1 Solve the equation $3\left(e^{2x} - \frac{5}{e^{2x}}\right) = -4$, giving your answer(s) in exact form. [5]

2 The equation of a polynomial is given by $f(x) = 2x^3 + x^2 - 8x + 21$.

(i) Show that $x+3$ is a factor of $f(x)$. [2]

(ii) Hence, show that the equation $f(x) = 0$ has only one real root. [5]

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3 **(i)** Differentiate $\dfrac{3\ln 2x}{x^3}$ with respect to x. [2]

(ii) Hence show that $\displaystyle\int_1^2 \frac{4\ln 2x}{x^4}\,dx = \frac{1}{18}(a + b\ln 2)$, where a and b are integer values to be determined. [5]

4 The fourth term in the binomial expansion of $\left(x-\frac{2}{x^2}\right)^n$, where n is a positive integer, is a constant a.

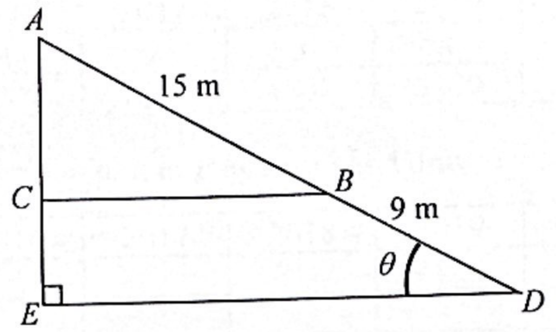
(i) Show that $n=9$ and hence find the value of a. [4]

(ii) Find the coefficient of x^5 in the expansion of $\left(x-\frac{2}{x^2}\right)^9(1+3x)^4$. [4]

(a) The equation of a quadratic curve is $y = -3x^2 + 4x - 5$. The line $y = mx - 2$ is a tangent to the curve at the point Q where $m > 0$. Find the value of the constant m and hence find the coordinates of Q. [5]

(b) Find the range of values of p such that the graph $y = px^2 - 5x + 9p$ lies entirely below the x-axis. [4]

6 Mark intends to fence up a triangular plot of land for his garden as shown in the diagram. He also intends to build fences along BC to form two separate plots of land such that they form a pair of similar triangles ABC and ADE. Point B lies on the straight line AD such that $AB = 15\,\text{m}$ and $BD = 9\,\text{m}$. AE is perpendicular to ED and angle $ADE = \theta$ where $0° \leq \theta \leq 90°$.



(i) Show that P m, the perimeter of the plot of land $BCED$, is given by $P = 9\sin\theta + 39\cos\theta + 9$. [3]

(ii) Express P in the form $9+R\sin(\theta+\alpha)$, where $R>0$ and $0°<\alpha<90°$. [3]

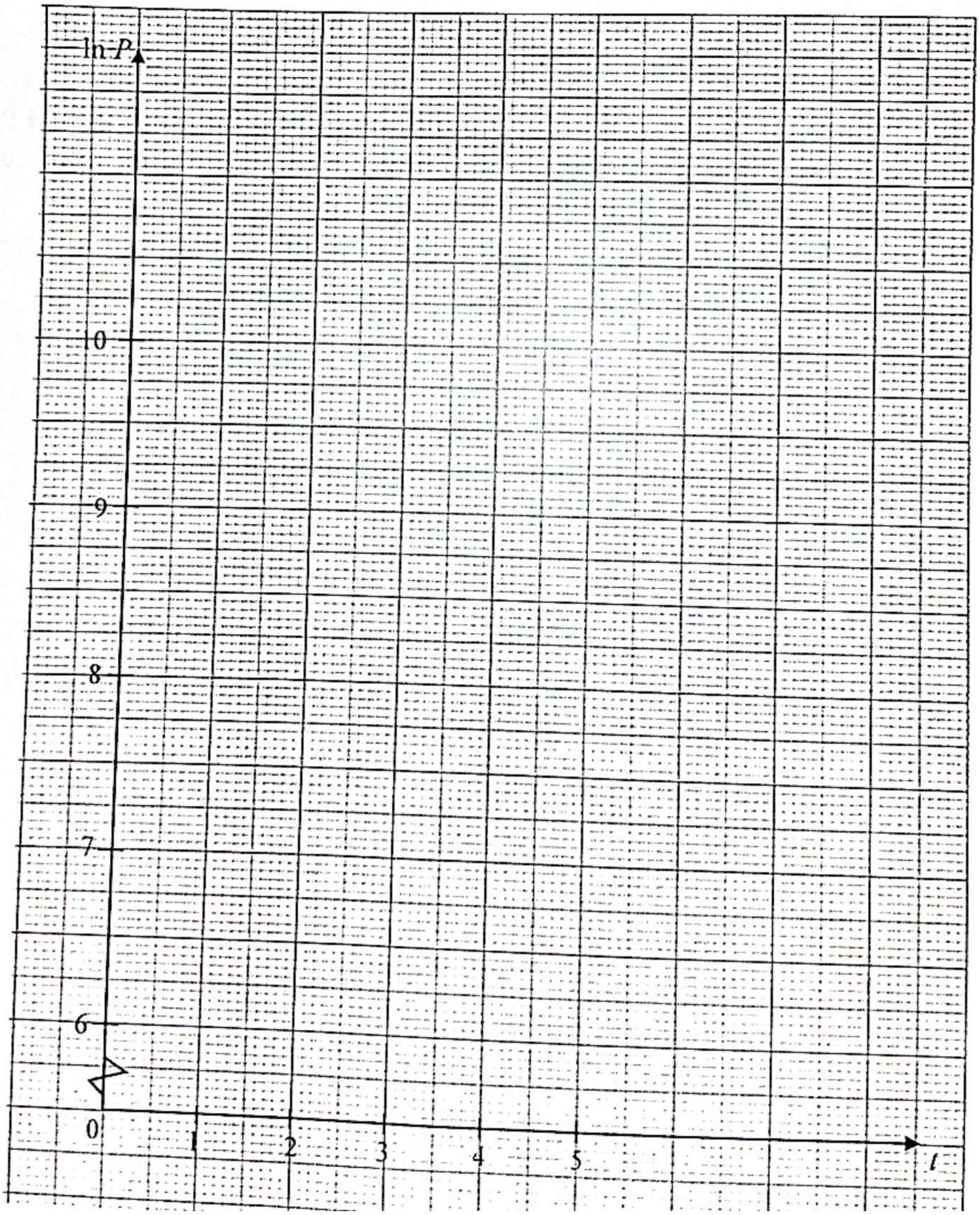
(iii) Find the value of P and the corresponding value of θ if Mark would like the plot of land $BCED$ to be as large as possible. [3]

7 The population of wild Red Pandas has been steadily decreasing over the years, facing the risk of extinction. The table shows the estimated population of wild Red Pandas from 2016 to 2020 where year 2016 is taken to be $t = 1$ and so on. A wildlife expert believes that these figures can be modelled using the formula $P = P_0 e^{-kt}$, where P_0 and k are constants.

Year	2016	2017	2018	2019	2020
t	1	2	3	4	5
P	9900	7200	4800	3200	2300

(i) Using the grid below, plot $\ln P$ against t and draw a straight line graph. [3]

Year	2016	2017	2018	2019	2020



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(ii) Use your graph to estimate the value of P_0 and of k. [3]

(iii) The Wildlife Expert uses this model to estimate the population of Red Pandas in 2030. Find the value of this estimation, correct to the nearest whole number, and explain if the estimation obtained is reliable. [3]

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8 A particle traveling in a straight line passes through O with a speed of 5 m/s. The acceleration a m/s^2, of the particle, t s after passing through O, is given by $a = -3e^{-0.5t}$. The particle comes to instantaneous rest at the point X.

(i) Find the time taken for the particle to reach X. [6]

(ii) Calculate the distance OX. [4]

(iii) Show that the particle is again at O at some instant during the twelfth second after passing through O. [2]

9 The equation of a circle is $(x-4r)^2+(y+3r)^2=kr^2$ where r and k are positive constants.

It is given that $k=9$.

(a) Explain why the x-axis is a tangent to the circle. [3]

(b) Find, in terms of r, the coordinates of the points on the circle at which the normal to the circle which passes through the centre of the circle, is perpendicular to the y-axis. [2]

It is now given that $k = 25$.

(c) Verify that the circle passes through the origin O. [1]

(d) Given that OD is the diameter of the circle, find in terms of r, the equation of the tangent to the circle at D and hence, find the coordinates of the point at which this tangent meets the x-axis. [6]

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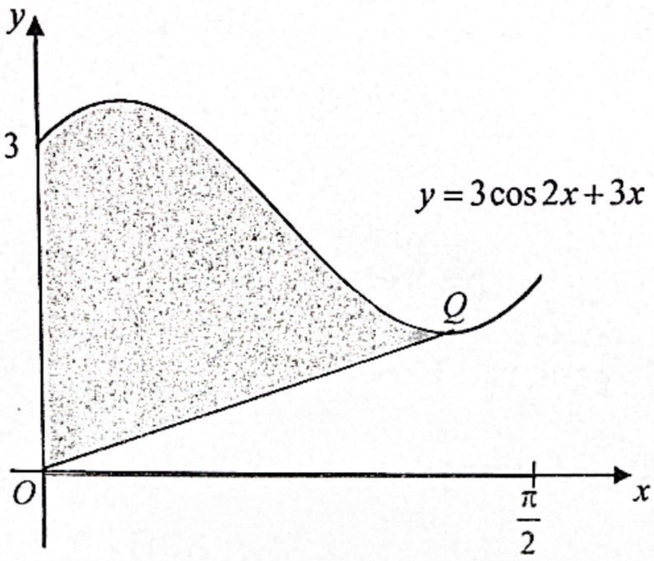

The diagram shows the curve of $y=3\cos 2x+3x$ for $0 \le x \le \frac{\pi}{2}$ radians. The point Q is the minimum point of the curve and OQ is a straight line.

Show that the area of the shaded region is $\frac{1}{16}\left(12+5\pi\sqrt{3}\right)$ units^2. [12]