Raffles Institution
H2 Mathematics
Solution for 2013 A-Level Paper 1

Question 1

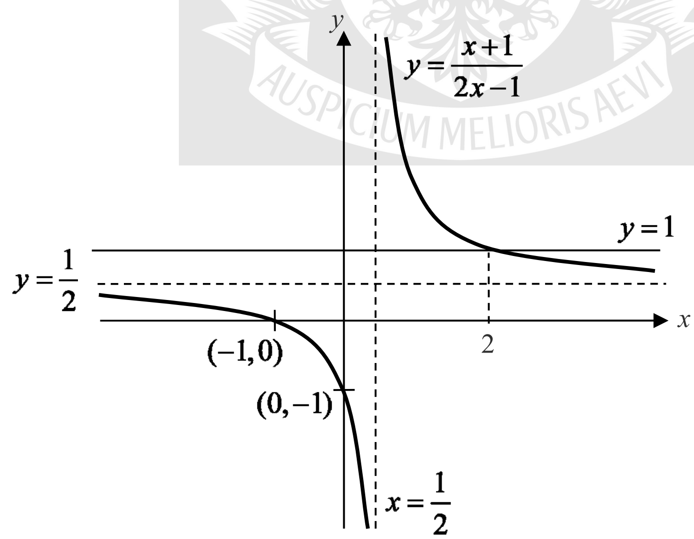
No.	Suggested Solution	Remarks for Student
(i)	When $\mu = 3,$ we have $$x - 2z = 4,$$ $$2x - 2y + z = 6,$$ $$5x - 4y + 3z = -9.$$ From GC, the point of intersection is $\left(-\frac{38}{3}, -\frac{119}{6}, -\frac{25}{3}\right)$.	
(ii)	When $\mu = 0,$ we have $$x - 2z = 4,$$ $$2x - 2y + z = 6,$$ $$5x - 4y = -9.$$ From GC, p, q and r have no point of intersection. Since no two of the planes are parallel, they form a triangular prism.	It's 3 marks, so obviously they are looking for more than "no solution". The normal can be read off the Cartesian form of the equation. Eg. $x - 2z = 4$ has normal $\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$. Hence it is easy to check that no two of the planes are parallel.

Question 2

No.	Suggested Solution	Remarks for Student
2	We have $$y = \frac{x^2 + x + 1}{x - 1} \quad \Rightarrow \quad xy - y = x^2 + x + 1$$ $$\Rightarrow \quad x^2 + (1 - y)x + (y + 1) = 0$$ If y is a value the expression can take, the final quadratic equation has a real solution for x. Therefore $$(1 - y)^2 - 4(1)(y + 1) \geq 0$$ $$\Rightarrow \quad y^2 - 6y - 3 \geq 0$$ $$\Rightarrow \quad y \leq 3 - 2\sqrt{3} \quad \text{or} \quad y \geq 3 + 2\sqrt{3}$$ Set of values y can take is $(-\infty, 3 - 2\sqrt{3}] \cup [3 + 2\sqrt{3}, \infty)$.	While this is the fastest method, it can sometimes be hard to see how to use the discriminant in such questions. If you have difficulty, you may want to try the alternative method.

	Alternative Method: We look for the range of the function $\mathrm{f}(x)=\frac{x^2+x+1}{x-1}=x+2+\frac{3}{x-1}$ by finding its maximum and minimum values. $\mathrm{f}'(x)=1-\frac{3}{(x-1)^2}=0$ when $x=1-\sqrt{3}$ or $x=1+\sqrt{3}$. These give the stationary points $(1-\sqrt{3},\ 3-2\sqrt{3})$ and $(1+\sqrt{3},\ 3+2\sqrt{3})$. $\mathrm{f}''(x)=\frac{6}{(x-1)^3}$, so $\mathrm{f}''(1-\sqrt{3})=-\frac{1}{2\sqrt{3}}<0$ and $\mathrm{f}''(1+\sqrt{3})=\frac{1}{2\sqrt{3}}>0$. Thus $(1-\sqrt{3},\ 3-2\sqrt{3})$ is a maximum point and $(1+\sqrt{3},\ 3+2\sqrt{3})$ is a minimum point. Therefore the set of values $y=\mathrm{f}(x)$ can take is $(-\infty,3-2\sqrt{3}]\cup[3+2\sqrt{3},\infty)$.	From here, it is best to use GC to check how the graph of $y=\mathrm{f}(x)$ looks to figure out the range.

Question 3

No.	Suggested Solution	Remarks for Student
(i)	$y=\frac{x+1}{2x-1}=\frac{1}{2}+\frac{3}{2(2x-1)}$ 	Simplifying the equation first will make it easier to find the asymptotes. Remember to follow the instructions of the question. It asks for the ***equations*** of asymptotes and ***coordinates*** of intercepts.

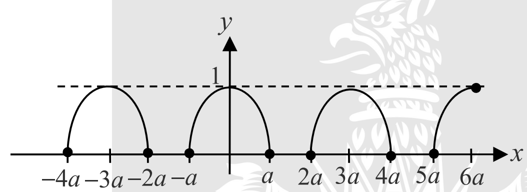
(ii)	$\frac{x+1}{2x-1}=1$ when $x=2$. From the graph in **(i)**, $\frac{x+1}{2x-1}<1$ when $x<\frac{1}{2}$ or $x>2$.	Don't waste time solving algebraically. Use the graph.

Question 4

No.	Suggested Solution	Remarks for Student
(i)	$w^3=(1+2\mathrm{i})^3=1+3(2\mathrm{i})+3(2\mathrm{i})^2+(2\mathrm{i})^3=-11-2\mathrm{i}$ **Alternative Method:** $w=1+2\mathrm{i}$ $w^2=(1+2\mathrm{i})^2=-3+4\mathrm{i}$ $w^3=(1+2\mathrm{i})(-3+4\mathrm{i})=-11-2\mathrm{i}$	Binomial Theorem. Usually this more tedious method is to be avoided. But in this case w^2 is need later, so there is no loss in time.
(ii)	Since $w=1+2\mathrm{i}$ is a root, $a(1+2\mathrm{i})^3+5(1+2\mathrm{i})^2+17(1+2\mathrm{i})+b=0$ $\Rightarrow\ a(-11-2\mathrm{i})+5(-3+4\mathrm{i})+17(1+2\mathrm{i})+b=0$ $\Rightarrow\ (-11a+b+2)+(-2a+54)\mathrm{i}=0$ Comparing imaginary and real parts, we have $a=27$ and $b=11a-2=295.$	It is possible to do **(ii)** and **(iii)** together. See below.
(iii)	Since a and b are real and $w=1+2\mathrm{i}$ is a root, $w^*=1-2\mathrm{i}$ is also a root. Therefore $27z^3+5z^2+17z+295=[z-(1+2\mathrm{i})][z-(1-2\mathrm{i})](cz+d)$ $=(z^2-2z+5)(cz+d),$ so clearly $c=27$ and $d=\frac{295}{5}=59.$ Hence the roots of this equation are $1+2\mathrm{i}$, $1-2\mathrm{i}$ and $-\frac{59}{27}.$ **Alternative Method:** Since a and b are real and $w=1+2\mathrm{i}$ is a root, $w^*=1-2\mathrm{i}$ is also a root. Therefore $az^3+5z^2+17z+b=[z-(1+2\mathrm{i})][z-(1-2\mathrm{i})](cz+d)$ $=(z^2-2z+5)(cz+d)$ $=cz^3+(-2c+d)z^2+(5c-2d)z+5d.$	Remember the trick to evaluate $(z-w)(z-w^*)$. This method deals with both **(ii)** and **(iii)** together, so we do not use the values of a and b from **(ii)**.

	Comparing coefficients of z^2 and z gives $$-2c+d=5$$ $$5c-2d=17.$$ Thus $c=27,\ d=59,$ and $a=c=27,\ b=5d=295.$ The roots of this equation are $1+2\text{i}, 1-2\text{i}$ and $-\dfrac{59}{27}.$	

Question 5

No.	Suggested Solution	Remarks for Student
(i)	Graph of $y=\text{f}(x)$ for $-4a \le x \le 6a$:  	We can assume $a>0$ here as we are given $-a \le x \le a$. Sketch the graph for $-a \le x \le 2a$ first, then use the condition $\text{f}(x+3a)=\text{f}(x)$, which says that f is periodic with period $3a$, to fill in the rest.
(ii)	Since $-a < \frac{1}{2}a < \frac{\sqrt{3}}{2}a < a$, $$\int_{\frac{1}{2}a}^{\frac{\sqrt{3}}{2}a} \text{f}(x)\ \text{d}x = \int_{\frac{1}{2}a}^{\frac{\sqrt{3}}{2}a} \sqrt{\left(1-\frac{x^2}{a^2}\right)}\,\text{d}x.$$ Using the substitution $x=a\sin\theta$, we have $$\sqrt{\left(1-\frac{x^2}{a^2}\right)}=\sqrt{1-\sin^2\theta}=\cos\theta \text{ and } \frac{\text{d}x}{\text{d}\theta}=a\cos\theta.$$ Also, $x=\dfrac{1}{2}a \Rightarrow \theta=\dfrac{\pi}{6}$ and $x=\dfrac{\sqrt{3}}{2}a \Rightarrow \theta=\dfrac{\pi}{3}.$ Therefore $$\int_{\frac{1}{2}a}^{\frac{\sqrt{3}}{2}a} \sqrt{\left(1-\frac{x^2}{a^2}\right)}\,\text{d}x = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos\theta(a\cos\theta)\ \text{d}\theta$$ $$=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} a\cos^2\theta\ \text{d}\theta$$ $$=\frac{a}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left(1+\cos 2\theta\right)\ \text{d}\theta$$ $$=\frac{a}{2}\left[\theta+\frac{\sin 2\theta}{2}\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}=\frac{\pi a}{12}.$$	For substitutions, change x, dx and the limits.

Question 6

No.	Suggested Solution	Remarks for Student
(i)	Since **a** and **b** are not parallel, a vector equation for the plane *OAB* is $\mathbf{r} = \lambda\mathbf{a} + \mu\mathbf{b},\ \lambda, \mu \in \mathbb{R}$. As *C* lies on this plane, we have $\mathbf{c} = \lambda\mathbf{a} + \mu\mathbf{b}$ for some real constants λ and μ.	
(ii)	$\overrightarrow{ON} = \frac{1}{7}(4\mathbf{a} + 3\mathbf{c})$	Ratio Theorem
(iii)	The area of triangle *ONC* is $$\frac{1}{2}\left\|\overrightarrow{ON} \times \overrightarrow{OC}\right\| = \frac{1}{2}\left\|\frac{1}{7}(4\mathbf{a} + 3\mathbf{c}) \times \mathbf{c}\right\| = \frac{2}{7}\left\|\mathbf{a} \times \mathbf{c}\right\| = \frac{2}{7}\left\|\mathbf{a} \times (\lambda\mathbf{a} + \mu\mathbf{b})\right\| = \frac{2\mu}{7}\left\|\mathbf{a} \times \mathbf{b}\right\|.$$ Similarly, the area of triangle *OMC* is $$\frac{1}{2}\left\|\overrightarrow{OM} \times \overrightarrow{OC}\right\| = \frac{1}{2}\left\|\frac{1}{2}\mathbf{b} \times \mathbf{c}\right\| = \frac{1}{4}\left\|\mathbf{b} \times (\lambda\mathbf{a} + \mu\mathbf{b})\right\| = \frac{\lambda}{4}\left\|\mathbf{b} \times \mathbf{a}\right\|.$$ Since $\left\|\mathbf{b} \times \mathbf{a}\right\| = \left\|\mathbf{a} \times \mathbf{b}\right\|$, we have $$\frac{2\mu}{7} = \frac{\lambda}{4} \quad \Rightarrow \quad \lambda = \frac{8\mu}{7}.$$	Note the use of properties of the cross product, in particular $\mathbf{a} \times \mathbf{a} = \mathbf{0}$. Note that μ and λ below can be taken out of the magnitude because we are told both are positive.

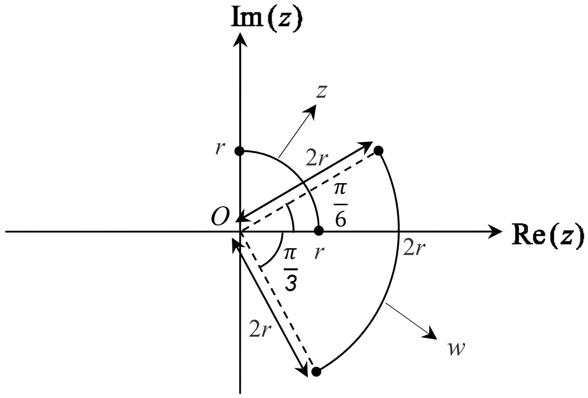
Question 7

No.	Suggested Solution	Remarks for Student
(i)	The length, a_k, of the *k*th piece of string cut off forms a geometric progression with first term 128 and common ratio $\frac{2}{3}$, so $p = a_n = 128\left(\frac{2}{3}\right)^{n-1}$. Hence $$\ln p = \ln\left[128\left(\frac{2}{3}\right)^{n-1}\right] = \ln 128 + (n-1)(\ln 2 - \ln 3) = \ln 2^7 + (n-1)(\ln 2 - \ln 3) = (n+6)\ln 2 + (-n+1)\ln 3,$$ giving $A = 1$, $B = 6$, $C = -1$ and $D = 1$.	

(ii)	The total length of string cut off cannot exceed $$\sum_{k=1}^{\infty} a_k = \frac{128}{1-\frac{2}{3}} = 384.$$	
(iii)	Let S_n denote the total length of the first n pieces of string cut off. Then $$S_n = \frac{128\left[1-\left(\frac{2}{3}\right)^n\right]}{1-\frac{2}{3}} = 384\left[1-\left(\frac{2}{3}\right)^n\right].$$ When this exceeds 380 we have $$384\left[1-\left(\frac{2}{3}\right)^n\right] > 380 \Rightarrow 1-\left(\frac{2}{3}\right)^n > \frac{95}{96}$$ $$\Rightarrow \left(\frac{2}{3}\right)^n < \frac{1}{96}$$ $$\Rightarrow n > \frac{\ln\frac{1}{96}}{\ln\frac{2}{3}} = 11.3.$$ Therefore 12 pieces must be cut off before the total length is greater than 380 cm.	The question asks for working to justify your answer, so you cannot just read the answer from the GC. In particular, this means no "Table of Values".

Question 8

No.	Suggested Solution	Remarks for Student
(i)	We have $1-\mathrm{i}\sqrt{3} = 2\mathrm{e}^{-\frac{\pi}{3}\mathrm{i}}$, so $$w = (1-\mathrm{i}\sqrt{3})z$$ $$= \left(2e^{-\frac{\pi}{3}\mathrm{i}}\right)re^{\mathrm{i}\theta} = 2re^{\mathrm{i}\left(\theta-\frac{\pi}{3}\right)}.$$ Therefore $\lvert w\rvert = 2r$ and $\arg w = \theta - \frac{\pi}{3}$. **Alternative Method:** $$\lvert w\rvert = \lvert(1-\mathrm{i}\sqrt{3})z\rvert = \lvert 1-\mathrm{i}\sqrt{3}\rvert\lvert z\rvert = 2r$$ $$\arg w = \arg\left((1-\mathrm{i}\sqrt{3})z\right) = \arg(1-\mathrm{i}\sqrt{3}) + \arg z = \theta - \frac{\pi}{3}$$	Be sure to consider the quadrant when finding arguments. Both presentations are fine, but the first is more convenient if you need to add or subtract 2π to the argument.

(ii)		Recall that the effect of multiplying a complex number z by $se^{i\phi}$ is to scale it by a factor of s and rotate it counter-clockwise about the origin by angle ϕ. Thus the locus of w is obtained from that of z by scaling it by a factor of 2 and rotating it clockwise by $\frac{\pi}{3}$.
(iii)	$\arg\left(\frac{z^{10}}{w^2}\right) = 10\arg z - 2\arg w$ $= 10\theta - 2\left(\theta - \frac{\pi}{3}\right) = 8\theta + \frac{2\pi}{3}$ Therefore $8\theta + \frac{2\pi}{3} = \pi$, giving $\theta = \frac{\pi}{24}$. **More complete solution:** Arguments of complex numbers can be changed by multiples of 2π without changing the number, so in fact $\arg\left(\frac{z^{10}}{w^2}\right) = 8\theta + \frac{2\pi}{3} + 2k\pi = \pi$ For $k \in \mathbb{Z}$. Therefore $\theta = \frac{\pi}{24} - \frac{k\pi}{4}$, and for $0 \le \theta \le \frac{\pi}{2}$, there are two answers, $\theta = \frac{\pi}{24}$ or $\theta = \frac{7\pi}{24}$, corresponding to $k = 0$ or $k = -1$.	This is the intended solution for this question, but if you are interested, there is a more complete discussion below.

Question 9

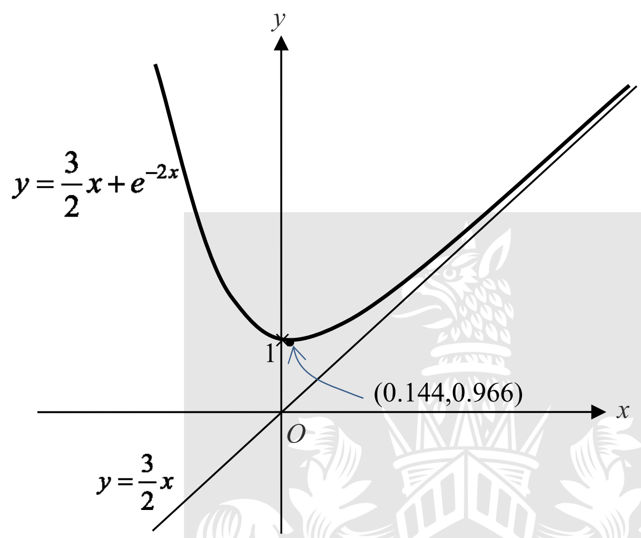
No.	Suggested Solution	Remarks for Student
(i)	Let P_n be the statement $\sum_{r=1}^{n} r(2r^2+1) = \frac{1}{2}n(n+1)(n^2+n+1),$ for $n \in \mathbb{Z}^+$. When $n = 1$, $\text{LHS} = 1(2(2)^2+1) = 3$ $\text{RHS} = \frac{1}{2}(1)(1+1)(1^2+1+1) = 3,$ so P_1 is true.	This is a straightforward, if somewhat tedious, induction question. Just keep your cool in the manipulations and you should be fine.

	Now assume that P_k is true for some $k \in \mathbb{Z}^+$, i.e. $\sum_{r=1}^{k} r(2r^2+1) = \frac{1}{2}k(k+1)(k^2+k+1).$ We need to show that P_{k+1} is true, i.e. $\sum_{r=1}^{k+1} r(2r^2+1) = \frac{1}{2}(k+1)[(k+1)+1][(k+1)^2+(k+1)+1)]$ $= \frac{1}{2}(k+1)(k+2)(k^2+3k+3).$ We have $\sum_{r=1}^{k+1} r(2r^2+1) = \sum_{r=1}^{k} r(2r^2+1) + (k+1)[2(k+1)^2+1]$ $= \frac{1}{2}k(k+1)(k^2+k+1) + (k+1)(2k^2+4k+3)$ $= \frac{1}{2}(k+1)[k(k^2+k+1) + 2(2k^2+4k+3)]$ $= \frac{1}{2}(k+1)(k^3+5k^2+9k+6)$ $= \frac{1}{2}(k+1)(k+2)(k^2+3k+3).$ Thus P_k is true $\Rightarrow P_{k+1}$ is true. Since P_1 is also true, P_n is true for all $n \in \mathbb{Z}^+$ by Mathematical Induction.	At this step, you **know** that the cubic must have a factor of $(k+2)$ as that is the answer you need! So just check it.
(ii)	We have $\mathrm{f}(r-1) = 2(r-1)^3 + 3(r-1)^2 + (r-1) + 24$ $= 2(r^3-3r^2+3r-1) + 3(r^2-2r+1) + r + 23$ $= 2r^3 - 3r^2 + r + 24,$ so $\mathrm{f}(r) - \mathrm{f}(r-1) = 6r^2$. Therefore $a = 6$. We have $6\sum_{r=1}^{n} r^2 = \sum_{r=1}^{n} [\mathrm{f}(r) - \mathrm{f}(r-1)]$ $= \{\ \mathrm{f}(1) - \mathrm{f}(0)$ $+\ \mathrm{f}(2) - \mathrm{f}(1)$ $+\ \mathrm{f}(3) - \mathrm{f}(2)$ $\vdots$ $+\ \mathrm{f}(n-2) - \mathrm{f}(n-3)$ $+\ \mathrm{f}(n-1) - \mathrm{f}(n-2)$ $+\ \mathrm{f}(n) - \mathrm{f}(n-1)\}$ $= \mathrm{f}(n) - \mathrm{f}(0).$	Students should recognize that this is clearly the set up for Method of Differences.

	Therefore $$\sum_{r=1}^{n} r^2 = \frac{1}{6}\left[\mathrm{f}(n) - \mathrm{f}(0)\right]$$ $$= \frac{1}{6}\left[(2n^3 + 3n^2 + n + 24) - 24\right]$$ $$= \frac{1}{6} n(n+1)(2n+1).$$	
(iii)	We have $\mathrm{f}(r) = r(2r^2+1) + 3r^2 + 24,$ so $$\sum_{r=1}^{n} \mathrm{f}(r) = \sum_{r=1}^{n} r(2r^2+1) + 3\sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} 24$$ $$= \frac{n(n+1)(n^2+n+1)}{2} + \frac{n(n+1)(2n+1)}{2} + 24n.$$	The key is to observe the connection with parts **(i)** and **(ii)**. Then just put the terms together. Be careful with $\sum_{r=1}^{n} 24$.

Question 10

No.	Suggested Solution	Remarks for Student
(i)	$$\frac{\mathrm{d}z}{\mathrm{d}x} = 3 - 2z \quad \Rightarrow \int \frac{1}{3-2z}\,\mathrm{d}z = \int 1\,\mathrm{d}x$$ $$\Rightarrow -\frac{1}{2}\ln(3-2z) = x + C,$$ where C is an arbitrary constant. Thus $z = \frac{3}{2} - A\mathrm{e}^{-2x},$ where $A = \frac{\mathrm{e}^{-2C}}{2}.$	Note that you are given $z < \frac{3}{2},$ so $3 - 2z > 0$ and there is no need for modulus in ln.
(ii)	Using (B), $\frac{\mathrm{d}y}{\mathrm{d}x} = z = \frac{3}{2} - A\mathrm{e}^{-2x} \quad \Rightarrow \quad y = \frac{3}{2}x + \frac{A}{2}\mathrm{e}^{-2x} + B,$ where B is another arbitrary constant.	
(iii)	$$y = \frac{3}{2}x + \frac{A}{2}e^{-2x} + B \quad \Rightarrow \quad \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{2} - A\mathrm{e}^{-2x}$$ $$\Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2A\mathrm{e}^{-2x}$$ $$= 2\left(\frac{3}{2} - \frac{\mathrm{d}y}{\mathrm{d}x}\right)$$ $$= -2\frac{\mathrm{d}y}{\mathrm{d}x} + 3$$ Therefore $a = -2$ and $b = 3.$	The question explicitly asked you to use the result for **(ii)**, so this is the solution. Otherwise it is trivial: What you need to proof is $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = a\frac{\mathrm{d}y}{\mathrm{d}x} + b.$ (B) allows us to write it as $\frac{\mathrm{d}}{\mathrm{d}x}(z) = az + b,$ which is (A).

(iv)	To get straight lines, we need $A = 0$, so two possible lines are $y = \frac{3}{2}x$ and $y = \frac{3}{2}x + 1$. Using $y = \frac{3}{2}x$ as the asymptote, we can pick $y = \frac{3}{2}x + e^{-2x}$ as a non-linear member. 	For this part, you need to ensure that *B* is the same for both the line and the non-linear solution in order to have the line be the asymptote. You are free to choose whatever value of *A* you like. We picked $A = 2$ here.

Question 11

No.	Suggested Solution	Remarks for Student
(i)	We have $x = 3t^2$, $y = 2t^3$, so $$\frac{dx}{dt} = 6t \quad \text{and} \quad \frac{dy}{dt} = 6t^2$$ $$\Rightarrow \quad \frac{dy}{dx} = \frac{6t^2}{6t} = t.$$ Therefore the equation of the tangent at the point with parameter *t* is $$y - 2t^3 = t(x - 3t^2) \quad \Rightarrow \quad y = tx - t^3.$$	Remember the point with parameter *t* has coordinates $(3t^2, 2t^3)$ according to the parametric equations.
(ii)	The equations of the tangents at *P* and *Q* are $$y = px - p^3 \quad \text{and} \quad y = qx - q^3$$ respectively. When they intersect at *R*, $$px - p^3 = qx - q^3 \quad \Rightarrow \quad (p - q)x = p^3 - q^3$$ $$\Rightarrow \quad x = \frac{(p^3 - q^3)}{p - q} = p^2 + pq + q^2$$ as $p \neq q$ (otherwise *P* and *Q* are the same point).	The rest of this question can be rather intimidating, and it is meant to be. If you have difficulty, take heart that you won't be the only one. Most papers will have one or two such questions, so simply try to do as many parts as you can.

	The corresponding y-coordinate is $$y = p(p^2+pq+q^2)-p^3 = pq(p+q).$$ When $pq=-1$, we have $$x = p^2+q^2-1 \quad \text{and} \quad y=-(p+q),$$ so $$y^2 = (-1)^2(p+q)^2 = p^2+q^2+2pq = x+pq = x-1.$$ Therefore R lies on the curve with equation $x = y^2+1$.	To show that R lies on a particular curve, we need to check that its coordinates satisfy the equation of the curve.
(iii)	For points on C, $x=3t^2$ and $y=2t^3$. For points on L, $x=y^2+1$. Therefore, for the point M, $$3t^2=(2t^3)^2+1 \Rightarrow 4t^6-3t^2+1=0.$$ We solve for t^2: $$4(t^2)^3-3t^2+1=0 \Rightarrow (t^2+1)\left(4(t^2)^2-4t^2+1\right)=0 \Rightarrow (t^2+1)(2t^2-1)^2=0$$ Since $t^2+1>0$, we have $t^2=\frac{1}{2} \Rightarrow t=\pm\frac{1}{\sqrt{2}}$. When $t=\pm\frac{1}{\sqrt{2}}$, $y=2\left(\pm\frac{1}{\sqrt{2}}\right)^3=\pm\frac{1}{\sqrt{2}}$. At M, $y\geq 0$, so $t=\frac{1}{\sqrt{2}}$, $y=\frac{1}{\sqrt{2}}$ and $x=3\left(\frac{1}{\sqrt{2}}\right)^2=\frac{3}{2}$. Therefore the coordinates of M are $\left(\frac{3}{2},\frac{1}{\sqrt{2}}\right)$.	To find the point of intersection between a curve given in parametric form and one in Cartesian form, simply put the parametric equations into the Cartesian one. Recall the Factor and Remainder theorems which help with factorization of polynomials. Or simply cheat with GC!
(iv)	L intersects the x-axis at $(1,0)$, so required area is $$\int_0^{\frac{3}{2}} y_C\,\mathrm{d}x-\int_1^{\frac{3}{2}} y_L\,\mathrm{d}x=\int_0^{\frac{1}{\sqrt{2}}} 2t^3(6t)\,\mathrm{d}t-\int_1^{\frac{3}{2}}\sqrt{x-1}\,\mathrm{d}x =\int_0^{\frac{1}{\sqrt{2}}}12t^4\,\mathrm{d}t-\int_1^{\frac{3}{2}}(x-1)^{\frac{1}{2}}\,\mathrm{d}x =\frac{12}{5}\left[t^5\right]_0^{\frac{1}{\sqrt{2}}}-\frac{2}{3}\left[(x-1)^{\frac{3}{2}}\right]_1^{\frac{3}{2}} =\frac{12}{5}\frac{1}{4\sqrt{2}}-\frac{2}{3}\frac{1}{2\sqrt{2}}=\frac{2\sqrt{2}}{15}.$$	Integrating parametric equatioins is like doing substitution: change y, dx and the limits to t and the corresponding values.