Dunman High School 德明政府中学

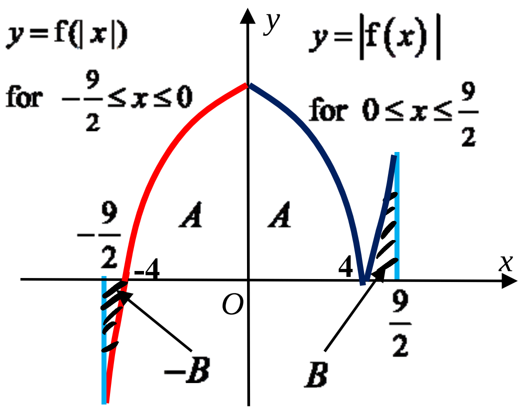
2012 Yr 6 H2 Math
Preliminary Examination Paper 1
Suggested Mark Scheme

Qn	Suggested Solution	Marking Scheme
1	$\frac{\lvert x\rvert+3}{x^2+1}>1$ $\Rightarrow \frac{y+3}{y^2+1}>1$ $\Rightarrow y+3>y^2+1$ i.e. $y^2-y-2<0$ $(y-2)(y+1)<0$ $-1<y<2$ $0\le\lvert x\rvert<2$ $-2<x<2$	**M1** – Cross-multiply or state that $y^2+1>0$ (in the case of combining into a single fraction) **B1** – Correct answer in terms of y **A1** **Total: 3 marks**

Qn	Suggested Solution	Marking Scheme
2	$\int_{\frac{1}{2}}^{n} \frac{\left(\tan^{-1} 2x\right)^2}{1+4x^2}\,\mathrm{d}x$	**M1** – Use $k\int f'(x)\left[f(x)\right]^n \mathrm{d}x$ and proceed to $\left(\tan^{-1} 2x\right)^{n+1}$
	$=\frac{1}{2}\int_{\frac{1}{2}}^{n} 2\frac{\left(\tan^{-1} 2x\right)^2}{1+4x^2}\,\mathrm{d}x=\frac{1}{6}\left[\left(\tan^{-1} 2x\right)^3\right]_{\frac{1}{2}}^{n}$	
	$=\frac{1}{6}\left[\left(\tan^{-1} 2n\right)^3-\left(\frac{\pi}{4}\right)^3\right]$	**A1 –** Evaluation with correct limits
	As $n\to\infty$, $\tan^{-1} 2n\to\frac{\pi}{2}$	**M1 –** Can show implicitly
	$\int_{\frac{1}{2}}^{\infty} \frac{\left(\tan^{-1} 2x\right)^2}{1+4x^2}\,\mathrm{d}x=\frac{1}{6}\left[\left(\frac{\pi}{2}\right)^3-\left(\frac{\pi}{4}\right)^3\right]$	
	$=\frac{7}{384}\pi^3$	**A1** – Exact answer
		Total: 4 marks

Qn	Suggested Solution	Marking Scheme
3	$\dfrac{2+x}{\sqrt{9-x}}$ $=(2+x)\dfrac{1}{3}\left(1-\dfrac{x}{9}\right)^{-\frac{1}{2}}$ $=\dfrac{1}{3}(2+x)\left(1+\left(-\dfrac{1}{2}\right)\left(-\dfrac{x}{9}\right)+\dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}\left(-\dfrac{x}{9}\right)^2+\dots\right)$ $=\dfrac{1}{3}(2+x)\left(1+\dfrac{x}{18}+\dfrac{x^2}{216}+\dots\right)$ $=\dfrac{1}{3}\left(2+\dfrac{x}{9}+\dfrac{x^2}{108}+x+\dfrac{x^2}{18}+\cdots\right)$ $=\dfrac{2}{3}+\dfrac{10}{27}x+\dfrac{7}{324}x^2+\dots$	**B1** – $k\left(1-\dfrac{x}{9}\right)^{-\frac{1}{2}}$ **M1** – Correct use of binomial theorem **A1**
	$\dfrac{2+x}{\sqrt{9-x}}\approx\dfrac{2}{3}+\dfrac{10}{27}x$ $\therefore\ \dfrac{2+\frac{1}{9}}{\sqrt{9-\frac{1}{9}}}\approx\dfrac{2}{3}+\dfrac{10}{27}\left(\dfrac{1}{9}\right)$ $\dfrac{19}{9}\times\dfrac{3}{4\sqrt{5}}\approx\dfrac{172}{243}$ $\sqrt{5}\approx\dfrac{19}{9}\times\dfrac{3}{4}\times\dfrac{243}{172}=\dfrac{1539}{688}$ i.e. $p=1539,\ q=688$ **<u>Alternatively</u>**, $\dfrac{19}{9}\times\dfrac{3}{4\sqrt{5}}\approx\dfrac{172}{243}$ $\dfrac{19}{12\sqrt{5}}\approx\dfrac{172}{243}$ $\dfrac{19\sqrt{5}}{60}\approx\dfrac{172}{243}$ $\sqrt{5}\approx\dfrac{3440}{1539}$ i.e. $p=3440,\ q=1539$	√**M1** – Correct substitution (to award once $\sqrt{5}$ is seen) **A1** **Total: 5 marks**

Qn	Suggested Solution	Marking Scheme
4	$\mathrm{f}(r-1)-\mathrm{f}(r)=\dfrac{r-1}{(r-2)!}-\dfrac{r}{(r-1)!}$ $=\dfrac{(r-1)^2-r}{(r-1)!}$ $=\dfrac{r^2-2r+1-r}{(r-1)!}$ $=\dfrac{r^2-3r+1}{(r-1)!}$	**M1** – Simplify with $(r-1)!$ in the denominator. **AG1**
(i)	$\displaystyle\sum_{r=2}^{n}\frac{r^2-3r+1}{(r-1)!}=\sum_{r=2}^{n}\left(\mathrm{f}(r-1)-\mathrm{f}(r)\right)$ $=\mathrm{f}(1)-\mathrm{f}(2)$ $+\mathrm{f}(2)-\mathrm{f}(3)$ $+\mathrm{f}(3)-\mathrm{f}(4)$ $\vdots$ $\mathrm{f}(n-1)-\mathrm{f}(n)$ $=\mathrm{f}(1)-\mathrm{f}(n)$ $=1-\dfrac{n}{(n-1)!}$	**M1** – List terms and show cancellation **B1** – $\mathrm{f}(1)-\mathrm{f}(n)$ **A1**
(ii)	As $n\to\infty$, $\dfrac{n}{(n-1)!}=\dfrac{n}{(n-1)(n-2)\ldots1}\to 0$. Thus $\displaystyle\sum_{r=2}^{\infty}\frac{r^2-3r+1}{(r-1)!}=1$	**B1** – Show $\dfrac{n}{(n-1)(n-2)\ldots1}\to 0$ **B1** **Total: 7 marks**

Qn	Suggested Solution	Marking Scheme
5(a)	Volume of solid $= \pi \int_0^{\ln 5} x^2 \mathrm{d}y$ $= \pi \int_0^{\ln 5} \left(5 - e^y\right)^2 \mathrm{d}y$ $= 38.44$ (2 dp) by GC	**B1** – Correct formulation and limits. **B1** – Answer to 2dp (accept 12.24π)
b(i)	$\{x \in \mathbb{R},\ 0 \le x \le 4\}$	**B1** – Condone w/o set notation
(ii)	 From the diagram $\int_0^{\frac{9}{2}} \lvert \mathrm{f}(x) \rvert \mathrm{d}x - \int_{-\frac{9}{2}}^{0} \mathrm{f}(\lvert x \rvert) \mathrm{d}x$ $= (A + B) - (A - B) = 2B$ Consider: $\int \ln(5-x)\, \mathrm{d}x$ $= x\ln(5-x) + \int \frac{x}{5-x} \mathrm{d}x$ $= [x\ln(5-x)] - \int \left(1 - \frac{5}{5-x}\right) \mathrm{d}x$ $= x\ln(5-x) - [x + 5\ln(5-x)] + c$ $= (x-5)\ln(5-x) - x + c$ $\therefore 2B = 2\left\lvert \left[(x-5)\ln(5-x) - x\right]_4^{\frac{9}{2}} \right\rvert = 2\left\lvert -\frac{1}{2}\ln\frac{1}{2} - \frac{1}{2} \right\rvert$ $= 1 - \ln 2$ $\therefore a = 1\ ,\ b = -1$	**M1** – Identify and simplify required sections <u>by symmetry</u> **M1** – Correct integration by parts applied **M1** – Split numerators and apply by parts. Must see $[x + k\ln(5-x)]$. Condone w/o $+c$ **B1**– Correct limits (or equivalents) **A1** – Both a and b correct **Total : 8 marks**

Qn	Suggested Solution	Marking Scheme
6 (i)	$y=\mathrm{e}^{\cos^{-1}x}$ $\ln y=\cos^{-1}x$ $\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{\sqrt{1-x^2}}$ $\left(1-x^2\right)\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=y^2$ $\left(1-x^2\right)\left(2\frac{\mathrm{d}y}{\mathrm{d}x}\right)\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)+(-2x)\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=2y\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ $\left(1-x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-x\frac{\mathrm{d}y}{\mathrm{d}x}=y$ (shown)	 **B1** **M1**– Differentiate wrt x again. Two out of three terms correct. **AG1** – All terms correct
	Alternative $y=\mathrm{e}^{\cos^{-1}x}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{\sqrt{1-x^2}}\mathrm{e}^{\cos^{-1}x}$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{1}{1-x^2}\mathrm{e}^{\cos^{-1}x}+\mathrm{e}^{\cos^{-1}x}\left(\left(\frac{1}{2}\right)\left(1-x^2\right)^{-\frac{3}{2}}(-2x)\right)$ $=\frac{1}{1-x^2}\mathrm{e}^{\cos^{-1}x}-x\left(1-x^2\right)^{-\frac{3}{2}}\mathrm{e}^{\cos^{-1}x}$ $\text{LHS}=\left(1-x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-x\frac{\mathrm{d}y}{\mathrm{d}x}$ $\left(1-x^2\right)\left(\frac{1}{1-x^2}\mathrm{e}^{\cos^{-1}x}-x\left(1-x^2\right)^{-\frac{3}{2}}\mathrm{e}^{\cos^{-1}x}\right)$ $-x\left(-\frac{1}{\sqrt{1-x^2}}\mathrm{e}^{\cos^{-1}x}\right)$ $=\mathrm{e}^{\cos^{-1}x}=y=\text{RHS}$ (shown)	 **B1** **M1**– Differentiate wrt x again. Correct application of chain rule or product rule. **AG1** – Show LHS = RHS
(ii)	$\left(1-x^2\right)\frac{\mathrm{d}^3y}{\mathrm{d}x^3}-2x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-\frac{\mathrm{d}y}{\mathrm{d}x}-x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{\mathrm{d}y}{\mathrm{d}x}$ $\left(1-x^2\right)\frac{\mathrm{d}^3y}{\mathrm{d}x^3}-3x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-2\frac{\mathrm{d}y}{\mathrm{d}x}=0$ When $x=0$, $y=\mathrm{e}^{\frac{\pi}{2}}$, $\frac{\mathrm{d}y}{\mathrm{d}x}=-\mathrm{e}^{\frac{\pi}{2}}$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\mathrm{e}^{\frac{\pi}{2}}$, $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=-2\mathrm{e}^{\frac{\pi}{2}}$, $y=\mathrm{e}^{\frac{\pi}{2}}\left(1-x+\frac{x^2}{2!}-\frac{2x^3}{3!}+...\right)$ $=\mathrm{e}^{\frac{\pi}{2}}\left(1-x+\frac{x^2}{2}-\frac{x^3}{3}+...\right)$	**M1** – Any pair of terms in LHS correct (as evident of correct implicit differentiation) **M1**– First 3 terms correct and provide value for $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$ **A1**

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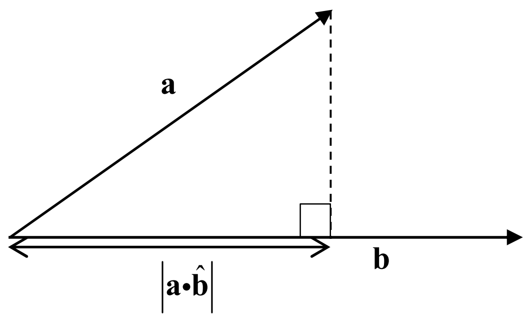
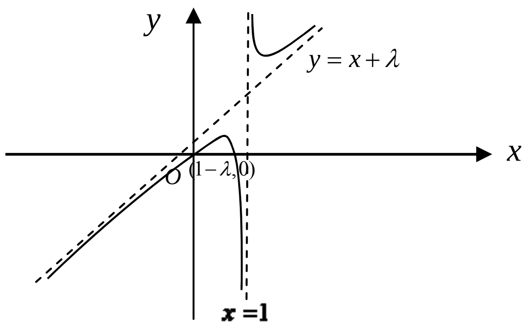
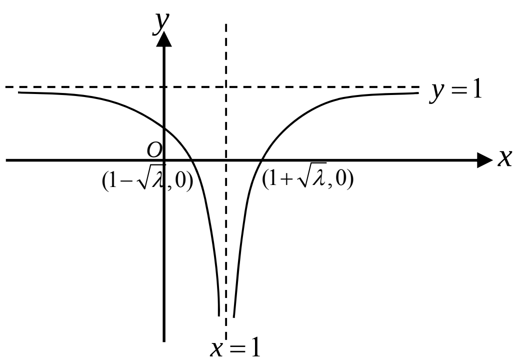
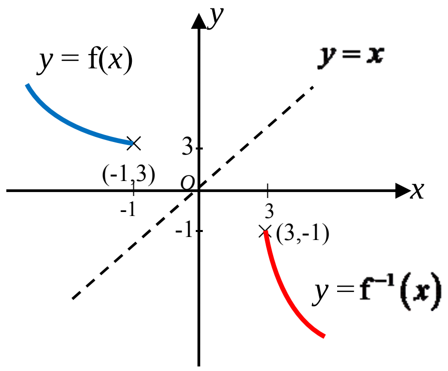
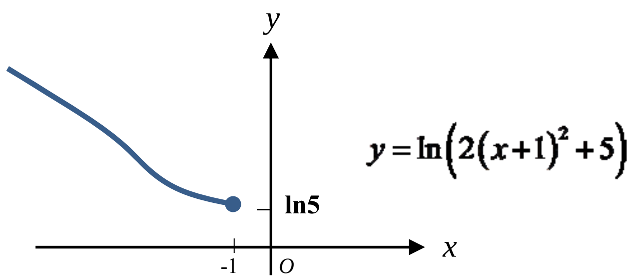
(iii)	$\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{e}^{\cos^{-1}x}=\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{e}^{\frac{\pi}{2}}\left(1-x+\frac{x^2}{2}-\frac{x^3}{3}+...\right)$	**M1**– Differentiate both sides of the equation in **(ii)**
	$\therefore -\frac{\mathrm{e}^{\cos^{-1}x}}{\sqrt{1-x^2}}=\mathrm{e}^{\frac{\pi}{2}}\left(-1+x-x^2+...\right)$	**B1**
	$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=0.5}=-\left.\frac{\mathrm{e}^{\cos^{-1}x}}{\sqrt{1-x^2}}\right\|_{x=0.5}\approx \mathrm{e}^{\frac{\pi}{2}}\left(-1+(0.5)-(0.5)^2\right)$	
	$=-\frac{3}{4}\mathrm{e}^{\frac{\pi}{2}}$ or -3.61 (3.s.f)	**A1**
		Total: 9 marks

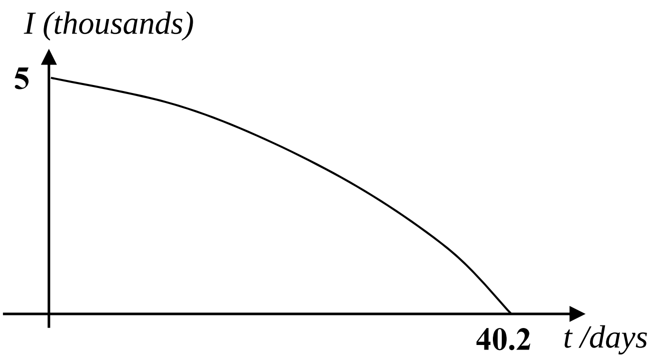
Qn	Suggested Solution	Marking Scheme
7 **(i)**	$\overrightarrow{OD}=\dfrac{3\mathbf{a}+2p\mathbf{b}}{5}$ $\overrightarrow{OE}=\dfrac{3\mathbf{a}+\mathbf{b}}{4}$	**B1** **B1**
(ii)	$\overrightarrow{OD}=q\overrightarrow{OE}$, where q is a constant $\dfrac{3\mathbf{a}+2p\mathbf{b}}{5}=q\left(\dfrac{3\mathbf{a}+\mathbf{b}}{4}\right)$ $\Rightarrow \dfrac{3}{5}=\dfrac{3}{4}q \Rightarrow q=\dfrac{4}{5}$ $\Rightarrow \dfrac{2}{5}p=\dfrac{1}{4}q \Rightarrow p=\dfrac{1}{2}$	**M1** – Collinear; form $\overrightarrow{OD}=q\overrightarrow{OE}$ using answer in **(i)** **M1** – Comparing coefficient **A1**
(iii)	Shortest distance from the point E to OB $=\left\lvert \overrightarrow{OE}\times\dfrac{\overrightarrow{OB}}{OB}\right\rvert$ $=\left\lvert \left(\dfrac{3\mathbf{a}+\mathbf{b}}{4}\right)\times\dfrac{\mathbf{b}}{5}\right\rvert$ $=\dfrac{1}{20}\left\lvert (3\mathbf{a}\times\mathbf{b})+(\mathbf{b}\times\mathbf{b})\right\rvert$ $=\dfrac{3}{20}\lvert\mathbf{a}\times\mathbf{b}\rvert \quad (\because \mathbf{b}\times\mathbf{b}=\mathbf{0})$ $k=\dfrac{3}{20}$	**M1** – Formula **M1** – Obtain the expression $k\lvert\mathbf{a}\times\mathbf{b}\rvert$ using properties of cross product **A1** – Correct k value, can be shown in the form $k\lvert\mathbf{a}\times\mathbf{b}\rvert$.
(iv)	It is the length of projection of **a** onto **b**. 	**B1** **Total: 9 marks**

Qn	Suggested Solution	Marking Scheme
8 (i)	$y=\dfrac{x^2+(\lambda-1)x}{x-1}=x+\lambda+\dfrac{\lambda}{x-1}$ The equations of asymptotes: $y=x+\lambda$ and $x=1$	**B1** – $y=x+\lambda$ **B1** – $x=1$
(ii)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=1-\dfrac{\lambda}{(x-1)^2}$ At stationary point, $\dfrac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow 1-\dfrac{\lambda}{(x-1)^2}=0$ $(x-1)^2=\lambda$ $x=1\pm\sqrt{\lambda}$ where $\lambda>0$ For C to have 2 stationary points for $x>0$, $1-\sqrt{\lambda}>0\Rightarrow\lambda<1$ $\therefore 0<\lambda<1$	**M1** – Correct differentiation based on expression in **(i)** **B1** – Correct simplification of $\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$ to a quadratic equation at stationary point **M1** – Use smaller root > 0 **A1** – $0<\lambda<1$
(iii)		**G1** – Asymptotes + shape **G1** – *x*-intercepts (condone if not written in coordinates form)
(iv)		**G1** – Asymptotes + *x*-intercepts (equidistance from vert asymptote & condone if not written in coordinates form) **G1**– Shape (symmetrical about vert asymptote) **Total: 10 marks**

Qn	Suggested Solution	Marking Scheme
9(a)	$z^4 = -4-4\sqrt{3}\,\mathrm{i}$ $z^4 = 8\mathrm{e}^{\mathrm{i}\left(-\frac{2\pi}{3}\right)}$ $z = 8^{\frac{1}{4}}\mathrm{e}^{\mathrm{i}\frac{1}{4}\left(-\frac{2\pi}{3}+2k\pi\right)},\ k=0,\pm1,2$ $z = 8^{\frac{1}{4}}\mathrm{e}^{\mathrm{i}\frac{\pi}{6}(3k-1)},\ k=0,\pm1,2$	**B1 –** Correct argument for z^4 **M1 –** Apply DM's Thm correctly **A1 –** Correct answer with correct *k* values or listing of roots
	$w^4 = -1+\sqrt{3}\,\mathrm{i} = \frac{1}{4}\left(z^4\right)^*$ $\Rightarrow w = \frac{z^*}{\sqrt{2}}$ $w = \frac{z^*}{\sqrt{2}} = \frac{8^{\frac{1}{4}}\mathrm{e}^{\mathrm{i}\frac{\pi}{6}(1-3k)}}{\sqrt{2}} = 2^{\frac{1}{4}}\mathrm{e}^{\mathrm{i}\frac{\pi}{6}(1-3k)},\ k=0,\pm1,2$	**M1 –** Attempt to make use of $\frac{1}{4}\left(z^4\right)^*$ or equivalent **B1** – Correct relationship between *z* and *w* **A1** – Correct answer
9(b)	$\left\|\frac{p^7}{q^3}\right\| = \frac{\|p\|^7}{\|q\|^3} = \frac{2^7}{7^3} = \frac{128}{343}$ Consider $7\arg(p)-3\arg(q) = 7\left(\frac{\pi}{3}\right)-3\left(-\frac{2\pi}{3}\right) = \frac{13\pi}{3}$ $\therefore \arg\left(\frac{p^7}{q^3}\right) = \frac{13\pi}{3}-4\pi = \frac{\pi}{3}$ $\therefore \frac{p^7}{q^3} = \frac{128}{343}\left[\cos\left(\frac{\pi}{3}\right)+\mathrm{i}\sin\left(\frac{\pi}{3}\right)\right] = \frac{64}{343}+\mathrm{i}\,\frac{64\sqrt{3}}{343}$	**M1 –** Award once $\frac{2^7}{7^3}$ is seen **M1** – Award once $7\arg(p)-3\arg(q)$ is seen **A1** – Correct answer $\frac{\pi}{3}$ **A1**
	Smallest integer value of *n* is 3.	**B1** **Total: 11 marks**

Qn	Suggested Solution	Marking Scheme
10 (i)	Let $y = x^2 + 2x + 4 = (x+1)^2 + 3$ $(x+1)^2 = y - 3$ $x = -\sqrt{y-3} - 1 \qquad (\because x \le -1)$ $\mathrm{f}^{-1}(x) = -\sqrt{x-3} - 1$ $\mathrm{D}_{\mathrm{f}^{-1}} = \mathrm{R}_\mathrm{f} = [3, \infty)$	**M1** – Express y in terms of x and solve for x (to award once x is expressed as the subject, no need to choose correct expression from the two choices) **A1** – Expression for $\mathrm{f}^{-1}(x)$ in terms of x **B1**
(ii)	 Since there is no intersection between the 2 graphs, there is **<u>no solution</u>** for $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$.	**G1** – f(x) : Shape, domain, range and min pt correct **G1** – Correct symmetry about $y = x$ [No follow-through] **B1** – Answer with reason
(iii)	$\mathrm{R}_\mathrm{f} = [3, \infty), \ \mathrm{D}_\mathrm{g} = \left(\frac{1}{2}, \infty\right)$ Since $\mathrm{R}_\mathrm{f} \subseteq \mathrm{D}_\mathrm{g}$, gf exists. $\mathrm{gf}(x) = \mathrm{g}\left((x+1)^2 + 3\right)$ $= \ln\left(2\left[(x+1)^2 + 3\right] - 1\right)$ $= \ln\left(2(x+1)^2 + 5\right)$ $(-\infty, -1] \xrightarrow{\mathrm{f}} [3, \infty) \xrightarrow{\mathrm{g}} [\ln 5, \infty)$ $\therefore \mathrm{R}_\mathrm{gf} = [\ln 5, \infty)$ <u>Alternatively</u> From graph of gf(x) for $x \le -1$,  $\mathrm{R}_\mathrm{gf} = [\ln 5, \infty)$	**B1** – Reason with details of R_f and D_g **M1** – Correct substitution with f(x) **A1** – Correct expression √**M1** – Show a two-step matching process **A1** – Exact range **Total: 11marks**

Qn	Suggested Solution	Marking Scheme
11 (a)	$z=\mathrm{e}^{2x}\dfrac{\mathrm{d}y}{\mathrm{d}x}$	
	$\dfrac{\mathrm{d}z}{\mathrm{d}x}=\mathrm{e}^{2x}\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+2\mathrm{e}^{2x}\dfrac{\mathrm{d}y}{\mathrm{d}x}$	**M1 –** Correct implicit differentiation
	$=\mathrm{e}^{2x}\left(\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+2\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)$	
	Given $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+2\dfrac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^{1-4x}$	
	$\mathrm{e}^{-2x}\dfrac{\mathrm{d}z}{\mathrm{d}x}=\mathrm{e}^{1-4x}$	
	$\Rightarrow \dfrac{\mathrm{d}z}{\mathrm{d}x}=\mathrm{e}^{1-4x}.\mathrm{e}^{2x}\Rightarrow\dfrac{\mathrm{d}z}{\mathrm{d}x}=\mathrm{e}^{1-2x}$ (shown)	**AG1** – Substitution & simplify to AG
	Hence, $\int \mathrm{d}z=\int \mathrm{e}^{1-2x}\mathrm{d}x$	
	$z=-\dfrac{1}{2}\mathrm{e}^{1-2x}+C$	**M1** – Condone wrong sign
	$\mathrm{e}^{2x}\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{1}{2}\mathrm{e}^{1-2x}+C$	**M1** – Replace z to obtain and solve 1st order DE
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{1}{2}\mathrm{e}^{1-4x}+C\mathrm{e}^{-2x}$	
	$y=\dfrac{1}{8}\mathrm{e}^{1-4x}-\dfrac{1}{2}C\mathrm{e}^{-2x}+D$	**A1**
	where C and D are arbitrary constants	

Qn	Suggested Solution	Marking Scheme
11 (b)	$\frac{\mathrm{d}I}{\mathrm{d}t}=\frac{1}{25}I-0.25$ $\frac{\mathrm{d}I}{\mathrm{d}t}=0.04(I-6.25)$ $\int\frac{1}{I-6.25}\,\mathrm{d}I=\int 0.04\,\mathrm{d}t$ $\ln\lvert I-6.25\rvert=0.04t+c$ $I-6.25=Ae^{0.04t}$ $\quad(A=\pm\mathrm{e}^{c})$ $I=A\mathrm{e}^{0.04t}+6.25$ when $t=0$, $I=5$, $5=A\mathrm{e}^{0}+6.25\Rightarrow A=-1.25$ $\therefore\ I=6.25-1.25\mathrm{e}^{0.04t}$  Since the curve $I=6.25-1.25\mathrm{e}^{0.04t}$ cuts the t-axis i.e. I = 0 at t = 40.2, it is possible for the insect population to be depleted. Number of days for this to happen is 41 days.	**AG1** – Correct formulation leading to answer **M1** – Separate variables & integrate **M1** – Correct result from integration, with arbitrary constant seen **A1** – I in terms of t **G1** – ecf; condone wrong t-intercept or no t-intercept **[Note: Do not award if graph includes negative *I* or *t* regions.]** **B1 –** To the nearest day **Total : 11 marks**

Qn	Suggested Solution	Marking Scheme
12 (a) (i)	$A=3h^2+\frac{\pi}{2}\left[\left(2r\right)^2-r^2\right]+7h^2$ $=10h^2+\frac{3}{2}\pi r^2$ (Shown)	**M1** – Correct method for area of "D" **AG1**
(ii)	$A=\frac{10k}{r}+\frac{3}{2}\pi r^2$ $\frac{\mathrm{d}A}{\mathrm{d}r}=-\frac{10k}{r^2}+3\pi r$ $\frac{\mathrm{d}A}{\mathrm{d}r}=0\Rightarrow r^3=\frac{10k}{3\pi}$ $r^2\frac{k}{h^2}=\frac{10k}{3\pi}$ $\frac{r}{h}=\sqrt{\frac{10}{3\pi}}$ $\frac{\mathrm{d}^2A}{\mathrm{d}r^2}=\frac{20k}{r^3}+3\pi>0$ since $k>0$ $\therefore$ A has a minimum value. min $A=14.7$ (3 s.f. from GC)	**M1** – Express in terms of a single variable √**M1** – Correct differentiation **B1** **A1** **B1** – Apply 2nd derivative test & provide correct conclusion **B1**
(b)	$y=2x+5+\frac{4}{x}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=2-\frac{4}{x^2}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow x=\pm\sqrt{2}$ Since the graph is that of a hyperbola, the stationary points correspond to turning points at $\left(-\sqrt{2},5-4\sqrt{2}\right)$ and $\left(\sqrt{2},5+4\sqrt{2}\right)$. Hence the set of values required is $\{y\in\mathbb{R}:5-4\sqrt{2}<y<5+4\sqrt{2}\}$.	**B1** **M1** – Find stationary points (must see some x-values computed) √**M1** – Compute y-coordinates of turning points **A1** **S.R.** 1 mark for correct method by GC (regardless of answer) **Total: 12 marks**

END OF SOLUTION