General Certificate of Education Ordinary Level
JUYING SECONDARY SCHOOL, SINGAPORE
Secondary Four Express/Five Normal Academic
Preliminary Examination

CANDIDATE NAME	

CENTRE NUMBER	S					INDEX NUMBER					

ADDITIONAL MATHEMATICS — 4049/02

Paper 2

27 August 2024
2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed in any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total number of marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

This document consists of **18** printed pages.

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Vetted by: Mdm Norhafiani Bte Abdul Majid

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Answer **ALL** the questions

1 **(a)** Express $3x^2 - 8x - 3$ in the form $a(x+b)^2 + c$, where a, b and c are constants to be determined. [3]

$3\left(x^2 - (2)\left(\frac{4}{3}\right)x + \left(\frac{4}{3}\right)^2 - \left(\frac{4}{3}\right)^2\right) \quad - 3$ M1

$= 3\left(x - \frac{4}{3}\right)^2 - 3\left(\frac{16}{9}\right) - 3$

$= 3\left(x - \frac{4}{3}\right)^2 - \frac{25}{3}$ A1

$a = 3 \qquad b = -\frac{4}{3} \qquad c = -\frac{25}{3}$ B1

(b) Hence, find the turning point and the value of x where it occurs. [2]

Turning Point $= \left(\frac{4}{3},\ -\frac{25}{3}\right)$

Value of $x = \frac{4}{3}$

2 **(a)** Write down and simplify the first three terms in the expansion, in ascending powers of x, of $\left(2+\frac{x}{2}\right)^n$, where n is a positive integer. [3]

$$\left(2+\frac{x}{2}\right)^n = 2^n + n(2)^{n-1}\left(\frac{x}{2}\right) + \frac{n(n-1)}{2}(2)^{n-2}\left(\frac{x}{2}\right)^2 + \cdots \qquad \text{M2}$$

$$= 2^n + \frac{xn(2)^n}{4} + \frac{n(n-1)x^2(2)^n}{32} + \cdots \qquad \text{A1}$$

(b) The first two terms in the expansion, in ascending powers of x, of $\left(\frac{4}{3}-3x\right)\left(2+\frac{x}{2}\right)^n$ are $a + bx^2$. Find the value of n. [3]

$$\left(\frac{4}{3}-3x\right)\left(2+\frac{x}{2}\right)^n = \left(\frac{4}{3}-3x\right)\left[2^n + \frac{xn(2)^n}{4} + \frac{n(n-1)x^2(2)^n}{32} + \cdots\right]$$

$$= \frac{4}{3}\times 2^n + \frac{xn(2)^n}{3} + \frac{n(n-1)x^2(2)^n}{24} - 3x2^n - \frac{3x^2n(2)^n}{4} + \cdots$$

B1

No x term:

$$\frac{n(2)^n}{3} - 3(2)^n = 0 \qquad \text{M1}$$

$$n = 9 \qquad \text{A1}$$

(c) Find the term independent of x in the expansion of $\left(x-\frac{1}{2x^2}\right)^9$. [3]

$$\boldsymbol{T_{r+1} = \binom{9}{r}(x)^{9-r}\left(-\frac{1}{2x^2}\right)^r}$$

Term independent of $\boldsymbol{x}$:

$$\boldsymbol{9-r-2r=0} \qquad \text{M1}$$

$$\boldsymbol{r=3} \qquad \text{A1}$$

Term independent of $\boldsymbol{x}$

$$= \boldsymbol{\binom{9}{3}(x)^{9-3}\left(-\frac{1}{2x^2}\right)^3}$$

$$= \boldsymbol{-\frac{21}{2}} \qquad \text{B1}$$

3 Radiocarbon dating, or carbon-14 dating, is a scientific method that can accurately determine the age of organic materials as old as approximately 60,000 years. The technique is based on the decay of the carbon-14 isotope. The time in which half of the original number of atom decay is defined as the half-life. It is modelled by the equation $N = N_0 e^{-kt}$, where k is a constant and t is the time in years. N_0 is the initial amount of material in an object.

Carbon-14 has a half life of 5730 years, meaning that 5730 years after an organism dies, half of its carbon-14 atoms have decayed.

(a) Find the value of k. [2]

$\frac{1}{2} = e^{-k(5730)}$ M1

$k = \frac{\ln\left(\frac{1}{2}\right)}{-5730}$ A1

$= 0.0001209681$

$= 0.000121$

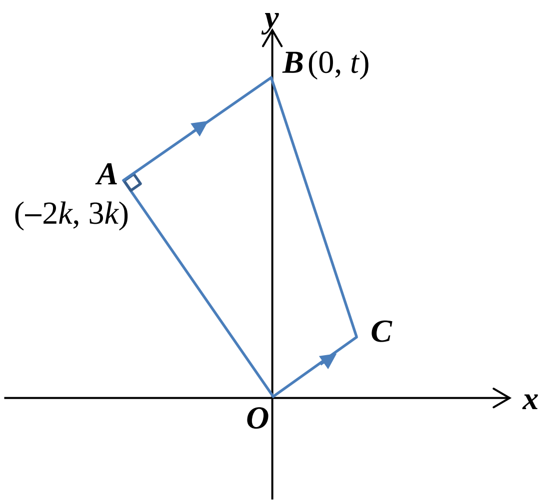
(b) A mummified body was found to have 8% of its original atoms left. How many years ago did the person die? Leave your answer to the nearest whole number. [2]

$\frac{8}{100} = e^{-0.0001209681t}$ M1

$t = \frac{\ln\left(\frac{8}{100}\right)}{-0.0001209681}$

$= 20879$ years A1

4 Solutions to this question by accurate drawing will not be accepted.



OABC is a trapezium with right angle *OAB*. The coordinates of *A* and *B* are $(-2k, 3k)$ and $(0, t)$ respectively, where $k > 0$, and $OA = \sqrt{117}$ units. Find

(a) the value of k and show that A is $(-6, 9)$, [2]

length $OA = \sqrt{4k^2 + 9k^2}$ M1

$117 = 13k^2$

$k = 3 \quad k = -3$ (rej) A1

Hence, A is $(-2 \times 3\,, 3 \times 3) = (-6, 9)$

(b) the equation of AB, [3]\

Gradient of $OA = \frac{9}{-6}$

$= -\frac{3}{2}$ M1

Gradient of $AB = \frac{2}{3}$ M1

Equation of AB:

$y - 9 = \frac{2}{3}(x + 6)$

$y = \frac{2}{3}x + 13$ A1

(c) the coordinates of C if $OC = \frac{1}{2}AB$, [1]

Coordinates of $B = (0, 13)$

Coordinates of $C = \left(\frac{6}{2}, \frac{4}{2}\right)$

$= (3, 2)$

(d) the area of $OABC$. [2]

Area $= \frac{1}{2}\begin{vmatrix} 0 & 3 & 0 & -6 & 0 \\ 0 & 2 & 13 & 9 & 0 \end{vmatrix}$ M1

$= \frac{1}{2}|(39) - (-78)|$

$= \frac{117}{2}$ units^2 A1

Triangle OAB lies in a circle C.

(e) Explain why OB is the diameter of the circle. [1]

By property of angles in a semicircle, since angle $OAB = 90°$,

OB is the diameter of the circle C_1.

(f) Find the equation of the circle C. [3]

Radius= 6.5 B1

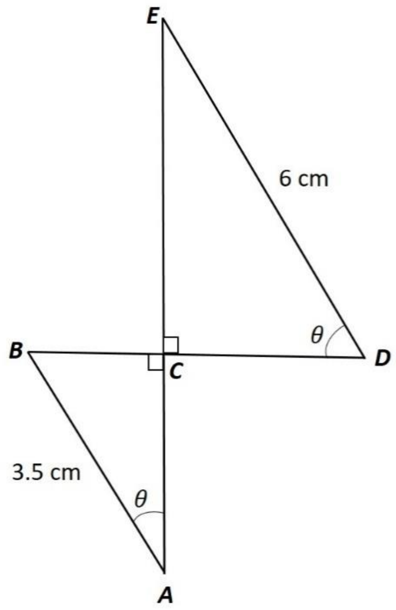
Center of circle $= \left(0, \frac{13}{2}\right)$ B1

Equation of C_1:

$(x - 0)^2 + \left(y - \frac{13}{2}\right)^2 = \left(\frac{13}{2}\right)^2$

$x^2 + \left(y - \frac{11}{2}\right)^2 = \frac{169}{4}$ B1

5 The diagram shows two right-angled triangles, *ABC* and *DEC*. $\angle BAC = \angle CDE = \theta$, *AB* = 3.5 cm and *DE* = 6 cm.



(a) Show that *BD* = $6\cos\theta + 3.5\sin\theta$. [2]

$\sin\theta = \frac{BC}{3.5}$ $\cos\theta = \frac{CD}{6}$

$BC = 3.5\sin\theta$ B1 $CD = 6\cos\theta$ B1

$BD = BC + CD$

$= 6\cos\theta + 3.5\sin\theta$ (shown)

(b) Express *BD* in the form $R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [3]

$R = \sqrt{3.5^2 + 36}$

$= \frac{\sqrt{193}}{2}$ M1

$\alpha = \tan^{-1}\left(\frac{3.5}{6}\right)$

$= 30.25643$

$= 30.3°$ M1

$BD = \frac{\sqrt{193}}{2}\cos(\theta - 30.3°)$ A1

(c) State the maximum length of *BD* and the corresponding value of θ. [2]

$Max\ BD = \frac{\sqrt{193}}{2}$

$= 6.94622$

$= 6.95$ cm B1

Occurs when $\cos(\theta - 30.256°) = 1$

$(\theta - 30.256°) = 0°$

$\theta = 30.3°$ B1

(d) Find the value of θ when $BD = 6$ cm. [2]

$6 = \frac{\sqrt{193}}{2}\cos(\theta - 30.3°)$

$\cos(\theta - 30.3°) = 0.8637789$ M1

$\theta - 30.25643 = 30.25643$

$\theta = 60.5°$ A1

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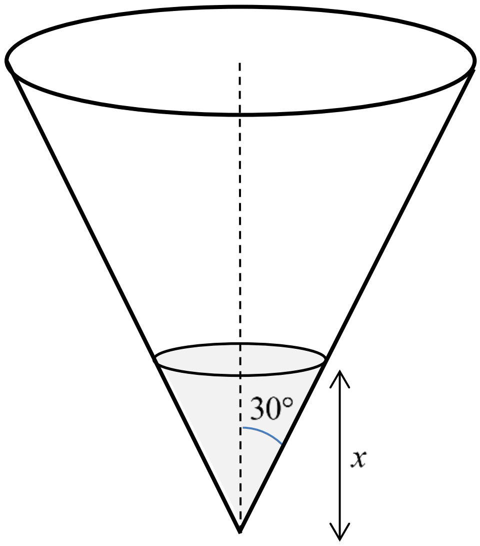
6 A conical vessel, whose vertical angle is $30°$ as shown in the diagram, has water poured into it at such a rate that after t seconds, the depth of the water is x cm and the radius of the water surface is r cm.



(a) Show that the volume of water, $V = \frac{\pi x^3}{9}$ cm^3. [3]

$\tan 30° = \frac{r}{x}$

$r = \frac{\sqrt{3}}{3}x$ B1

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi\left(\frac{\sqrt{3}}{3}x\right)^2 x$ M1

$= \frac{\pi x^3}{9}$ cm^3 A1

(b) Given further that the volume of water, $V = 8t$, find the rate of which the water level is increasing after 3 seconds. [5]

When $t = 3$,

$V = 24$

$24 = \frac{\pi x^3}{9}$

$x = \frac{6}{\sqrt[3]{\pi}}$ M1

$\frac{dV}{dt} = 8$ M1

$\frac{dV}{dx} = \frac{\pi x^2}{3}$ M1

$\frac{dV}{dx} \times \frac{dx}{dt} = \frac{dV}{dt}$ M1

Sub $x = \frac{6}{\sqrt[3]{\pi}}$,

$\frac{dx}{dt} = \frac{24}{\pi x^2}$

$= 0.455$ cm/s A1

7 Show that $2^{n+5} + 2^{n+3} + 81(2^n)$ is a multiple of 11. [3]

$2^{n+5} + 2^{n+3} + 81(2^n)$

$= 2^5(2^n) + 2^3(2^n) + 81(2^n)$ M1

$= (2^n)(32 + 8 + 81)$

$= (2^n)(121)$ M1

$= (11^2)(2^n)$ A1

8. Evaluate $\int_0^{\frac{\pi}{3}} \left(\frac{1}{2}\sin 3x - \cos^2 x\right)\, dx$, correct to 2 decimal places. [4]

$\int_0^{\frac{\pi}{3}} \left(\frac{1}{2}\sin 3x - \cos^2 x\right)\, dx$

$= \int_0^{\frac{\pi}{3}} \left(\frac{1}{2}\sin 3x - \left(\frac{\cos 2x + 1}{2}\right)\right)\, dx$ M1

$= \left[-\frac{\cos 3x}{6} - \frac{\sin 2x}{4} - \frac{x}{2}\right]_0^{\frac{\pi}{3}}$ M2

$= \frac{1}{6} - \frac{\sqrt{3}}{8} - \frac{\pi}{6} - \left(-\frac{1}{6}\right)$

$= -0.41$ A1

9. A particle moves in a straight line and its initial velocity is 21 cm/s. After t seconds, its acceleration, a cm/s^2, is given by $a = 2t - 10$. When $t = 0$, its displacement, s cm, from a fixed point O is 4 cm.

Find the distance travelled by the particle during the first 7 seconds. [8]

$v = \int a\, dt$

$v = t^2 - 10t + c$ M1

When $t = 0, v = 21$

$c = 21$

$v = t^2 - 10t + 21$ M1

$s = \int v\, dt$

$s = \frac{t^3}{3} - 5t^2 + 21t + c_1$ M1

When $t = 0, s = 4$

$c_1 = 4$

$s = \frac{t^3}{3} - 5t^2 + 21t + 4$ M1

Turning point

$v = 0$

$t^2 - 10t + 21 = 0$ M1

$t = 3\ s \qquad t = 7\ s$ A1

When $t = 3\ s$

$s = 9 - 45 + 63 + 4$

$s = 31\ cm$

When $t = 7\ s$

$s = \frac{343}{3} - 245 + 147 + 4$

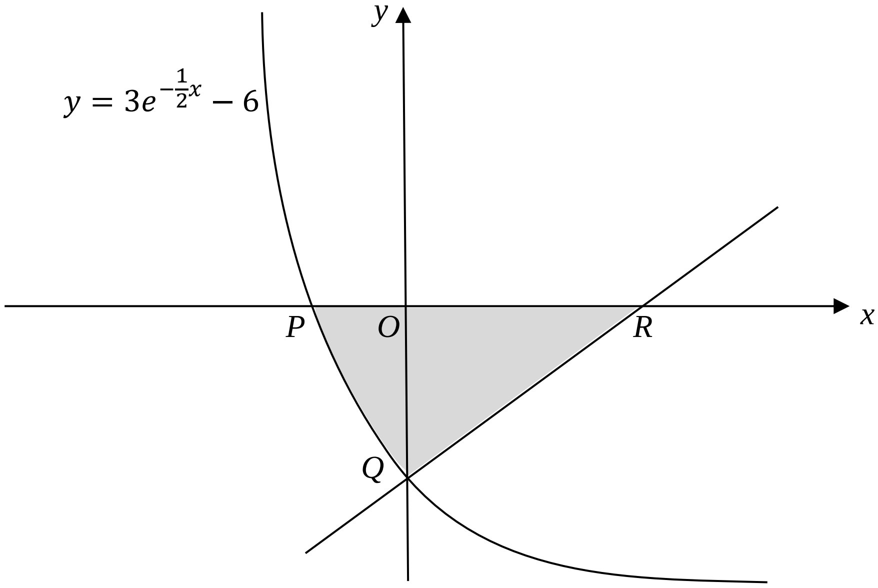
$s = \frac{61}{3}\ cm$

Distance travelled in the first 7 seconds

$= (31 - 4) + \left(31 - \frac{61}{3}\right)$ M1

$= \frac{113}{3}\ cm$ A1

10.



The diagram shows part of the curve $y = 3e^{-\frac{1}{2}x} - 6$ which crosses the x-axis at P and the y-axis at Q. The normal to the curve at Q meets the x-axis at R. Find

(a) the equation of the normal, [4]

When $x = 0$,

$y = -3$ M1

$\frac{dy}{dx} = -\frac{3}{2}e^{-\frac{1}{2}x}$

$\frac{dy}{dx} = -\frac{3}{2}$ M1

Gradient of normal $= \frac{2}{3}$ A1

Equation of normal:

$y + 3 = \frac{2}{3}(x - 0)$

$y = \frac{2}{3}x - 3$ A1

(b) the area of the shaded region. [6]

when $y = 0$,

$3e^{-\frac{1}{2}x} - 6 = 0$

$e^{-\frac{1}{2}x} = 2$

$-\frac{1}{2}x = \ln 2$

$x = -\ln 4$

$P = (-\ln 4\ , 0)$ M1

$\frac{2}{3}x - 3 = 0$

$x = \frac{9}{2}$

$R = \left(\frac{9}{2}, 0\right)$ M1

$Q = (0, -3)$ M1

Area of shaded region

$= \left|\int_{-\ln 4}^{0} \left(3e^{-\frac{1}{2}x} - 6\right)\ dx\right| + \frac{1}{2}\left(\frac{9}{2}\right)(3)$ M1

$= \left|\left[\frac{3e^{-\frac{1}{2}x}}{-\frac{1}{2}} - 6x\right]_{-\ln 4}^{0}\right| + \frac{27}{4}$

$= \left|\left[-6e^{-\frac{1}{2}x} - 6x\right]_{-\ln 4}^{0}\right| + \frac{27}{4}$ M1

$= |[-6 + 12 - 6\ln 4]| + \frac{27}{4}$

$= 9.07$ A1

11. The function f is defined, for $0 \leq x \leq 2\pi$, by $f(x) = 1 - 2\cos 2x$.

(a) State the period and amplitude of f. [2]

Period $= \pi$

Amplitude $= 2$

(b) Sketch the graph of $y = f(x)$ for $0 \leq x \leq 2\pi$. [3]

(c) On the same diagram in part **(b)**, sketch the graph of $y = \frac{4x}{\pi} - 2$. [1]

(d) Hence, state the number of solutions, $0 \leq x \leq 2\pi$, to the equation

$2\cos 2x - 3 + \frac{4x}{\pi} = 0$. [1]

12. The gradient function of a curve is given by $\frac{dy}{dx} = -\frac{1}{x^2} + 6\cos 3x$. The coordinates of the point P, which lies on the curve, is $\left(\frac{\pi}{2}, \pi\right)$. A point Q exists such that the mid-point of PQ is $\left(\frac{3}{4}\pi, 2\pi\right)$. Find

(a) the coordinates of the point Q, [3]

$\left(\frac{3}{4}\pi, 2\pi\right) = \left(\frac{\frac{\pi}{2}+x}{2}, \frac{\pi+y}{2}\right)$ M1

$\frac{3}{4}\pi = \frac{\frac{\pi}{2}+x}{2}$ $\quad$ $2\pi = \frac{\pi+y}{2}$

$\frac{3\pi}{2} = \frac{\pi}{2} + x$ $\quad$ $y = 3\pi$

$x = \pi$ A1

Coordiantes of $Q = (\pi, 3\pi)$ A1

(b) the equation of the curve. [2]

$y = \int \left(-\frac{1}{x^2} + 6\cos 3x\right) dx$

$\mathrm{y} = \frac{1}{x} + 2\sin 3x + c$ M1

When $x = \frac{\pi}{2}, y = \pi$

$\pi = \pi - 2 + c$

$c = 2$

Equation of curve:

$y = \frac{1}{x} + 2\sin 3x + 2$ A1

13. Without using the calculator, evaluate the following trigonometric functions.

(a) $\sin\left(x + \frac{4\pi}{3}\right)$ [2]

$= \sin x \cos\frac{4\pi}{3} + \cos x \sin\frac{4\pi}{3}$ M1

$= -\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x$ A1

(b) $\cos 75°$ [2]

$= \cos(30° + 45°)$

$= \cos 30° \cos 45° - \sin 30° \sin 45°$

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} - \frac{1}{2} \times \frac{\sqrt{2}}{2}$ M1

$= \frac{\sqrt{2}(\sqrt{3}-1)}{4}$ A1

(c) $\tan(\theta - 45°)$ [2]

$= \frac{\tan\theta - \tan 45°}{1+\tan\theta\tan 45°}$ M1

$= \frac{\tan\theta - 1}{1+\tan\theta}$ A1