H

TEMASEK JUNIOR COLLEGE

2023 PRELIMINARY EXAMINATION

Higher 2

TEMASEK
JUNIOR COLLEGE

CANDIDATE NAME

CENTRE NUMBER | S | | | | |

INDEX NUMBER | | | | |

MATHEMATICS   **9758/02**

Paper 2   **13 September 2023**

                                           3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name, centre number and index number on the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Write your answers in the spaces provided in the Question Paper.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

| For Examiner's Use ||
Question Number	Marks Obtained
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total Marks	

This document consists of **23** printed pages and **1** blank page.

[Turn over

Section A: Pure Mathematics [40 marks]

1 **(a)** Prove the identity $\left(n+\dfrac{1}{2}\right)^4 - \left(n-\dfrac{1}{2}\right)^4 \equiv 4n^3 + n$. [1]

(b) The sum S_N is defined by $S_N = \displaystyle\sum_{n=1}^{N} n^3$. Using the identity in part **(a)** find S_N as a polynomial in terms of N. [5]

(c) Explain why $N^{-4}S_N$ converges and state the value that it converges to. [2]

2 The function f is defined by
$$\text{f}:x \mapsto \begin{cases} 2+\sqrt{x} & \text{for } 0 < x < 4, \\ 12x - x^2 - 28 & \text{for } 4 \le x \le a, \end{cases}$$
where a is a constant.

(a) State the largest value of $a \in \mathbb{Z}$ for which the function f^{-1} exists. [1]

For the rest of the question, let the domain of f be restricted to $(0, 5]$.

(b) Sketch the graph of f, indicating clearly the coordinates of the end point(s). [2]

(c) Find f^{-1} in similar form. [4]

The function g is defined by $\text{g}(x) = 1 + |x-3|$ for $x \in \mathbb{R}^+$.

(d) Show that gf exists and find its corresponding range. [2]

3 **(a)** By using suitable small angle approximations and standard series from the List of Formulae (MF26), find the Maclaurin's series for $\dfrac{2\cos 2x}{1+\sin 2x}$ up to the x^2 term. [3]

(b) It is given that $y = \ln(1+\sin 2x)$, for $-\dfrac{\pi}{4} < x \le \dfrac{\pi}{4}$.

(i) Show that $e^y \left(\dfrac{d^2 y}{dx^2} + \left(\dfrac{dy}{dx}\right)^h \right) = k \sin 2x$, where h and k are constants to be determined. [3]

(ii) Find the first three non-zero terms of the Maclaurin expansion of $\ln(1+\sin 2x)$. [3]

(c) Using the result in part **(b)(ii),** verify the correctness of your answer in part **(a).** [1]

4 **(a)** With reference to the origin O, the points A, B and C are such that $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = \mathbf{c}$. The length of OA and OB are 1 and 2 respectively.

    (i) Give the geometrical meaning of $|\mathbf{a} \times \mathbf{b}|$. [1]

    (ii) Given that $\mathbf{c}$ is perpendicular to $\mathbf{a} \times \mathbf{b}$, explain why $\mathbf{c}$ can be expressed as $\mathbf{c} = \mu\mathbf{a} + \lambda\mathbf{b}$ for some constants μ and λ. [1]

    (iii) Given that angle $AOC = \dfrac{\pi}{2}$ radians and angle $AOB = \dfrac{\pi}{3}$ radians and the length of OC is $\sqrt{3}$, show that $\mu = -\lambda$ and hence find all possible value(s) of λ. [4]

(b) Planes p_1 and p_2 have vector equations $\mathbf{r} \cdot \begin{pmatrix} \sqrt{11} \\ 2 \\ -1 \end{pmatrix} = 4$ and $\mathbf{r} \cdot \begin{pmatrix} \sqrt{11} \\ 2 \\ -1 \end{pmatrix} = 12$

respectively. The point A has coordinates $\left(2\sqrt{11}, 4, 10\right)$.

    (i) Find the shortest distance from A to p_1. [2]

    (ii) By finding the distance between p_1 and p_2, or otherwise, determine with clear justification whether A is in between the planes p_1 and p_2. [2]

    (iii) Given that the point B lies on p_1 and has coordinates $\left(0, -4, -12\right)$, find the position vector of the point C on p_2 such that the line BC is perpendicular to both p_1 and p_2. [3]

Section B: Statistics [60 marks]

5 A group of 12 artistes, comprises 7 men and 5 women are invited to participate in an episode of TV variety show. In one of the challenges, 3 pairs of artistes, each consisting of a man and a woman, are to be selected to play a game in a night market.
(a) Find the number of ways the 3 pairs can be selected and grouped into pair *A*, *B* and *C*. [2]

In the game, the woman is required to generate a number, 1 to 9, by using a number generator with the probability of obtaining a number *x* is $\frac{1}{45}x$. If the number is an even number, the male artiste must draw a ball from a red box with 3 black and 5 yellow balls. If the number is odd, he must draw a ball from a blue box with 8 black and 4 yellow balls. If a yellow ball is picked, the pair will win. Otherwise, the pair will lose.
(b) Pair *A* plays the game once. Show that the probability that the pair loses the game is $\frac{29}{54}$. [2]

Pair *B* plays a match with Pair *C*. Each pair takes turns to play the game. The ball is replaced before the next pair plays the game. The match will stop only when either one of them wins a game. Pair *C* starts first.

(c) Find the probability that Pair *B* is the winner. [3]

6 A multiple choice test consists of 30 questions, with each question having 4 choices of which only 1 is correct. Each correct answer gains 3 marks but each incorrect answer loses 1 mark.
Charlie's strategy for this test is to answer all 30 questions. For each question, he will choose his answer at random with each choice having equal chance of being chosen. Let *C* denote the number of correct answers that Charlie achieves.
(a) State, in context, what must be assumed for *C* to be well modelled by a binomial distribution. [1]

Assume now that *C* has the binomial distribution $B(30, 0.25)$.
(b) Find the probability that Charlie answers at most 10 questions correctly. [1]

(c) Show that the expected number of marks that Charlie can achieve is zero and find the variance of Charlie's marks. [4]

The passing mark for this test is 32.
(d) Find the probability that Charlie will pass the test. [2]

7   A vending machine is set to dispense cups of drink of β ml. It is known that the volume of drink dispensed per cup is distributed normally with standard deviation of 6.3 ml. The company manager does a check by taking a random sample of 8 cups of drink to check if the volume of drink dispensed for each cup is less than β ml. He finds the volume of drink, in ml, are as follows.

<div align="center">

190   190   192   194   195   200   203   207

</div>

(a)   Find the mean of the sample of these 8 cups of drink.   [2]

(b)   State suitable hypotheses for the test, defining any symbols that you use.   [2]

(c)   Given that the company manager concludes that the vending machine is dispensing less than β ml at 5% level of significance, find the range of possible values of β.   [3]

The company repairs the machine and sets it to dispense a bigger cup of drink of 210ml. The company manager decides to perform a hypothesis test on a random sample of 40 cups of drink to find out if the machine is dispensing the correct volume of drink.

(d)   Explain why the company manager takes a sample of 40 cups of drink for this test when he only took a sample of 8 cups of drink in his earlier test.   [2]

8   The random variable X has the probability function

$$P(X = x) = \begin{cases} kx & \text{for } x = 3, 4, 5, \\ k(11 - x) & \text{for } x = 6, 7, 8. \end{cases}$$

where k is a constant.

(a)   Show that $k = \dfrac{1}{24}$.   [1]

(b)   Find the exact values of $E(X)$ and $Var(X)$.   [3]

(c)   Fifty independent observations of X are taken. Using a suitable approximation, estimate the probability that the mean of these observations is at less than 4.9.   [2]

(d)   Given that $Y = mX - 1$ where m is a constant, find $E(Y^2)$ in terms of m.   [3]

(e)   Given now that $Y = 3X - 1$, find the largest integer a such that $P(Y \geq a) \geq 0.5$.   [3]

9 A stall owner sells 2 kinds of poultry namely chicken and turkey.
The masses, in kg, of chickens and turkeys are modelled as having independent normal distributions with means and standard deviations as shown in the table.

	Mean mass	Standard deviation
Chickens	2.2	0.5
Turkeys	10.5	2.1

Chickens and turkeys with masses exceeding 3.5 kg and 14.5 kg respectively are considered as too big. Chickens and turkeys with masses smaller than 1 kg and 5 kg respectively are considered as too small.

(a) Find the probability that a randomly chosen chicken is neither too small nor too big. [1]

(b) Given that a randomly chosen chicken has a mass between 3.2 kg and 3.7 kg, find the probability that it is too big. [3]

(c) Two chickens and one turkey are chosen randomly. Find the probability that at most one of them is too big, giving your answer correct to 5 significant figures. [4]

(d) Find the probability that the difference in masses between a turkey and thrice the mass of a chicken is more than 0.3kg. [4]

10 To own a car in Singapore, one will need to purchase a certificate of entitlement (COE). The price of COE is determined based on a bidding system. To control the vehicle population in Singapore given the limited road space, there is a quota set for the number of COE given out each year. The table below shows the yearly quota, x (in thousands) for Category B cars (i.e. Cars above 1600cc or 97kW), and the annual average price of COE, y thousands dollar, for some years from 2003 to 2020.

Year	2003	2004	2007	2008	2010	2011	2012	2013	2014	2020
x	22	25	28	27	13	10	8	8	11	18
y	29	25	16	13	40	65	84	k	73	38

Source derived from https://coe.sgcharts.com/

(a) Given that the estimated least squares regression line of y on x is $y = -3.19x + 100.43$ show that the value of k is 79. [3]

(b) Draw a scatter diagram relating x and y. [2]

(c) Calculate the product moment correlation coefficient and comment on the value obtained. [2]

The model of the form $y = a + b \ln x$ where a and b are constants, is being considered to relate x and y.

(d) Explain why the model $y = a + b \ln x$ will give a better relationship between x and y than the regression line in **(a)**. [2]

(e) Use the model $y = a + b \ln x$ to predict the average price of COE when the yearly quota is 20000, correcting your answer to the nearest thousand. [2]

(f) Determine whether it is appropriate to use either the regression line in **(a)** or the model $y = a + b \ln x$ to predict the average price of COE if the yearly quota is 35000. [1]

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