Name:_____ **Class:**_____

JURONG PIONEER JUNIOR COLLEGE

JC2 Preliminary Examination 2022

MATHEMATICS
Higher 1

8865/01
16 September 2022
3 hours

Paper 1

Candidates answer on the Question Paper.

Additional materials: List of Formulae (MF 26)

READ THESE INSTRUCTIONS FIRST

Write your name and civics class on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or question.

Qn	Marks
1	/4
2	/6
3	/6
4	/9
5	/15
6	/6
7	/6
8	/7
9	/7
10	/10
11	/10
12	/14
Accuracy	
Total	/100

This document consists of **6** printed pages.

[Turn over

Section A: Pure Mathematics [40 marks]

1 Without using a calculator, find the set of values of the real constant k for which the expression

$$kx^2 + x^2 - 4x + k - 1$$

is always positive for all real values of x. [4]

2 **(i)** Find $\int \frac{2}{3\sqrt{1-4x}}\,\mathrm{d}x$. [3]

(ii) Differentiate $\left(3\mathrm{e}^{2x} - \frac{5}{\mathrm{e}^{2x}}\right)^2$ with respect to x, giving your answer in the form $p\mathrm{e}^{4x} + q\mathrm{e}^{-4x}$, where p and q are constants to be determined. [3]

3 A restaurant sells three set meals *A*, *B* and *C*. Set Meal *A* is the cheapest while Set Meal *C* is the most expensive. Recently, the restaurant offers a sales promotion. For every two set meals sold, the set meal which is cheaper or of equivalent price is sold at half price.

- The original price of one Set Meal *C* is \$11.00 cheaper than the total original prices of one Set Meal *A* and one Set Meal *B*.
- The original prices of one Set Meal *A*, one Set Meal *B* and one Set Meal *C* adds up to be \$51.80.
- A group of five friends intends to order one Set Meal *A*, two Set Meals *B* and two Set Meals *C* at the restaurant. If they use this sales promotion to pay for their order in a single receipt, the restaurant will charge them a total of \$73.40.

(i) Write this information as three linear equations, and hence find the original prices of Set Meals *A*, *B* and *C*. [4]

(ii) One of the five friends remembers that he has membership at this restaurant and is able to get a 20% discount on the total amount before discount spent at the restaurant. Justify, with a reason, if they should use the sales promotion offer or the membership discount to pay for the food that they intend to order. [2]

4 The curve *C* has the equation $y = 5 + \ln(2 - x)$.

(i) Sketch the graph of *C*, giving the exact coordinates of any points
the axes and the equation of the asymptote.

(ii) Find the equation of the tangent to *C* at the point where $x = -1$, giving your answer in the form $y = mx + c$, where m and c are exact constants. [4]

(iii) Find the area bounded by *C*, the axes and the line $x = -3$. [2]

5 A group of wildlife biologists models the population, *P* thousand, of a certain species of birds in a particular forest by

$$P = \frac{1}{2}\left(a + \mathrm{e}^{bt}\right),$$

where t is the time in years and a and b are constants.

The initial population of birds in the forest is 2 thousand. At $t = 1$, the population is 1.75 thousand.

(i) Without the use of a calculator, find the values of a and b. [4]

(ii) Find the rate of change of the population at $t = 5$. Describe, in context of this question, the meaning of this value calculated. [2]

(iii) Sketch the graph of P against t. [1]

(iv) Describe, in context of this question, what will happen to the population in the long run. [1]

The biologists model the population, Q thousand, of another species of birds in the same forest by

$$Q = \frac{1}{10}t\left(1 + \frac{5}{t} + \frac{5}{t^3}\right) \quad \text{for} \quad 1 \le t \le 15.$$

(v) Use differentiation to find the coordinates of any stationary point on the graph of Q against t, justifying whether it is a minimum or a maximum point. [4]

(vi) Sketch the graph of Q against t. [2]

(vii) Find t, to the nearest whole number, when the 2 species of birds have equal populations. [1]

Section B: Statistics [60 marks]

6 **(i)** Find the number of ways in which 5 letters can be selected from the 9 letters of the word TRIANGLES, given that at most two vowel must be included. [3]

(ii) Find the number of different arrangements of the 9 letters of the word TRIANGLES in which the S and the T can be in either order and there are exactly 4 letters between them. [3]

7 A survey was conducted with a large number of office workers who were asked to select one skill that they would like to pick up. The survey showed that 45% of office workers would like to pick up coding skills.

Twelve office workers who took part in the survey were randomly selected and asked which skill they would like to pick up.

(a) Find the probability that

(i) less than half the number of office workers said that they would like to pick up coding skills. [2]

(ii) at least four office workers said that they would like to pick up coding skills. [2]

Six groups of twelve office workers were randomly selected.

(b) Find the probability that each group has at least half the number of office workers who said that they would like to pick up coding skills, giving your answer to 4 significant figures. [2]

8 A new college has two year groups, Year 1 and Year 2. There are 800 students in Year 1 and 700 students in Year 2. Each student is either a Citizen, a Permanent Resident or a Foreigner, and the numbers of each are summarised in the table below.

	Citizen	Permanent Resident	Foreigner
Year 1	623	115	62
Year 2	525	123	52

A student is chosen at random from the college.

(i) Find the probability that the student is in Year 1 or a Citizen (or both). [1]

(ii) Find the probability that the student is in Year 2 given that the student is a Citizen. [1]

(iii) Determine whether or not the events 'the student is a foreigner' and 'the student is in Year 2' are independent. [2]

On another occasion, 3 of these students are chosen at random, without replacement.

(iv) Find the probability that exactly 2 are Permanent Residents, giving your answers to 5 decimal places. [3]

9 A bag contains 3 red balls, 2 yellow balls and 1 green ball. Two balls are drawn at random, one by one and without replacement.

(i) Draw a tree diagram to represent this situation, showing all possible outcomes. [2]

(ii) Find the probability that both balls are of the same colour. [2]

A third ball is drawn from the bag without replacing the first 2 balls drawn.

(iii) Find the probability that there are two red balls now. [3]

10 A restaurant keeps track of the number of customers, x, as well as the total amount of profits generated, y dollars for 7 randomly chosen days. The table below gives a summary of the results.

x	157	247	188	127	197	267	175
y	2956	4825	3824	2325	3974	5765	3754

(i) Give a sketch of the scatter diagram for the data, as shown on your calculator. [2]

(ii) Find the product moment correlation coefficient and comment on its value in the context of the question. [2]

(iii) Find the equation of the regression line y on x, in the form $y = mx + c$, giving the values of m and c correct to 2 decimal places. Sketch this line on your scatter diagram. [2]

(iv) Give an interpretation of the meaning of m and c in the context of the question. [2]

(v) Calculate an estimate of the profit generated when there are 200 customers, giving your answer to the nearest dollar. Comment on the reliability of this estimate. [2]

11 A farmer claims that the mean mass of chickens in his farm is at least 1200 g. To test this claim, a random sample of 30 chickens is weighed and the masses of chickens, x g, are summarised by

$$\sum(x-1200) = -264 \quad \text{and} \quad \sum(x-1200)^2 = 18462.$$

(i) Test at the 5% significance level whether the farmer's claim is supported by the data. [6]

(ii) State, with a reason, whether it is necessary to assume that the masses of chickens are distributed normally for this test to be valid. [1]

It is known that the masses of ducks in the farm have a population variance of 200 g^2. A random sample of 40 ducks is selected and the mean mass is 1500 g. A test at the α % significance level gives sufficient evidence that the population mean mass of ducks is not 1495 g.

(iii) Find the set of possible values of α. [3]

12 There are three bus services between the towns of Arcadia and Elysium. The journey times, in minutes by each bus service have independent normal distributions. The means and standard deviations of these distributions are shown in the following table.

Bus Service	Mean	Standard Deviation
A	27	10
B	35	6
C	32	σ

(i) The probability that a randomly chosen journey by Bus Service C will take more than 36 minutes is 0.24. Find σ. [3]

(ii) Find the probability that two randomly chosen journeys by Bus Service A will each take more than 30 minutes. [2]

(iii) The probability that the total time for two randomly journeys by Bus Service A is more than 60 minutes is denoted by p. Without calculating its value, explain why p will be greater than your answer to part **(ii)**. [1]

(iv) Find the probability that the total journey time of 5 randomly chosen journeys by Bus Service A is more than the total journey time of 4 randomly chosen journeys by Bus Service B by less than 10 minutes. [4]

Due to changes in bus routes, the journey time of Bus Service A is increased by 10% while the journey time for Bus Service B is decreased by 10%.

(v) Find the probability that the total journey times for 6 randomly chosen journeys by Bus Service A and 4 randomly chosen journeys by Bus Service B is at least 5 hours. [4]