Topic 13
Electric Fields

Data

permittivity of free space,	ε_o	=	8.85×10^{-12} F m^{-1} $(1/(36\pi)) \times 10^{-9}$ F m^{-1}
elementary charge,	e	=	1.60×10^{-19} C
unified atomic mass constant	u	=	1.66×10^{-27} kg
rest mass of electron,	m_e	=	9.11×10^{-31} kg
rest mass of proton,	m_P	=	1.67×10^{-27} kg

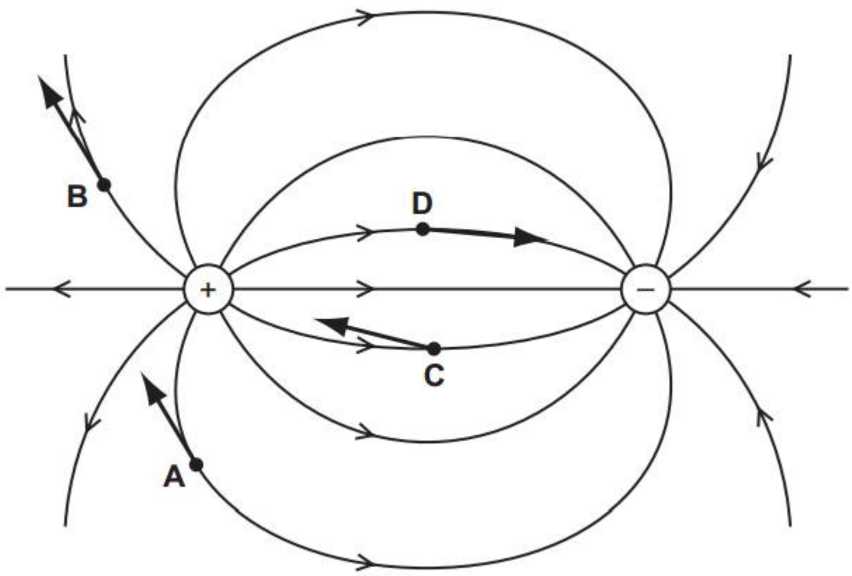
Formulae

electric potential, $V = \dfrac{Q}{4\pi\varepsilon_o r}$

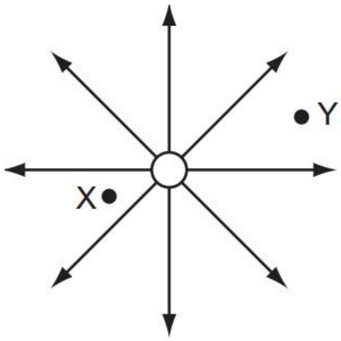
Multiple Choice Questions

1 The diagram shows a non-uniform electric field near a positively charged and a negatively charged sphere. Four electrons, A, B, C and D, are shown at different positions in the field.

On which electron is the direction of the force on the electron shown correctly?



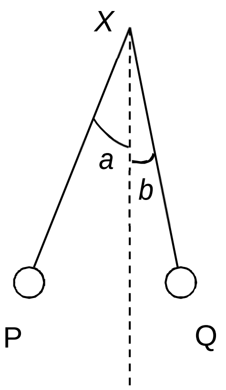
2 The diagram shows the electric field near a point charge and two electrons X and Y.



Which row describes the forces acting on X and Y?

	direction of force	magnitude of force on X
A	radially inwards	less than force on Y
B	radially inwards	greater than force on Y
C	radially outwards	less than force on Y
D	radially outwards	greater than force on Y

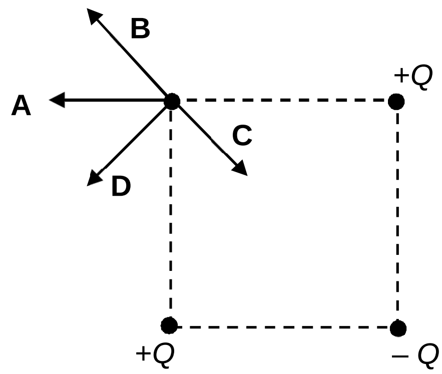
3 The diagram below shows two small charged spheres P and Q of small mass which are hung by identical fine nylon threads from a fixed point X. It is found that, in equilibrium, the angle *a* is greater than the angle *b*.



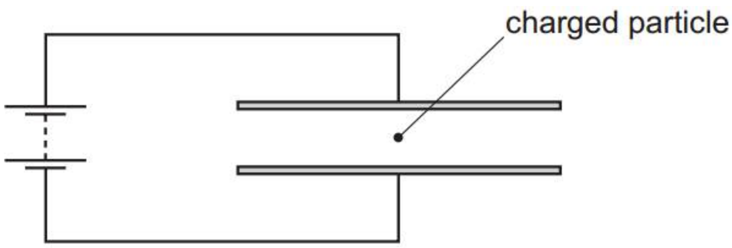
Which of the following statements must be correct?

A The mass of P is less than that of Q.
B The mass of P is greater than that of Q.
C The charge of P is numerically smaller than that of Q.
D The charge of P is numerically greater than that of Q.

4 Point charges, each of magnitude Q, are placed at three corners of a square as shown in the diagram. What is the direction of the resultant electric field at the fourth corner?



5 A charged particle is in the electric field between two horizontal metal plates connected to a source of constant potential difference, as shown. There is a force F on the particle due to the electric field.



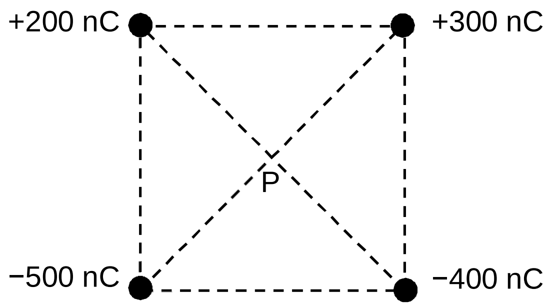
The separation of the plates is doubled. What will be the new force on the particle?

A $F/4$ **B** $F/2$ **C** F **D** $2F$

6 A particle has a charge of 4.8×10^{-19} C. The particle remains at rest between a pair of horizontal, parallel plates having a separation of 15 mm. The potential difference between the plates is 660 V. What is the weight of the particle?

A 1.1×10^{-23} N
B 2.1×10^{-17} N
C 2.1×10^{-15} N
D 2.1×10^{-14} N

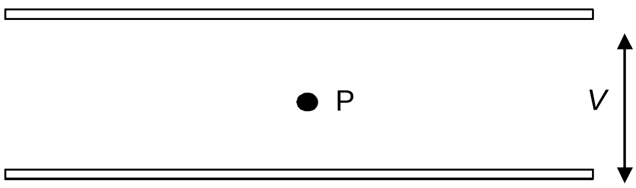
7 Four charges are arranged at the corners of a square as shown. Point P is located at the centre of the square. The diagonal of the square is 8.0 cm.



How much work is done by the field in bringing a −3.0 nC charge from infinity to point P without any change in its kinetic energy?

A 270 μJ **B** 135 μJ **C** −135 μJ **D** −270 μJ

8 A small negatively charged particle P is balanced halfway between two horizontal plates where a potential difference of *V* is applied between the plates.



When *V* is increased, P rises towards the upper plate.
When *V* is decreased, P falls towards the lower plate.

Which statement is correct?

A The change of electric potential energy of the particle must equal the change in gravitational potential energy of the particle.

B Increasing *V* increases both the gravitational and electric potential energy of the particle.

C Decreasing *V* decreases both the gravitational and electric potential energy of the particle.

D Decreasing *V* decreases the gravitational potential energy and increases the electric potential energy of the particle.

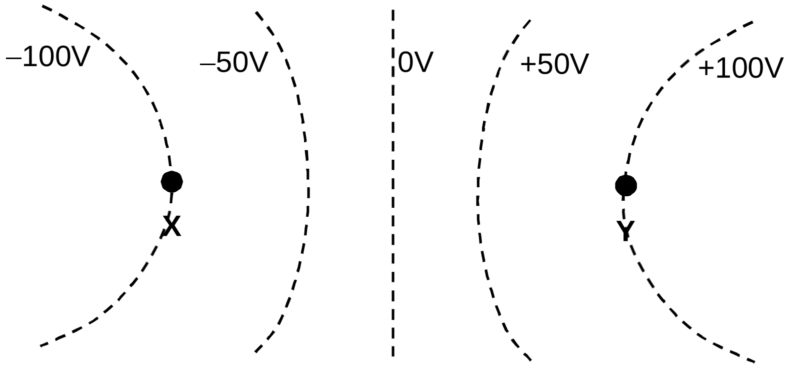
9 The electric potentials *V* are measured at distances *x* from P along a line PQ. The results are:

V / V	13	15	18	21	23
x / m	0.020	0.030	0.040	0.050	0.060

The component along PQ of the electric field for *x* = 0.040 m is approximately

A 250 V m^{-1} towards P
B 250 V m^{-1} towards Q
C 300 V m^{-1} towards P
D 300 V m^{-1} towards Q

10 The diagram below describes a region in an electric field.



Which of the following correctly describes the change in electric potential energy and force on an electron as it moves from X to Y?

	Potential Energy	Force
A	Increases, then decreases	Decreases, then increases
B	Increases	Constant
C	Decreases	Increases, then decreases
D	Decreases, then increases	Constant

Structured Questions

11 (a) State what is meant by *an electric field*.

It is a region of space where an electric charge experiences a force (due to other charged bodies).

(b) The electric field between an earthed metal plate and two charged metal spheres is illustrated in Fig. 11.1.

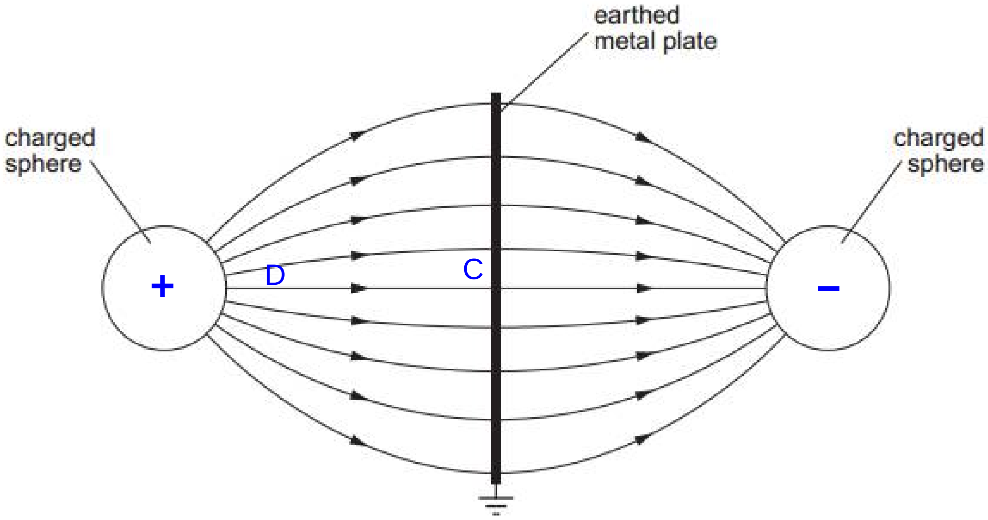


Fig. 11.1

(i) On Fig. 11.1, label each sphere with (+) or (–) to show its charge.

(ii) On Fig. 11.1, mark a region where the magnitude of the electric field is

1. constant (label this region C),

2. decreasing (label this region D).

(c) Fig. 11.2 shows the configuration of two isolated conductors. Sketch, on Fig. 11.2, the electric field lines for the region of space between two-conductors.

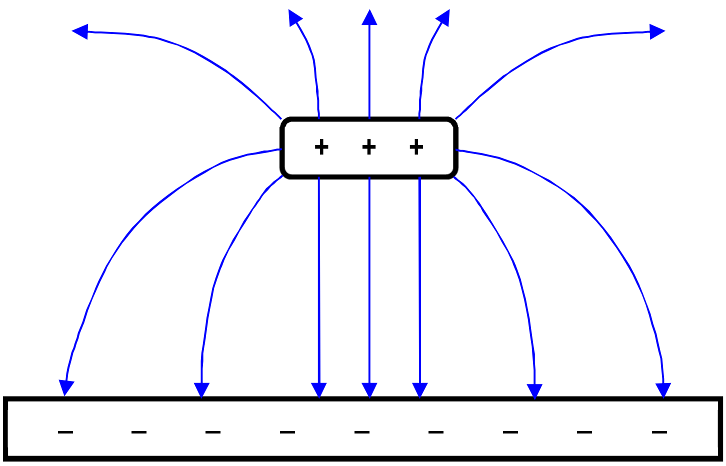


Fig. 11.2

12 Two ions A and B are separated by a distance of 0.72 nm in a vacuum, as shown in Fig. 12.1. A has a charge of $+4.8 \times 10^{-19}$ C and B has a charge of -3.2×10^{-19} C.

(a) Without making detailed calculations, draw labelled arrows on the figure to represent

(i) the field E_A at the point X due to A only,

(ii) the field E_B at X due to B only,

(iii) the resultant field E_R at X due to both charges.

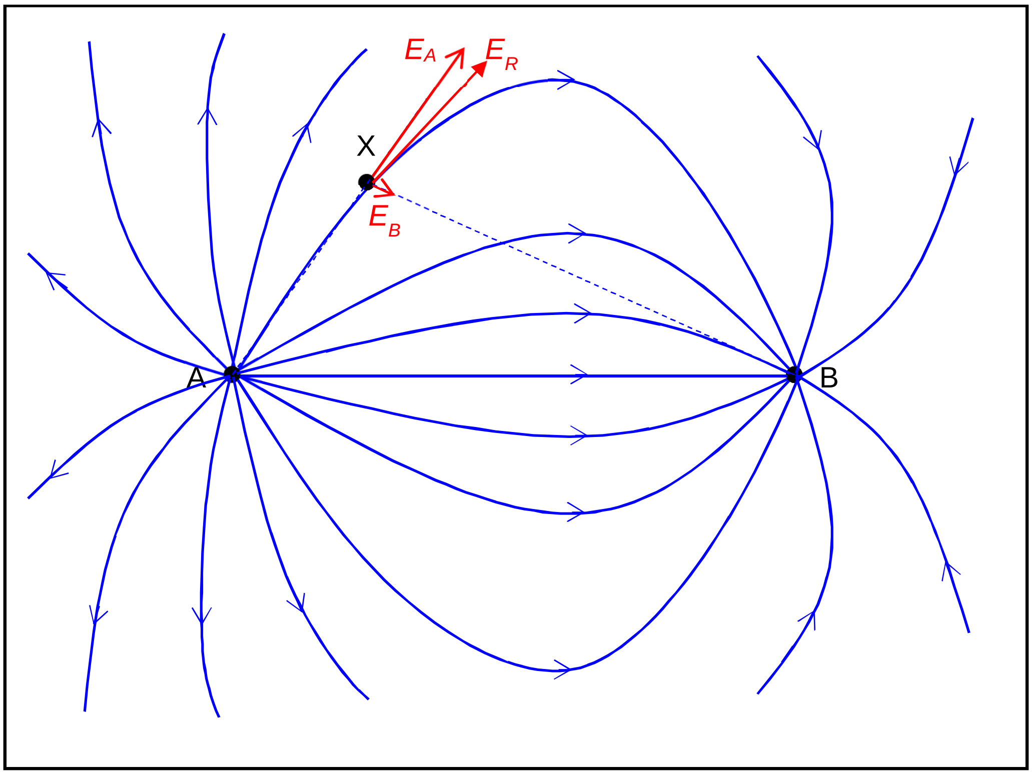


Fig. 12.1

(b) Sketch on the diagram lines representing the electric field caused by the two ions in the region within the rectangle. Include the field line passing through X.

13 A molecule has its centre P of positive charge situated a distance of 2.8×10^{-10} m from its centre N of negative charge, as illustrated in Fig. 13.1.

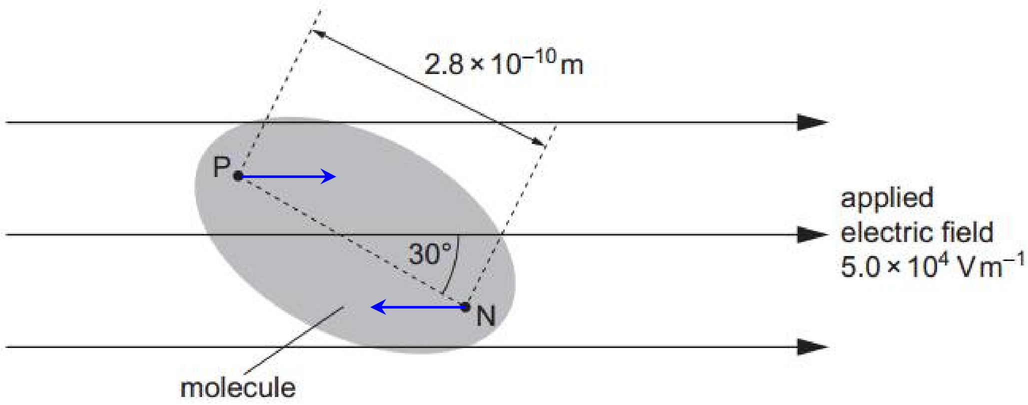


Fig. 13.1

The molecule is situated in a uniform electric field of field strength 5.0×10^4 V m^{-1}. The axis NP of the molecule is at an angle of 30° to this uniform applied electric field. The magnitude of the charge at P and at N is 1.60×10^{-19} C.

(a) On Fig. 13.1, draw an arrow at P and an arrow at N to show the directions of the forces due to the applied electric field at each of these points.

(b) Calculate the torque on the molecule produced by the forces in **(a)**.

$$\text{Torque } = F_E d_{perpendicular}$$
$$= \left(1.60 \times 10^{-19}\right)\left(5.0 \times 10^{4}\right)\left(2.8 \times 10^{-10} \sin 30^{o}\right)$$
$$= 1.12 \times 10^{-24} \text{ Nm}$$

14 Two fixed charges +1.0 μC and –3.0 μC are 10.0 cm apart. Determine a point where a third charge be located so that no net force acts on it?

Since the 2 are unlike charges and the magnitude of the negative charge is larger, the neutral point must lie in the region beyond the positive charge.

Let x be the distance from the positive charge.

At the neutral point, $|E_{+Q}| = |E_{-Q}|$

$$\left|\frac{+1.0 \times 10^{-6}}{4\pi\varepsilon_o (x)^2}\right| = \left|\frac{-3.0 \times 10^{-6}}{4\pi\varepsilon_o (x+0.100)^2}\right|$$
$$(x+0.1)^2 = 3x^2$$
$$2x^2 + 0.2x + 0.1^2 = 0$$
$$x = 0.137 \text{ m} \quad \text{or} \quad -0.0366 \text{ m (inadmissible)}$$

15 Fig. 15.1 shows 3 equal charges of +10 μC placed at the corners of an equilateral triangle ABC of sides 10 cm. P is a position midway along the line joining B and C.

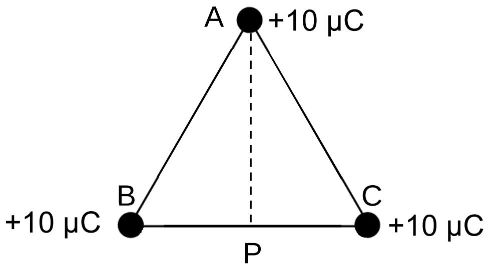


Fig. 15.1

(a) Determine at point P,

(i) the resultant electric field, and

Since charge B and C are both positive, equal in magnitude and equidistant from P, the electric field at P due to these two charges cancel each other.

Hence

$$\begin{aligned} E_{@\text{P}} &= E_{@\text{P due to A}} \\ &= \frac{Q_A}{4\pi\varepsilon_o(\text{AP})^2} \\ &= \frac{10\times10^{-6}}{4\pi(8.85\times10^{-12})(0.10^2-0.050^2)} = 1.20\times10^7\ \text{N C}^{-1}\text{, in direction AP} \end{aligned}$$

(ii) the resultant potential.

$$\begin{aligned} V_\text{P} &= \frac{Q_A}{4\pi\varepsilon_o(\text{AP})} + \frac{Q_B}{4\pi\varepsilon_o(\text{BP})} + \frac{Q_C}{4\pi\varepsilon_o(\text{CP})} \\ &= \left(\frac{10\times10^{-6}}{4\pi(8.85\times10^{-12})}\right)\left(\frac{1}{\sqrt{0.10^2-0.050^2}} + \frac{1}{0.050} + \frac{1}{0.050}\right) \\ &= 4.64\times10^6\ \text{V} \end{aligned}$$

(b) Hence, determine the work done by the electric field if a –5 μC charge were to be brought from infinity to the point P.

$$\begin{aligned} \Delta U &= q\Delta V \\ &= q(V_p - V_\infty) \\ &= (-5\times10^{-6})(4.64\times10^6 - 0) \\ &= -23.2\ \text{J} \end{aligned}$$

Hence $W_\text{field} = -W_\text{ext}$

$$\begin{aligned} &= -\Delta U \\ &= 23.2\ \text{J} \end{aligned}$$

16 Consider two widely separated isolated conducting spheres P and R, with R having twice the diameter of P. P initially has a positive charge q and R is initially uncharged. The two spheres are now connected with a long thin wire.

(a) State and explain How are the final potentials of the spheres related?

The final potentials of the spheres must be the same to ensure that electrostatic equilibrium is achieved.

(b) What are the final charges on the spheres, in terms of q?

At electrostatic equilibrium, $V_{\text{@ surface of P}} = V_{\text{@ surface of R}}$

$$\frac{Q_P}{4\pi\varepsilon_o r} = \frac{Q_R}{4\pi\varepsilon_o 2r}$$

$$Q_R = 2Q_P$$

By conservation of charge, $Q_R + Q_P = q$

Hence, $Q_P = \dfrac{q}{3}$

$$Q_R = \frac{2q}{3}$$

17 **(a)** Define *electric potential* at a point.

It is the work done per unit positive charge by an external agent in bringing a small test charge from infinity to that point, without any change in kinetic energy of the charge

(b) A charged particle is accelerated from rest in a vacuum through a potential difference V. Show that the final speed v of the particle is given by the expression

$$v = \sqrt{\left(\frac{2Vq}{m}\right)}$$

where $\dfrac{q}{m}$ is the ratio of the charge to the mass (the specific charge) of the particle.

By conservation of energy, gain in E_K = loss in Electric E_P

$$\frac{1}{2}mv^2 - 0 = qV$$

$$v = \sqrt{\frac{2Vq}{m}}$$

(c) A particle of specific charge $+9.58 \times 10^{7}$ C kg^{-1} is projected towards a fixed conducting sphere with a speed of 2.5×10^{5} m s^{-1}, as illustrated in Fig. 17.1. The surface of the sphere is at a potential of +470 V.

Fig. 17.1

Use the expression in **(b)** to determine if the particle will strike the surface of the sphere.

The expression in (b) can also be used to relate the potential difference V required to bring a particle of specific charge $\dfrac{q}{m}$ and speed v to rest.

Hence, $v_{\text{min to strike surface of sphere}} = \sqrt{2 \times 470 \times 9.58 \times 10^{7}}$

$$= 3.0 \times 10^{5} \text{ m s}^{-1}$$

Since the speed of the particle is less than 3.0×10^{5} m s^{-1}, the particle will be brought to rest before reaching the sphere and hence will not strike the surface of the sphere.

18 (a) (i) State what is meant by a *field of force*.

It is a region of space where an object with a physical quantity (i.e. mass, charge) experiences a force due to the presence of other objects possessing the same physical quantity.

(ii) Define *electric field strength*.

It is the electric force per unit positive charge experienced by a charge.

(iii) Suggest why, when defining electric field strength, the test particle must be stationary.

A moving charge will also experience a magnetic force due to the presence of a magnetic field. Thus in order to ensure the force per unit charge acting on the test particle to be solely due to the electric field, the test particle must be stationary.

(b) Two charged solid metal spheres A and B are situated in a vacuum. Their centres are separated by a distance of 30.0 cm, as illustrated in Fig. 18.1. The diagram is not to scale.

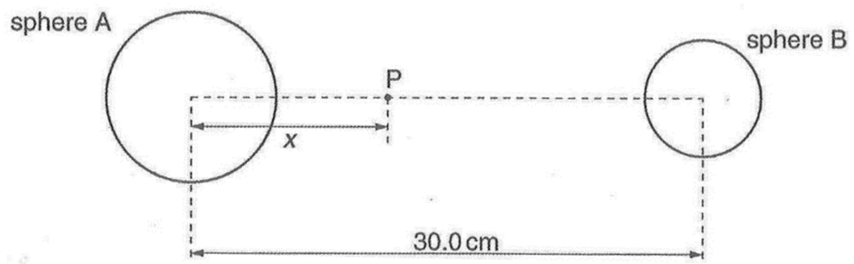


Fig. 18.1

Point P is a point on the line joining the centres of the two spheres. Point P is a distance *x* from the centre of A.

The variation with distance *x* of the electric field strength *E* at point P is shown in Fig. 18.2.

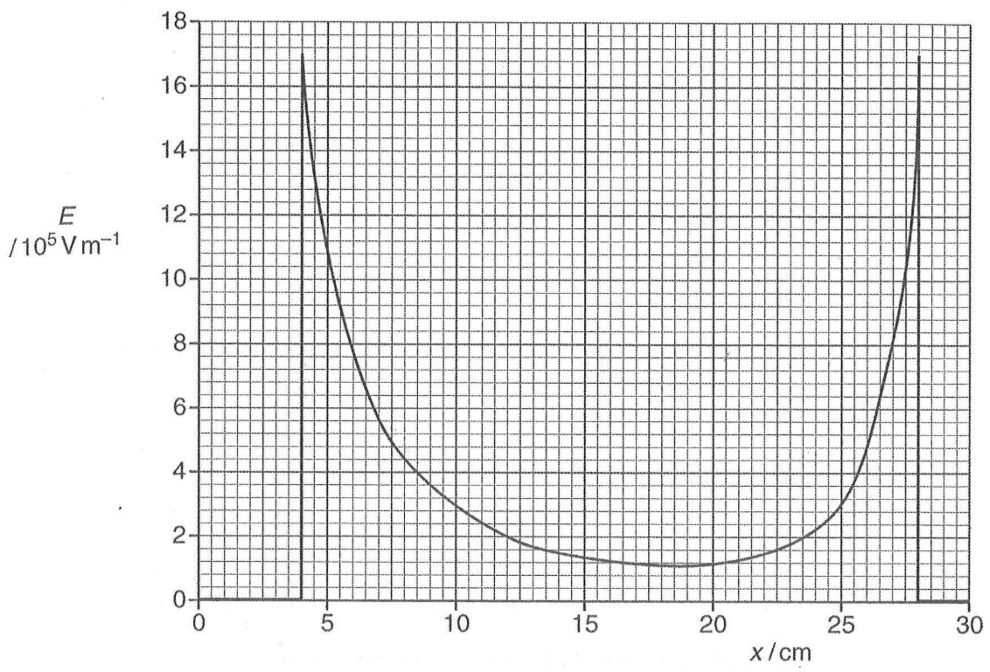


Fig. 18.2

(i) Suggest why the electric field strength is zero for two regions of *x*.

The two regions are within the metal spheres. As the spheres are conductors, the electric potential within them must be uniform and hence have zero electric field strength.

(ii) Use Fig. 18.2 to

1. determine the radius of each sphere

Radius of sphere A is 4.0 cm
Radius of sphere B is 2.0 cm

2. state and explain whether the spheres have charges of the same, or opposite, sign.

Since the graph of Fig. 18.2 is always positive / does not have negative values, this will imply that the direction of E due to both spheres in the region between the two spheres must be the same. Hence the spheres will have charges of opposite signs.

(iii) A lithium-7 $\left({}^{7}_{3}Li\right)$ nucleus moves along the line joining the centres of the two spheres.

1. Estimate the energy gained by this nucleus as it moves from point P where x = 16.0 cm to the point P where x = 21.0 cm.

change in electric potential ΔV = area under the graph

$\approx (1.2 \times 10^{5})(5.0\times 10^{-2}) = 6000$ V

change in electric E_P $= q\,\Delta V$

$= (3 \times 1.60 \times 10^{-19})(6000) = 2.88 \times 10^{-15}$ J

2. Calculate the acceleration of the nucleus at point P where x = 25.0 cm.

$$F_E = ma$$
$$q_{Li}E = m_{Li}\, a$$
$$(3 \times 1.60 \times 10^{-19})(3.0 \times 10^{5}) = (7 \times 1.66 \times 10^{-27})a$$
$$a = 1.24 \times 10^{13} \text{ m s}^{-2}$$

3. The nucleus is at rest at point P where x = 4.0 cm.

Describe qualitatively the variation with x of the acceleration of the nucleus for x = 4.0 cm to x = 28.0 cm.

The magnitude of the acceleration decreases from a maximum at x = 4.0 cm to a minimum at x = 18.5 cm. The magnitude of the acceleration then increases to a maximum at x = 28.0 cm.

[N14/III/6 (mod)]

19 (a) (i) State the relation between electric field strength E and potential V.

The negative rate of change of electric potential V in the direction of increasing r at a point in the field is equal to the component of the field strength E in the direction of r at that point.

(ii) Two charged metal spheres A and B, of diameters 18 cm and 12 cm respectively, are isolated in space, as shown in Fig. 19.1.

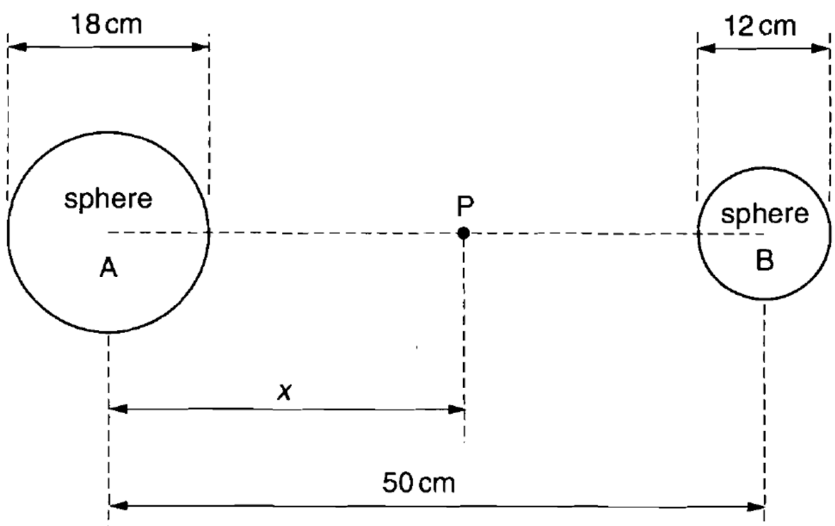


Fig. 19.1

The centres of the spheres are separated by a distance of 50 cm. Point P is at a distance x from the centre of sphere A along the line joining the centres of the two spheres.

The variation with x of the electric potential V at P is shown in Fig. 19.2.

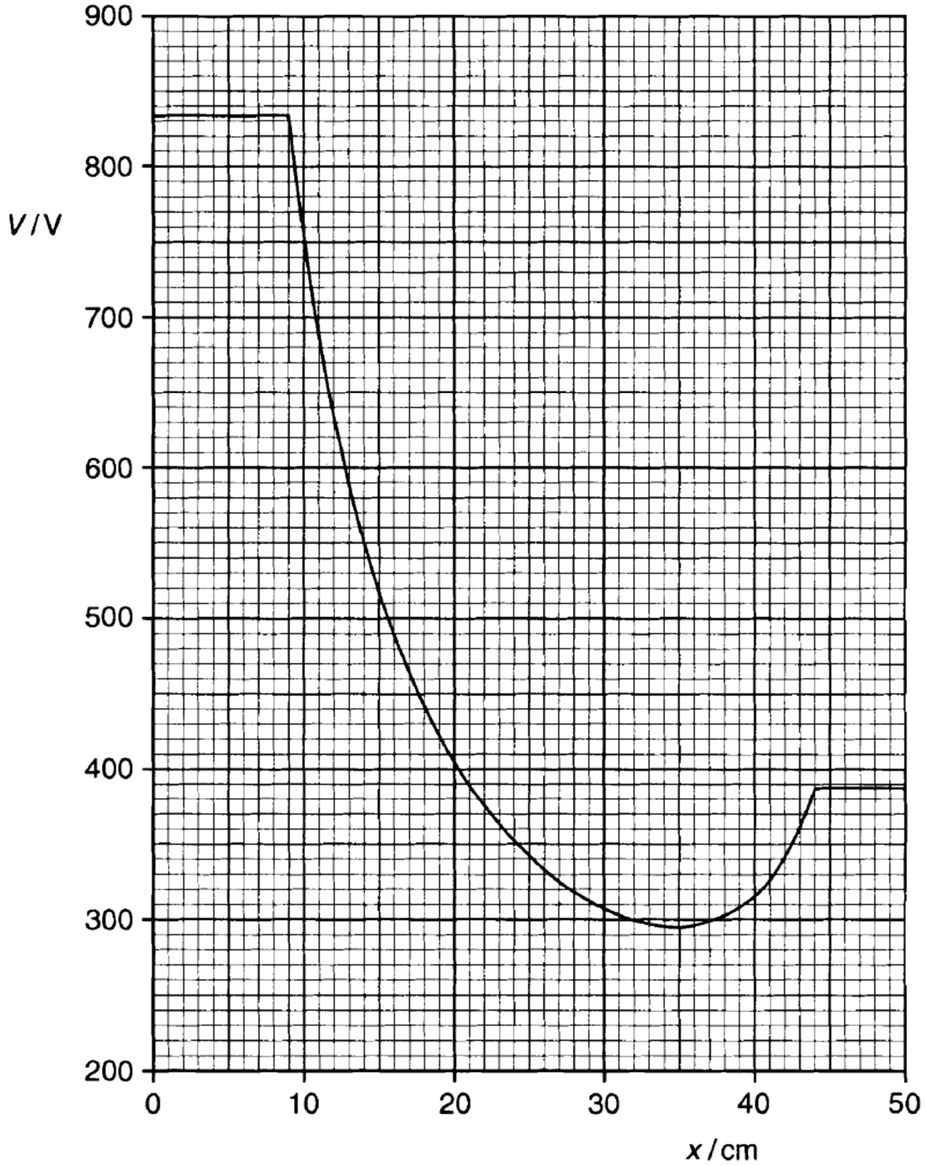


Fig. 19.2

1 State and explain the direction of the electric field at the point P, where x = 25.0 cm.

Since the direction of electric field is in the direction of decreasing potential, the direction of the electric field at x = 25.0 cm must towards the centre of sphere B.

2 Use Fig. 19.2 to determine the force on an electron placed at point P, where x = 35.0 cm.

The gradient of the tangent to a point on the *V*–*x* graph is equal to the electric field strength. At x = 35.0 cm, the tangent is horizontal, implying that the field strength is zero. Thus the force on an electron placed there is also zero.

3 By making reference to electric fields, explain why the potential is constant for distances between x = 0 and x = 9.0 cm.

The electric field strength inside a charged conducting sphere is always zero, implying that the potential within the sphere must be a constant.

(b) A student states that the potential V decreases with distance x for distances between x = 10 cm and x = 25 cm according to the expression

$$Vx = \text{constant}.$$

(i) Without drawing a graph, use data from Fig. 19.2 to show whether the student is correct.

For x = 10.0 cm, V = 750 V → Vx = 7500 V cm
For x = 25.0 cm, V = 340 V → Vx = 8500 V cm

Since from the two data points, the product of Vx differs from the average by more than 5%, the expression proposed by the student is not correct.

(ii) Suggest an explanation for your conclusion in **(i)**.

The expression proposed by the student is valid only when a single charged sphere is present, whereas in this situation there are two charged spheres.

[N12/III/7 (part)]

20 **(a)** Two flat parallel metal plates, each of length 12.0 cm, are separated by a distance of 1.5 cm, as shown in Fig. 20.1.

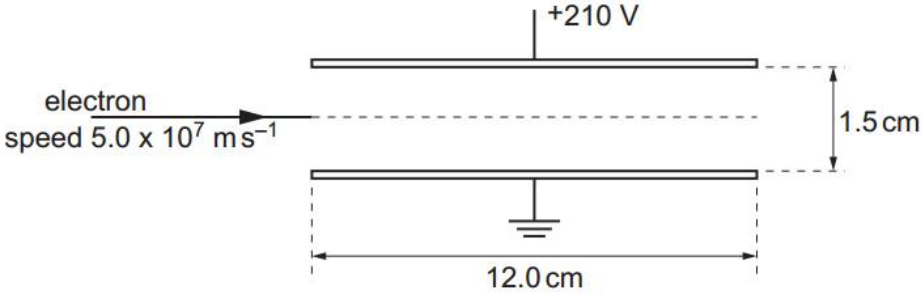


Fig. 20.1

The space between the plates is a vacuum. The potential difference between the plates is 210 V. The electric field may be assumed to be uniform in the region between the plates and zero outside this region.

Calculate the magnitude of the electric field strength between the plates.

$$E = \frac{\Delta V}{d}$$
$$= \frac{210}{1.5 \times 10^{-2}} = 1.4 \times 10^{4} \text{ V m}^{-1}$$

(b) An electron initially travels parallel to the plates along a line mid-way between the plates, as shown in Fig. 20.1. The speed of the electron is 5.0×10^7 m s^{-1}.

For the electron between the plates,

(i) determine the magnitude and direction of its acceleration,

$$a = \frac{F_E}{m_e}$$
$$= \frac{(1.60 \times 10^{-19})(1.4 \times 10^{4})}{9.11 \times 10^{-31}} = 2.5 \times 10^{15} \text{ m s}^{-2} \text{ (upwards / towards the positive plate)}$$

(ii) calculate the time for the electron to travel a horizontal distance equal to the length of the plates.

$$s_x = u_x t$$
$$t = \frac{0.120}{5.0 \times 10^{7}} = 2.4 \times 10^{-9} \text{ s}$$

(iii) determine if the electron will hit the plates or exit the region between the plates without hitting the plates.

$$s_y = u_y t + \frac{1}{2} a_y t^2$$
$$= 0 + \frac{1}{2}(2.5 \times 10^{15})(2.4 \times 10^{-9})^2 = 7.2 \times 10^{-3} \text{ m}$$

Hence the electron will exit the region between the plates without hitting them.

Data Analysis Question

21 When some substances are in solid state, they exist as positively-charged and negatively-charged ions arranged in a cubic lattice as illustrated in Fig. 21.1.

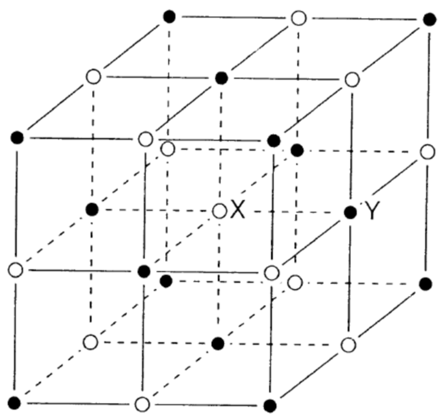


Fig. 21.1

A starting point for the understanding of lattice energies is to consider the potential energy E_P between two ions X and Y.

Fig. 21.2 shows the variation with distance r between X and Y of E_p.

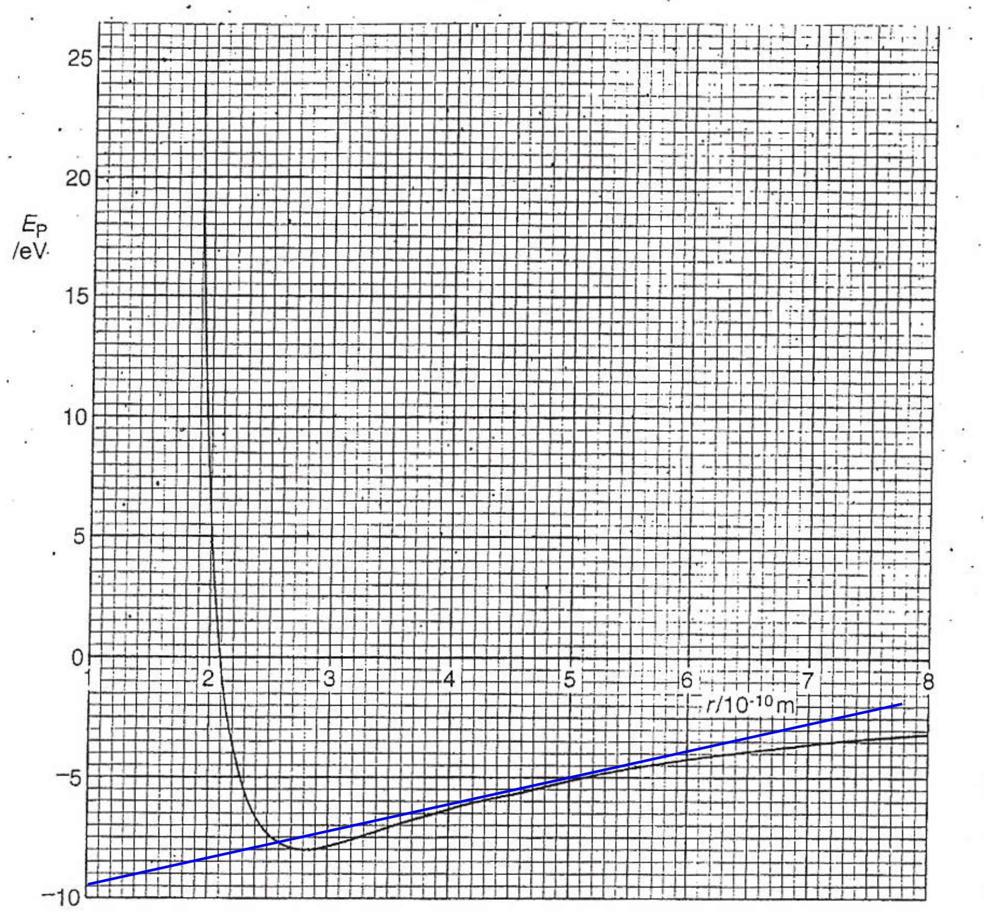


Fig. 21.2

(a) (i) The gradient *G* of the graph varies with the distance *r*. Show that, starting from the definition of work done, for any value of *r* the magnitude of the force *F* between X and Y is given by the expression

$$F = G$$

Work done = Force × distance moved in the direction of the force

$$\delta W = \int F_{Field}\, dr$$

$$F_{Field} = \frac{dW}{dr} = \text{ gradient of } W - r \text{ graph} = \text{G}$$ [2]

(ii) Suggest how Fig. 21.2 indicates that, for some values of *r*, the force between X and Y is attractive and, for other distances, the force is repulsive.

When the gradient of the graph is positive sloping, the force is attractive.

When the gradient of the graph is negative sloping, the force is repulsive.

[2]

(iii) Use Fig. 21.2, the expression in **(i)** and that $1\ \text{eV} = 1.6 \times 10^{-19}\ \text{J}$ to determine the magnitude of the force, in newton, for values of the distance *r* equal to

1. 2.8×10^{-10} m,

The gradient of the tangent to a point on the E_P –*x* graph is equal to the electric force. Since the tangent is horizontal, the magnitude of the force is zero.

force = ……………………………0…. N [1]

2. 5.0×10^{-10} m,

Force $=$ gradient of the tangent at r $= 5.0 \times 10^{-10}$ m

$$= \frac{\left[-9.00-(-2.50)\right]\times 1.60\times 10^{-19}}{(1.40-7.20)\times 10^{-10}}$$

$$= 1.8(\pm 0.2)\times 10^{-9}\ \text{N}$$

force = …………………………….. N [3]

(b) The variation with distance *r* of the potential energy E_P may be represented by the expression

$$E_P = -\frac{A}{r} + \frac{B}{r^8}$$

where *A* and *B* are constants.

By reference to Fig. 21.2, state two features of the force represented by the term $\frac{B}{r^8}$ in this expression.

E_P is only positive for r < 2.1×10^{-10} m. This suggests that the force is only significant only when the ion separation is small.

The E_P–*r* graph is negative sloping for r < 2.1×10^{-10} m. This suggests that the force is repulsive. [2]

(c) Fig. 21.3 shows part of Fig. 21.2, drawn on a larger scale.

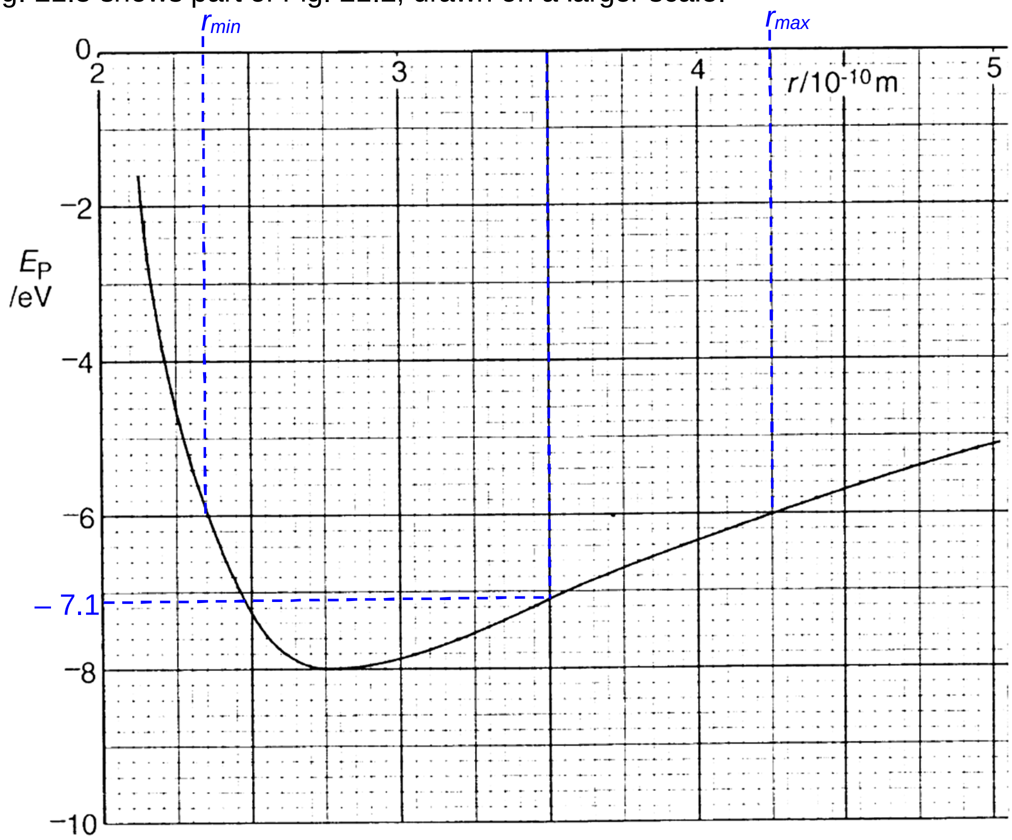


Fig. 21.3

Thermal energy of the ions causes them to vibrate.

The ions have a total energy of –6.0 eV.

(i) Use Fig. 21.3 to determine, for these ions,

1. the value of r between which they vibrate,

minimum value of r = ……2.35×10^{-10}…… m

maximum value of r = ……4.25×10^{-10}…… m [2]

2. the kinetic energy of the ions at a distance r = 3.5×10^{-10} m.,

$E_k = E_T - E_P$

$= -6.0 - (-7.1) = 1.1$ eV   kinetic energy = …………………… eV [2]

(ii) State why, although the ions are oscillating, their motion is **not** simple harmonic.

The force, and hence acceleration, of the ions is not proportional to their displacements from the equilibrium position as seen from the non-symmetry of the graph about the equilibrium position. Thus the oscillations do not follow the governing relationship $a = -\omega^2 x$ for an S.H.M. [1]

(d) By reference to Fig. 21.3, suggest why the dimensions of the whole lattice increases as it is heated.

When the lattice is heated, the total energy and hence maximum potential energy of each ion increases. From Fig. 21.3, the maximum value of *r* increases while the minimum value of *r* will decrease. With the ions in the lattice vibrating with a larger range of *r*, this suggests that the dimensions of the whole lattice increases. [3]

[N07/II/7]

Answer Key

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
A	B	A	B	B	D	D	D	C	C

13 **(b)** 1.1×10^{-24} N m

14 13.7 cm from +1.0 μC charge

15 **(a)(i)** 1.20×10^{7} V m^{-1} in the direction of AP **(ii)** 4.64×10^{6} V
(b) 23.2 J

16 **(b)** $Q_P = q/3$, $Q_R = 2q/3$

18 **(b)(ii)1.** $R_A = 4.0$ cm, $R_2 = 2.0$ cm **(iii)1.** 2.88×10^{-15} J **2.** 1.24×10^{13} m s^{-2}

19 **(a)(ii)1.** towards centre of sphere B **2.** 0 N

20 **(a)** 1.4×10^{4} N C^{-1}
(b)(i) 2.5×10^{15} m s^{-2} , upwards **(ii)** 2.4×10^{-9} s

21 **(a)(iii)1.** 0 N **2.** 1.8 (± 0.2) × 10^{-9} N
(c)(i)1. 2.35×10^{-10} m, 4.25×10^{-10} m **2.** 1.1 eV