ $\frac{(3x-2)(x+2)-4(x-1)}{(x-1)(x+2)} \le 0$ $\frac{3x^2}{(x-1)(x+2)} \le 0$  $-2 < x < 1$	
	$\frac{3\lvert x\rvert-2}{\lvert x\rvert-1} \le \frac{4}{\lvert x\rvert+2}$ Replace x by $\lvert x\rvert$, $-2 < \lvert x\rvert < 1$ $\Rightarrow \lvert x\rvert < 1$ (since $-1 < \lvert x\rvert$ always true) $\Rightarrow -1 < x < 1$	
2(i)	$y^2 = 1 + \tan x$ Differentiating w.r.t x: $2y\frac{dy}{dx} = \sec^2 x$ $2y\frac{dy}{dx} = 1 + \tan^2 x$ $2y\frac{dy}{dx} = 1 + (y^2-1)^2$ (shown) Differentiating w.r.t x: $2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 = 2(y^2-1)(2y)\left(\frac{dy}{dx}\right)$ $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 2y(y^2-1)\left(\frac{dy}{dx}\right)$ (shown)	
2(ii)	When $x=0: y=1$ $2(1)\frac{dy}{dx} = 1 + (1-1)^2 \Rightarrow \frac{dy}{dx} = \frac{1}{2}$ $(1)\frac{d^2y}{dx^2} + \left(\frac{1}{2}\right)^2 = 0 \Rightarrow \frac{d^2y}{dx^2} = -\frac{1}{4}$ The Maclaurin expansion for y is: $y = 1 + \frac{1}{2}x - \frac{1}{4}\left(\frac{x^2}{2}\right) + \ldots = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \ldots$	

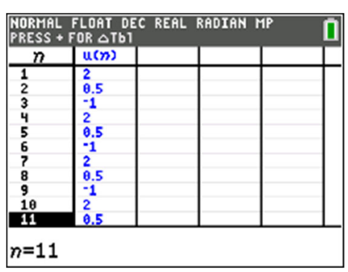
3(i)		
3(ii)	$y=\mathrm{f}(x)\xrightarrow{A}y=\mathrm{f}(x+k)$ $\xrightarrow{B}y=\mathrm{f}(-x+k)$ $\xrightarrow{C}y=\mathrm{f}(-(x-k)+k)=\mathrm{f}(-x+2k)$ $a=-1,b=2k$	
3(iii)	From (i), we see that the curve $y=\mathrm{f}(-x+2k)$ is a reflection of the curve $y=\mathrm{f}(x)$ about the line $x=k$. Hence if $\mathrm{g}(x)=\mathrm{g}(4-x)$ for all *x*, then the curve $y=\mathrm{g}(x)$ is the same as when it is reflected about the line $x=2$. Hence line of symmetry is $x=2$.	
4(i)		
4(ii)	$x=0$, a minimum point $x=6$, a (stationary) point of inflexion/inflection	
4(iii)	From the graph, $\mathrm{f}'(2)=5$, so the tangent has equation $y=5x$. Thus, the tangent passes through the point $(2,10)$. Hence, the equation of the normal is: $y-10=-\frac{1}{5}(x-2)$ or $y=-\frac{1}{5}x+\frac{52}{5}$	

5(i)		
5(ii)	$\mathrm{g}(x)=3x^2-12x+13=3(x-2)^2+1$. Hence $R_\mathrm{g}=[1,\infty)$ $D_\mathrm{f}=\left(\frac{2}{3},\infty\right)$. Therefore $R_\mathrm{g}\subset D_\mathrm{f}$, thus fg exists. Put R_g as the domain of f. therefore $R_{\mathrm{fg}}=[0,0.695]$	
5(iii)	$\mathrm{g}(x)=3(x-2)^2+1$ Hence turning point is at $x=2$, therefore largest *k* is 2. $y=3(x-2)^2+1, x\le 2$ $x-2=\pm\sqrt{\frac{y-1}{3}}\Rightarrow x=2\pm\sqrt{\frac{y-1}{3}}$ Since $x\le 2$, $x=2-\sqrt{\frac{y-1}{3}}$ $\mathrm{h}^{-1}:x\mapsto 2-\sqrt{\frac{x-1}{3}}, x\ge 1$	
6(i)	$\overrightarrow{OC}=\overrightarrow{OA}+\lambda\overrightarrow{AN}$ $=\mathbf{a}+\lambda\left(-\mathbf{a}+\frac{1}{3}\mathbf{b}\right)$ $=(1-\lambda)\mathbf{a}+\frac{\lambda}{3}\mathbf{b}$ $\overrightarrow{OC}=\overrightarrow{OB}+\mu\overrightarrow{BM}$ $=\mathbf{b}+\mu\left(\frac{1}{2}\mathbf{a}-\mathbf{b}\right)$ $=\frac{\mu}{2}\mathbf{a}+(1-\mu)\mathbf{b}$ Comparing the coefficients of **a** and **b**, $1-\lambda=\frac{\mu}{2} \quad \Rightarrow \lambda=1-\frac{\mu}{2}$ $\frac{\lambda}{3}=1-\mu \quad \Rightarrow \lambda=3-3\mu$	

	$1-\frac{\mu}{2}=3-3\mu$ $\frac{5}{2}\mu=2$ $\lambda=\frac{3}{5},\ \mu=\frac{4}{5}$ $\overrightarrow{OC}=\mathbf{c}=\left(1-\frac{3}{5}\right)\mathbf{a}+\frac{1}{5}\mathbf{b}=\frac{2}{5}\mathbf{a}+\frac{1}{5}\mathbf{b}$	
6(ii)	$\lvert\mathbf{a}\cdot\hat{\mathbf{c}}\rvert$ is the length of projection of vector **a** onto the line with direction vector **c**.	
6(iii)	Since OA is the diameter of the circle, $\angle OCA=\frac{\pi}{2}$.  $\overrightarrow{OC}\cdot\overrightarrow{AC}=0$ $\mathbf{c}\cdot(\mathbf{c}-\mathbf{a})=0$ $\lvert\mathbf{c}\rvert^2=\mathbf{a}\cdot\mathbf{c}$ $\lvert\mathbf{c}\rvert^2=\mathbf{a}\cdot\left(\frac{2}{5}\mathbf{a}+\frac{1}{5}\mathbf{b}\right)$ $=\frac{2}{5}\lvert\mathbf{a}\rvert^2+\frac{1}{5}\mathbf{a}\cdot\mathbf{b}$ $=\frac{2}{5}\lvert\mathbf{a}\rvert^2+\frac{1}{5}\left(\frac{1}{2}\lvert\mathbf{a}\rvert^2\right)$ $=\frac{1}{2}\lvert\mathbf{a}\rvert^2$ $\frac{1}{2}\lvert\mathbf{a}\rvert^2=\lvert\mathbf{c}\rvert^2 \Rightarrow \frac{\lvert\mathbf{a}\rvert}{\lvert\mathbf{c}\rvert}=\sqrt{2} \Rightarrow \lvert\mathbf{a}\rvert:\lvert\mathbf{c}\rvert=\sqrt{2}:1$ **Alternatively,** Since OA is the diameter of the circle, $\angle OCA=\frac{\pi}{2}$. $\lvert\mathbf{a}\cdot\hat{\mathbf{c}}\rvert=\lvert\mathbf{c}\rvert$ $\frac{1}{\lvert\mathbf{c}\rvert}\left\lvert\mathbf{a}\cdot\left(\frac{2}{5}\mathbf{a}+\frac{1}{5}\mathbf{b}\right)\right\rvert=\lvert\mathbf{c}\rvert$ $\left\lvert\frac{2}{5}\lvert\mathbf{a}\rvert^2+\frac{1}{5}\mathbf{a}\cdot\mathbf{b}\right\rvert=\lvert\mathbf{c}\rvert^2$ $\left\lvert\frac{2}{5}\lvert\mathbf{a}\rvert^2+\frac{1}{5}\left(\frac{1}{2}\lvert\mathbf{a}\rvert^2\right)\right\rvert=\lvert\mathbf{c}\rvert^2$ $\frac{1}{2}\lvert\mathbf{a}\rvert^2=\lvert\mathbf{c}\rvert^2$ $\frac{\lvert\mathbf{a}\rvert}{\lvert\mathbf{c}\rvert}=\sqrt{2}$ $\lvert\mathbf{a}\rvert:\lvert\mathbf{c}\rvert=\sqrt{2}:1$	

	Alternatively, $\mathbf{c}=\frac{2}{5}\mathbf{a}+\frac{1}{5}\mathbf{b}$ $\mathbf{c}\cdot\mathbf{a}=\left(\frac{2}{5}\mathbf{a}+\frac{1}{5}\mathbf{b}\right)\cdot\mathbf{a}$ $\mathbf{c}\cdot\mathbf{a}=\frac{2}{5}\lvert\mathbf{a}\rvert^2+\frac{1}{5}\mathbf{a}\cdot\mathbf{b}$ $\mathbf{c}\cdot\mathbf{a}=\frac{2}{5}\lvert\mathbf{a}\rvert^2+\frac{1}{5}\left(\frac{1}{2}\lvert\mathbf{a}\rvert^2\right)$ $\mathbf{c}\cdot\mathbf{a}=\frac{1}{2}\lvert\mathbf{a}\rvert^2$ $\lvert\mathbf{c}\rvert\lvert\mathbf{a}\rvert\cos\theta=\frac{1}{2}\lvert\mathbf{a}\rvert^2$, where $\theta=\angle AOC$. $\cos\theta=\frac{\lvert\mathbf{a}\rvert}{2\lvert\mathbf{c}\rvert}$ $\cos\theta=\frac{\lvert\mathbf{c}\rvert}{\lvert\mathbf{a}\rvert}$, from the right-angle triangle *OCA*. $\frac{\lvert\mathbf{a}\rvert}{2\lvert\mathbf{c}\rvert}=\frac{\lvert\mathbf{c}\rvert}{\lvert\mathbf{a}\rvert}$ $\lvert\mathbf{a}\rvert^2=2\lvert\mathbf{c}\rvert^2$ $\frac{\lvert\mathbf{a}\rvert}{\lvert\mathbf{c}\rvert}=\sqrt{2}$ $\lvert\mathbf{a}\rvert:\lvert\mathbf{c}\rvert=\sqrt{2}:1$	
6(iv)	Triangle *OCA* is a right-angle triangle with $\lvert\mathbf{a}\rvert:\lvert\mathbf{c}\rvert=\sqrt{2}:1$. By Pythagoras' Theorem, $\lvert\mathbf{a}\rvert:\lvert\mathbf{c}\rvert:\lvert\overline{CA}\rvert=\sqrt{2}:1:1$, and hence triangle *OCA* is an isosceles triangle. Area of Triangle *OCA* $=\frac{1}{2}\lvert\mathbf{c}\rvert^2=\frac{1}{2}\left(\frac{1}{\sqrt{2}}\lvert\mathbf{a}\rvert\right)^2=\frac{1}{4}\lvert\mathbf{a}\rvert^2$ **Alternatively,** Area of Triangle *OCA* $=\frac{1}{2}\lvert\mathbf{a}\times\mathbf{c}\rvert=\frac{1}{2}\lvert\mathbf{a}\rvert\lvert\mathbf{c}\rvert\sin\theta$, where $\theta=\angle AOC$. Since $\cos\theta=\frac{\lvert\mathbf{c}\rvert}{\lvert\mathbf{a}\rvert}=\frac{1}{\sqrt{2}}$ from **(iii)**, $\theta=\frac{\pi}{4}$. Area of Triangle *OCA* $=\frac{1}{2\sqrt{2}}\lvert\mathbf{a}\rvert\lvert\mathbf{c}\rvert=\frac{1}{2\sqrt{2}}\lvert\mathbf{a}\rvert\left(\frac{\lvert\mathbf{a}\rvert}{\sqrt{2}}\right)=\frac{1}{4}\lvert\mathbf{a}\rvert^2$	
7(i)	$\frac{dx}{dt}=-2a\sin\left(t+\frac{\pi}{6}\right)$ and $\frac{dy}{dt}=a\cos t$	

	$\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$ $=\frac{a\cos t}{-2a\sin\left(t+\frac{\pi}{6}\right)}$ $=-\frac{\cos t}{2\left(\frac{\sqrt{3}}{2}\sin t+\frac{1}{2}\cos t\right)}$ $=-\frac{\cos t}{\cos t+\sqrt{3}\sin t}$ Gradient of normal when $t=\theta$: $\frac{\cos\theta+\sqrt{3}\sin\theta}{\cos\theta}=1+\sqrt{3}\tan\theta$ (shown)	
7(ii)	To find Q, let $x=0$: $\cos\left(t+\frac{\pi}{6}\right)=0\Rightarrow t+\frac{\pi}{6}=\frac{\pi}{2}$ or $\frac{3\pi}{2}\Rightarrow t=\frac{\pi}{3}$ or $\frac{4\pi}{3}$ $y=a\sin\left(\frac{\pi}{3}\right)$ or $a\sin\left(\frac{4\pi}{3}\right)=\frac{\sqrt{3}}{2}a$ (rej) or $-\frac{\sqrt{3}}{2}a$ Thus, at $Q\left(0,-\frac{\sqrt{3}}{2}a\right)$, $t=\frac{4\pi}{3}$. (Shown)	
7(iii)	Using parts (i) and (ii): Gradient of normal at Q is $1+\sqrt{3}\tan\left(\frac{4\pi}{3}\right)=1+\left(\sqrt{3}\right)^2=4$ Equation of normal at $Q\left(0,-\frac{\sqrt{3}}{2}a\right)$: $y-\left(-\frac{\sqrt{3}}{2}a\right)=4(x-0)$ $\therefore y=4x-\frac{\sqrt{3}}{2}a$	
7(iv)	When the normal intersects the x-axis, sub $y=0$: $0=4x-\frac{\sqrt{3}}{2}a\Rightarrow x=\frac{\sqrt{3}}{8}a$ Thus, $T\left(\frac{\sqrt{3}}{8}a,0\right)$. To find P, let $y=0$: $\sin t=0\Rightarrow t=0$ or π $x=2a\cos\left(\frac{\pi}{6}\right)$ or $2a\cos\left(\pi+\frac{\pi}{6}\right)=\sqrt{3}a$ or $-\sqrt{3}a$ (rej) Thus, $P\left(\sqrt{3}a,0\right)$.	

	$\frac{OT}{OP}=\frac{\sqrt{3}}{8}a\Big/\left(\sqrt{3}a\right)$ $=\frac{\sqrt{3}/8}{\sqrt{3}}$ $=\frac{1}{8}$ (independent of a, shown)	
8(a)(i)	$\sum_{r=1}^{n}\frac{1}{4r^2-1}=\frac{n}{2n+1}$ $=\frac{1}{2}-\frac{1}{2(2n+1)}$ As $n\to\infty$, $\frac{1}{2(2n+1)}\to 0$. $\sum_{r=1}^{\infty}\frac{1}{4r^2-1}=\frac{1}{2}$. **Alternative:** $\sum_{r=1}^{\infty}\frac{1}{4r^2-1}=\lim_{n\to\infty}\frac{n}{2n+1}=\lim_{n\to\infty}\frac{1}{2+\frac{1}{n}}=\frac{1}{2}$ Since $\frac{1}{2}$ is a constant, the series converges.	
8(a)(ii)	Replace r with $r-1$: $\sum_{r=6}^{N}\frac{1}{(2r+1)(2r+3)}=\sum_{r-1=6}^{r-1=N}\frac{1}{(2(r-1)+1)(2(r-1)+3)}$ $=\sum_{r=7}^{N+1}\frac{1}{(2r-1)(2r+1)}$ $=\sum_{r=1}^{N+1}\frac{1}{4r^2-1}-\sum_{r=1}^{6}\frac{1}{4r^2-1}$ $=\frac{N+1}{2(N+1)+1}-\frac{6}{12+1}$ $=\frac{N+1}{2N+3}-\frac{6}{13}$ $=\frac{N-5}{13(2N+3)}$	
8(b)	$u_2=1-\frac{1}{2}=\frac{1}{2}$ $u_3=1-\frac{1}{\frac{1}{2}}=-1$ $u_4=1-\frac{1}{-1}=2$ **Alternative: Use GC**  $\sum_{r=1}^{50}u_r=2\times17+\frac{1}{2}\times17+(-1)\times16$ $=26.5$	

9(i)

$\tan \angle APQ = \dfrac{3.05-1.75}{x} \Rightarrow \angle APQ = \tan^{-1}\left(\dfrac{1.3}{x}\right)$

$\tan \angle BPQ = \dfrac{3.45-1.75}{x} \Rightarrow \angle BPQ = \tan^{-1}\left(\dfrac{1.7}{x}\right)$

Hence, $\theta = \angle BPQ - \angle APQ = \tan^{-1}\left(\dfrac{1.7}{x}\right) - \tan^{-1}\left(\dfrac{1.3}{x}\right)$

9(ii)

$$\frac{d\theta}{dx} = \frac{\left(-\dfrac{1.7}{x^2}\right)}{1+\left(\dfrac{1.7}{x}\right)^2} - \frac{\left(-\dfrac{1.3}{x^2}\right)}{1+\left(\dfrac{1.3}{x}\right)^2}$$

$$= \frac{-1.7}{x^2+1.7^2} + \frac{1.3}{x^2+1.3^2}$$

At stationary point, $\dfrac{d\theta}{dx} = 0$:

$$\frac{-1.7}{x^2+1.7^2} + \frac{1.3}{x^2+1.3^2} = 0$$

$$1.7\left(x^2+1.3^2\right) = 1.3\left(x^2+1.7^2\right)$$

$$x^2 = \frac{1.7^2 \times 1.3 - 1.3^2 \times 1.7}{1.7-1.3} = 2.21$$

$x = 1.487$ (as x is positive)

9(iii)

Method 1: First Derivative Test

	$x = 1.48$	$x = 1.487$	$x = 1.49$
$\dfrac{d\theta}{dx}$	$0.000398 > 0$	0	$-0.000202 < 0$
Graph	/	—	\

By the First Derivative Test, this gives a maximum point.

Method 2: Second Derivative Test

$$\frac{d^2\theta}{dx^2} = \frac{3.4x}{\left(x^2+1.7^2\right)^2} - \frac{2.6x}{\left(x^2+1.3^2\right)^2}$$

When $x = 1.487$, $\dfrac{d^2\theta}{dx^2} = -0.0597 < 0$

By the Second Derivative Test, this gives a maximum point.

It gives the distance which maximises the angle *APB*, so it makes for easier throwing / more chance to throw object through the hole / gives the widest leeway / better accuracy.

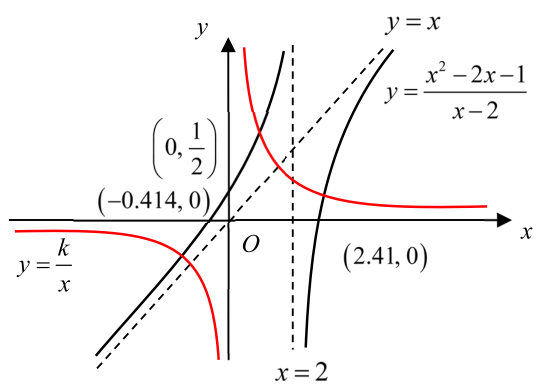
9(iv)

By chain rule: $\dfrac{d\theta}{dt} = \dfrac{d\theta}{dx} \times \dfrac{dx}{dt} = \left(\dfrac{-1.7}{x^2+1.7^2} + \dfrac{1.3}{x^2+1.3^2}\right) \times 0.1$

When $x = 1$: $\dfrac{d\theta}{dt} = 0.04625 \times 0.1 = 0.00463$ rad/s

10(i)

$$y = \frac{ax^2 - 2ax + a - 2}{x-2}$$

	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x-2)(2ax-2a)-(ax^2-2ax+a-2)}{(x-2)^2}$ $=\frac{2ax^2-6ax+4a-(ax^2-2ax+a-2)}{(x-2)^2}$ $=\frac{ax^2-4ax+3a+2}{(x-2)^2}$ **Alternatively:** $y=\frac{ax^2-2ax+a-2}{x-2}=ax+\frac{a-2}{x-2}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=a-\frac{a-2}{(x-2)^2}$	
10(ii)	For C to have no stationary points, $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ has no solutions. $\frac{ax^2-4ax+3a+2}{(x-2)^2}=0 \Rightarrow ax^2-4ax+3a+2$ No solutions, therefore $D<0$ $(-4a)^2-4a(3a+2)<0$ $(4a)(4a-3a-2)<0$ $4a(a-2)<0$ $0<a<2$ **Alternatively:** $\frac{\mathrm{d}y}{\mathrm{d}x}=a-\frac{a-2}{(x-2)^2}=0$ $\Rightarrow a=\frac{a-2}{(x-2)^2}\Rightarrow (x-2)^2=\frac{a-2}{a}$ For no solutions, $\frac{a-2}{a}<0$ Hence $0<a<2$.	
10(iii)		

10(iv)	$x^3-2x^2-x=k(x-2)\Rightarrow\dfrac{x^2-2x-1}{x-2}=\dfrac{k}{x}$ Sketch $y=\dfrac{k}{x}, k>0$. From diagram, there are 2 positive roots and 1 negative root.	
11(i)	$l_2: \dfrac{1-y}{2}=z-3,\ x=5 \Rightarrow \mathbf{r}=\begin{pmatrix}5\\1\\3\end{pmatrix}+\lambda\begin{pmatrix}0\\-2\\1\end{pmatrix}$ $l_1: \mathbf{r}=\begin{pmatrix}a\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}0\\1\\b\end{pmatrix}$ Since l_1 is perpendicular to l_2, $\begin{pmatrix}0\\-2\\1\end{pmatrix}\cdot\begin{pmatrix}0\\1\\b\end{pmatrix}=0\Rightarrow b-2=0\Rightarrow b=2$ (shown) Since l_1 intersects p_1 at $(a,1,0)$, $(a)+2(1)+4(0)=4\Rightarrow a+2=4\Rightarrow a=2$ (shown)	
11(ii)	$\begin{pmatrix}2\\2\\2\end{pmatrix}=\begin{pmatrix}2\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}0\\1\\2\end{pmatrix}$ for $\lambda=1$, therefore A lies on l_1.	
11(iii)	$\overrightarrow{OF}=\begin{pmatrix}2\\2\\2\end{pmatrix}+\lambda\begin{pmatrix}1\\2\\4\end{pmatrix}$ for some $\lambda\in\mathbb{R}$. $\begin{pmatrix}2+\lambda\\2+2\lambda\\2+4\lambda\end{pmatrix}\cdot\begin{pmatrix}1\\2\\4\end{pmatrix}=4$ $21\lambda=-10 \quad\Rightarrow\quad \lambda=-\dfrac{10}{21}$ $\therefore\overrightarrow{OF}=\begin{pmatrix}2\\2\\2\end{pmatrix}+\left(-\dfrac{10}{21}\right)\begin{pmatrix}1\\2\\4\end{pmatrix}=\dfrac{1}{21}\begin{pmatrix}32\\22\\2\end{pmatrix}$ $F\left(\dfrac{32}{21},\dfrac{22}{21},\dfrac{2}{21}\right)$ $\overrightarrow{AF}=\dfrac{1}{21}\begin{pmatrix}32\\22\\2\end{pmatrix}-\begin{pmatrix}2\\2\\2\end{pmatrix}=\dfrac{1}{21}\begin{pmatrix}-10\\-20\\-40\end{pmatrix}=-\dfrac{10}{21}\begin{pmatrix}1\\2\\4\end{pmatrix}$ $\left\|\overrightarrow{AF}\right\|=\sqrt{\dfrac{10^2}{21^2}\left(1^2+2^2+4^2\right)}=\dfrac{10}{\sqrt{21}}$ units	

	Alternatively to find $\overrightarrow{OF}$ and $\left\|\overrightarrow{AF}\right\|$: Let $(2,1,0)$ be point B, and $\mathbf{n}=\begin{pmatrix}1\\2\\4\end{pmatrix}$, the normal of plane p_1. Then $\overrightarrow{AB}=\begin{pmatrix}2\\1\\0\end{pmatrix}-\begin{pmatrix}2\\2\\2\end{pmatrix}=\begin{pmatrix}0\\-1\\-2\end{pmatrix}$ $\overrightarrow{AF}=\left(\frac{\overrightarrow{AB}\cdot\mathbf{n}}{\|\mathbf{n}\|}\right)\frac{\mathbf{n}}{\|\mathbf{n}\|}=\frac{1}{\sqrt{21}}\left[\begin{pmatrix}0\\-1\\-2\end{pmatrix}\cdot\begin{pmatrix}1\\2\\4\end{pmatrix}\right]\frac{1}{\sqrt{21}}\begin{pmatrix}1\\2\\4\end{pmatrix}=-\frac{10}{21}\begin{pmatrix}1\\2\\4\end{pmatrix}$ $\overrightarrow{OF}=\overrightarrow{OA}+\overrightarrow{AF}=\begin{pmatrix}2\\2\\2\end{pmatrix}-\frac{10}{21}\begin{pmatrix}1\\2\\4\end{pmatrix}=\frac{1}{21}\begin{pmatrix}32\\22\\2\end{pmatrix}$ $\left\|\overrightarrow{AF}\right\|=\left\|\frac{\overrightarrow{AB}\cdot\mathbf{n}}{\|\mathbf{n}\|}\right\|=\frac{1}{\sqrt{21}}\left\|\begin{pmatrix}0\\-1\\-2\end{pmatrix}\cdot\begin{pmatrix}1\\2\\4\end{pmatrix}\right\|=\frac{10}{\sqrt{21}}$ units	
11(iv)	Two direction vectors parallel to plane p_2 are $\begin{pmatrix}0\\1\\2\end{pmatrix}$ and $\begin{pmatrix}1\\2\\4\end{pmatrix}$. $\begin{pmatrix}0\\1\\2\end{pmatrix}\times\begin{pmatrix}1\\2\\4\end{pmatrix}=\begin{pmatrix}0\\2\\-1\end{pmatrix}$ $\mathbf{r}\cdot\begin{pmatrix}0\\2\\-1\end{pmatrix}=\begin{pmatrix}2\\1\\0\end{pmatrix}\cdot\begin{pmatrix}0\\2\\-1\end{pmatrix}=2$ P: $\mathbf{r}\cdot\begin{pmatrix}0\\2\\-1\end{pmatrix}=2$	
12(a)(i)	$u_n\geq 7500$ $4500+(n-1)350\geq 7500$ $n-1\geq\frac{7500-4500}{350}$ $n\geq 9.57$ Hence, student A first earns 15 health points on the 10$^{\text{th}}$ day. **Alternatively:**	

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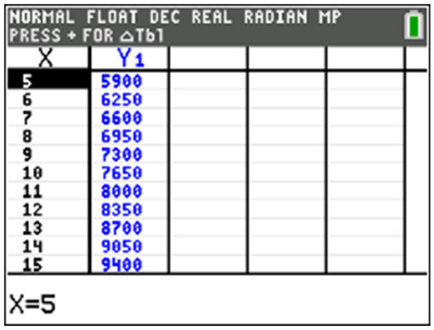
$$u_n \geq 7500$$
$$4500+(n-1)350 \geq 7500$$



Using GC,
$u_9 = 7300 < 7500$
$u_{10} = 7650 > 7500$

Hence, student A first earns 15 health points on the 10th day.
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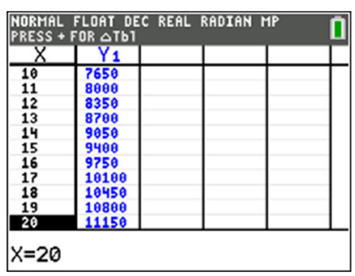
<tr><td>12(a)(ii)</td><td>

By observation,
$u_2 = 4850$
$u_3 = 5200$

$$u_n \geq 10000$$
$$4500+(n-1)350 \geq 10000$$
$$n-1 \geq \frac{10000-4500}{350}$$
$$n \geq 16.7$$

Hence, student A to first earn 5 health points on day 3 and 25 health points on day 17 respectively.

Alternatively:



Using GC,
$u_3 = 5200 > 5000$
$u_{17} = 10100 > 10000$

Hence, student A to first earn 5 health points on day 3 and 25 health points on day 17 respectively.

$$\text{Total number of points earned} \begin{aligned} &= 5\times 7+15\times 7+25\times 4 \\ &= 240 \end{aligned}$$
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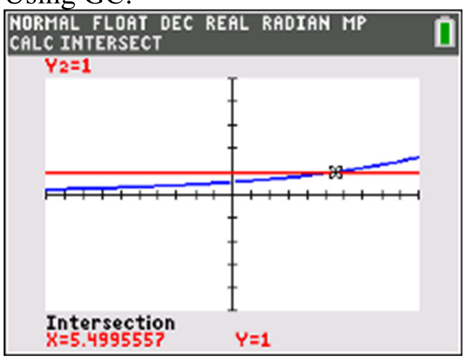
$$S_{20} \geq 261500$$

$$\frac{7500\left(\left(1+\frac{x}{100}\right)^{20}-1\right)}{\frac{x}{100}} \geq 261500$$

$$\frac{1500\left(\left(1+\frac{x}{100}\right)^{20}-1\right)}{523x} \geq 1$$

Using GC:



$x \geq 5.4995$
Hence, minimum $x = 5.50$
</td><td></td></tr>
<tr><td>12(b)(ii)</td><td>

$U_n = 7500(1.038)^{n-1} \geq 10000$

$$n-1 \geq \frac{\ln 1.3333}{\ln 1.038}$$

$$n \geq 8.71$$

Total number of points $= 8 \times 15 + 12 \times 25$
$= 420$
</td><td></td></tr>
<tr><td>12(c)</td><td>

$$S_\infty = \frac{2}{1-0.91}$$
$$= \$22.22$$
$$< \$23$$

The maximum coupon value he can get is $22.22, which is less than $23.
</td><td></td></tr>
</table>