Tutorial: Gravitational Field

Summary

1	**Gravitational field strength** g at a point is the gravitational force per unit mass exerted on a _______________ mass placed at that point. i.e. g = ______ Force acting on an object of mass m placed there, $F = mg$
2	**Newton's law of gravitation** states that the gravitational force between two point masses is directly proportional to the product of the masses and _______________ proportional to the _____________ of their separation. i.e. F = _____________
3	**Gravitational field strength g due to a point mass M** at a distance r from it is given by $g = -$____________ where $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2} (negative sign indicates that the field strength is directed towards M) Force acting on an object of mass m placed there, $F = -$_____________
4	**Gravitational potential** ϕ at a point is the work done per unit mass in bringing a ___________________ mass from ______________ to that point. i.e. $\phi = \dfrac{W}{m}$
5	**Gravitational potential** ϕ due to a point mass M at a distance r from it is given by $\phi = -$_____________
6	**Gravitational potential energy** U of an object of mass m placed at a point is given by $U = m\phi$
7	**Gravitational potential energy** U of an object of mass m placed at a point distance r from M is given by $U = -$_____________
8	**Satellite of mass m round the Earth (mass M_E) with orbit radius r :**

	Gravitational force acting on the satellite by the Earth, $F = \frac{GM_E m}{r^2}$ This gravitational force **provides** the centripetal force: $$\frac{GM_E m}{r^2} = mr\omega^2 = mr\left(\frac{2\pi}{T}\right)^2$$ → $T^2 = \left(\frac{4\pi^2}{GM_E}\right)r^3$ (Kepler's 3rd law)
9	**Energies of satellite of mass *m* in orbit of radius *r* round the Earth (mass M_E):** From $\frac{mv^2}{r} = \frac{GM_E m}{r^2}$ → KE, $K = \frac{mv^2}{2} = \frac{GM_E m}{2r}$ PE, $U = -\frac{GM_E m}{r}$ → TE, $E = K + U = \frac{GM_E m}{2r} + \left(-\frac{GM_E m}{r}\right) = -\frac{GM_E m}{2r}$ i.e. $E = \frac{U}{2}$
10	**Geostationary orbit**, for satellite to appear stationary when viewed from a fixed point on the rotating Earth, (a) has period T = Earth's period of rotation = ____________ (b) is in the plane of ______________ (c) moves from ________ to ________, same sense as self-rotation of the Earth
11	**Relation between gravitational force *F* and potential energy *U*** $$F = -\frac{dU}{dr}$$ Divide both sides by *m*, $g = -$ ______

Self-Attempt Questions

S1 The force of attraction between two masses 3 kg and 2 kg is *F* when the distance between them is *r*. Express the new force of attraction, in terms of *F*, if the distance between the two masses is doubled. (*F*/4) [2]

S2 The radius of the Earth may be taken as 5760 km. A satellite is in circular orbit 144 km above the Earth's surface. What is the approximate value of the gravitational force on the satellite compared with that when it is at the Earth's surface?

A greater by 10%

B greater by 5%

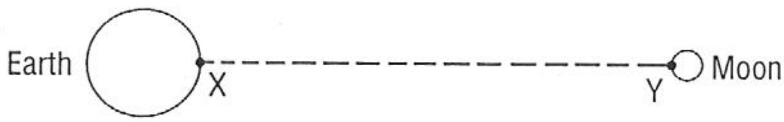
C less by 5%

D less by 10%

S3 If a body of mass *m* was released in a vacuum just above the surface of a planet of mass *M* and radius *R*, what would be its gravitational acceleration?

A $\frac{GmM}{R}$ **B** $\frac{Gm}{R}$ **C** $\frac{GM}{R^2}$ **D** $\frac{GmM}{R^2}$ **E** $\frac{GM}{R}$

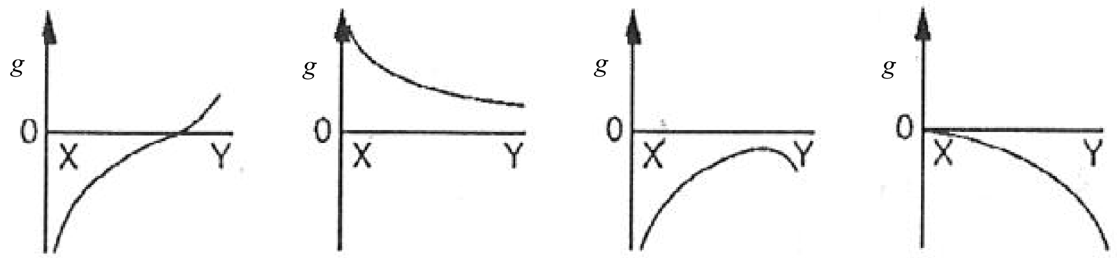
S4 The diagram (not to scale) represents the relative positions of the Earth and the Moon.



The line XY joins the surface of the Earth and the surface of the Moon.

Which graph represents the variation of gravitational field strength *g* along the line XY?

A **B** **C** **D**



S5 A body is moved from a point P on the Earth's surface to another point Q further from the Earth's centre. Which one of the following statements about the gravitational potential energy of the body at the two points is correct? [Take the gravitational potential energy of the body as zero when it is at an infinite distance from the Earth.]

A It is positive at both points and greater at Q than at P.

B It is positive at both points and lower at Q than at P.

C It is zero at P but positive at Q.

D It is negative at both points and greater at Q than at P.

E It is negative at both points and lower at Q than at P.

S6 A satellite of mass 50 kg is initially placed at a point where the gravitational potential due to the Earth is -20 MJ kg^{-1}. It is then moved to another point where the gravitational potential is -60 MJ kg^{-1}.

In which direction did it move and what is its change in potential energy?

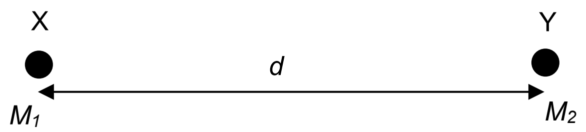
A closer to the Earth and a loss of 2000 MJ of potential energy

B closer to the Earth and a loss of 40 MJ of potential energy

C further from the Earth and a gain of 2000 MJ of potential energy

D further from the Earth and a gain of 40 MJ of potential energy

S7 Two stationary particles of masses M_1 and M_2 are at distance d apart as shown below.



(a) **(i)** Draw on the figure above, the direction of the gravitational field which M_1 causes at Y. [1]

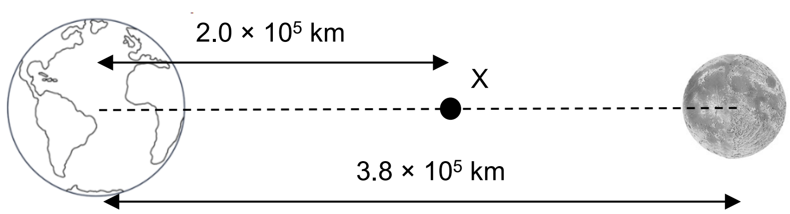
(ii) What is the gravitational field strength which M_1 causes at Y? [1]

(iii) What is the force which M_1 exerts on M_2? [1]

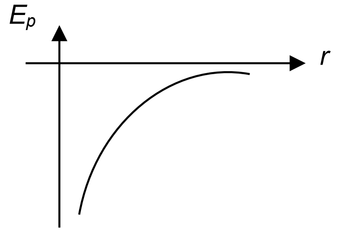
(b) A third particle, M_3 lying on the line joining the two particles, experiences no resultant gravitational force. The distance between M_1 and M_3 is x. Express x in terms of d, M_1 and M_2. [2]

S8 Earth has a mass of 6.0×10^{24} kg and Moon has a mass of 7.4×10^{22} kg. The distance between their centres is 3.8×10^5 km. Find the resultant gravitational field strength due to Earth and Moon at point X which is 2.0×10^5 km away from Earth as shown below.

(9.85×10^{-3} N kg^{-1} towards Earth) [2]



S9 The diagram below shows the variation of the gravitational potential energy E_p of a body with distance r from the centre of the planet.



What is represented by the gradient at any point on the curve?

A The gravitational potential at that value of r

B The gravitational field strength at that value of r

C The force pulling the body towards the planet

D The acceleration of the body towards the planet

S10 The radius of the Earth's orbit around the Sun is 1.5×10^{11} m. The mass of the Earth is 6.0×10^{24} kg and period of revolution of the Earth around the Sun is 1 year.

(a) Calculate

(i) the speed of the Earth in its orbit around the Sun, (2.99×10^{4} m s^{-1}) [2]

(ii) the acceleration of the Earth, (5.96×10^{-3} m s^{-2}) [2]

(iii) the force the Sun exerts on the Earth, (3.58×10^{22} N) [2]

(iv) the gravitational field strength of the Sun at the Earth,

(5.96×10^{-3} N kg^{-1}) [1]

(b) The radius of the orbit of Mars around the Sun is 2.3×10^{11} m. Determine the period of revolution of Mars about the Sun in terms of years. (1.90 years) [3]

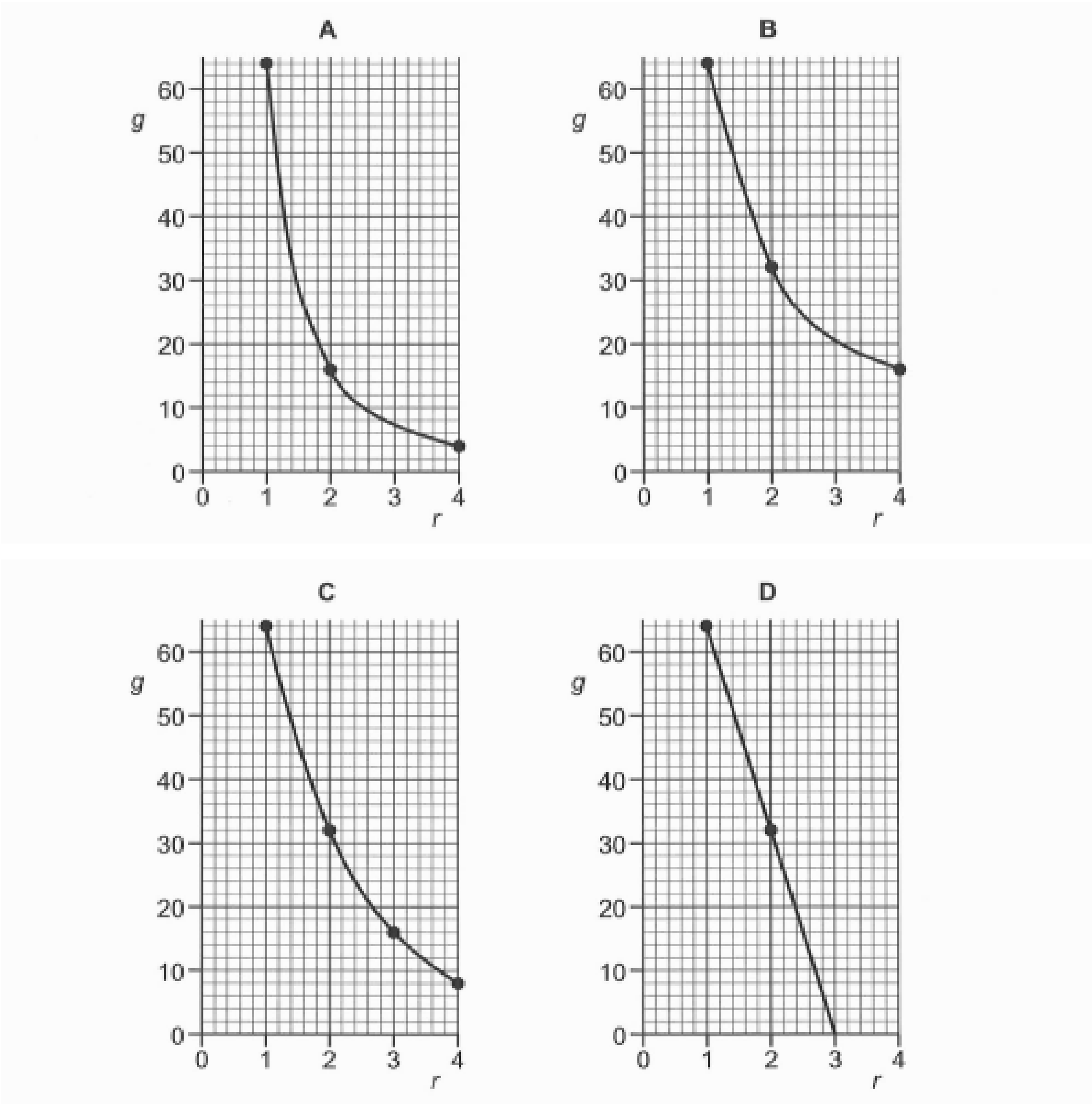
<u>Discussion Questions</u>

D1 The Earth may be assumed to be spherical with radius r and density ρ. Which equation correctly relates the gravitational field strength g at its surface to these quantities and the gravitational constant G?

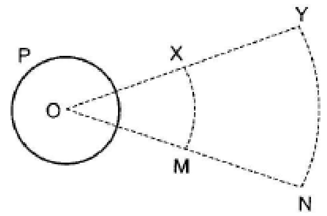
A $g = \dfrac{G\rho}{r^2}$ **B** $g = \dfrac{3G}{4\pi r \rho}$ **C** $g = \dfrac{4\pi r \rho G}{3}$ **D** $g = \dfrac{4\pi r^2 \rho G}{3}$

D2 Which graph shows how the gravitational field strength g due to a point mass varies with the distance r from the point mass?

(Arbitrary values are used on the graph axes. On all the graphs, g has the value 64 units when r = 1 unit.)



D3 P is a planet with centre O, as shown in the figure below. X and M are two points of equal gravitational potential, ϕ_A. Y and N are two other points of equal potential, ϕ_B.



Which of the following statements is incorrect?

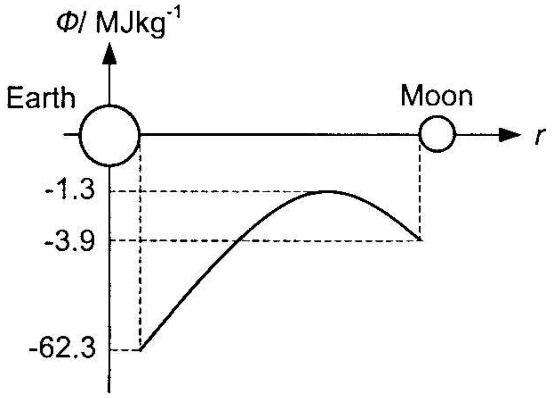
A The work done by an external agent to move a mass from Y to X is negative.

B The work done by the gravitational field to move a mass from X to N is the same as that needed to move the same mass from M to Y.

C The work done by the gravitational field to move a mass from M to N is positive.

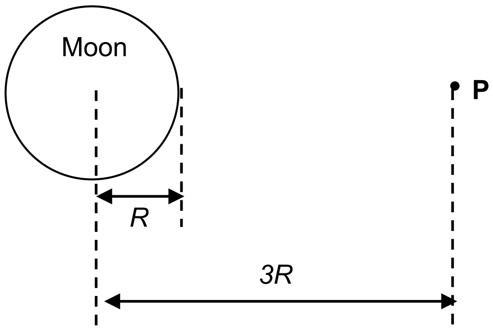
D $\dfrac{\phi_A}{\phi_B} = \dfrac{OY}{OX}$

D4 The figure below shows the variation of gravitational potential between the Earth and the Moon.



Determine the minimum energy required to launch an object of mass 2.0 kg from the Earth's surface to the Moon's surface. (122 MJ) [2]

D5 A stationary object of mass m is released from a point P at a distance $3R$ from the centre of the Moon which has radius R and mass M.



Determine the speed of the object when it hits the Moon in terms of G, M and R. [2]

D6 **(a)** Explain what is meant by *gravitational potential*. [1]

(b) Explain why the gravitational potential has a negative value. [2]

(c) The Earth may be considered to be an isolated sphere with a radius of 6.4×10^3 km and a mass of 6.0×10^{24} kg which is concentrated at its centre. An object, projected vertically from the surface of the Earth, reaches an altitude of 1.3×10^4 km. Determine

(i) the change in gravitational potential, (41.9 MJ kg^{-1}) [2]

(ii) the projection speed from the Earth's surface, assuming negligible air resistance. (9.15×10^3 m s^{-1}) [2]

(iii) the escape velocity. (1.12×10^4 m s^{-1}) [2]

(d) Why is the equation $v^2 = u^2 + 2as$ not appropriate for the calculation in (c)(ii)? [1]

D7 The Earth may be taken to have a mass M of 6.0×10^{24} kg. Its gravitational field may be considered to be the same as that of a point mass M at its centre. A satellite, initially orbiting the Earth with a radius R of 6610 km at a speed v of 7780 m s^{-1} is boosted into a higher orbit of radius 6890 km.

(a) Show that the satellite in its orbit of radius R, has a linear speed v given by $v = \sqrt{\frac{GM}{R}}$, where G is the gravitational constant. [2]

(b) Hence or otherwise, show that in the new orbit, the satellite's speed is 7620 m s^{-1}. [2]

(c) **(i)** A satellite of mass 120 kg moves from one orbit to the other. For this satellite, use the data provided to determine the change in its [5]

1. kinetic energy, (-1.48×10^{8} J)

2. gravitational potential energy, (2.95×10^{8} J)

3. total energy (1.47×10^{8} J)

(ii) Write down if the change in total energy is an increase or a decrease. [1]

D8 **(a)** The Earth spins on its axis with a period of one day.

(i) Show that the angular velocity of a point on the Earth's surface is 7.27×10^{-5} rad s^{-1}. [1]

(ii) Calculate the centripetal acceleration of a point on the Earth's equator. The radius of the Earth's equator is 6.38×10^{6} m. (3.38×10^{-2} m s^{-2}) [2]

(b) The acceleration of free fall g at the equator is not equal to the acceleration of free fall at the poles. Explain

(i) why they are different, [2]

(ii) why the difference is small. [1]

(c) A satellite is in geostationary orbit.

(i) Explain what is meant by the term *geostationary orbit*. Explain why a geostationary satellite must be placed vertically above the equator and move from West to East. [3]

(ii) The mass M of the Earth may be considered to be concentrated at its centre. The radius of the Earth is R. Derive in terms of M and R, the equation relating the Earth's gravitational field strength g to the gravitational constant G. [2]

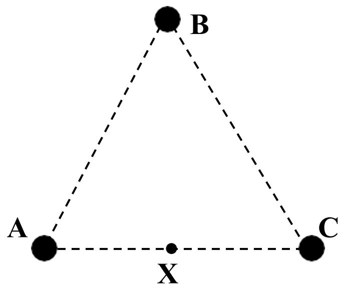
(iii) Hence, show that the geostationary satellite needs to be 4.23×10^{7} m away from the centre of the Earth for its angular velocity to be equal to the angular velocity of the Earth. [3]

(iv) Calculate the speed of the satellite. (3.08×10^{3} m s^{-1}) [2]

(v) Calculate the acceleration of the satellite. (0.226 m s^{-2}) [2]

(d) Many systems, such as the Global Positioning System, use several satellites in low orbits that pass over the Earth's poles. Suggest two advantages of these low polar orbits and two advantages of geostationary orbits. [4]

D9 Three identical point masses **A**, **B** and **C**, each of mass 2.0×10^5 kg form the vertices of an equilateral triangle with a side length of 1000 m. What is (a) the net gravitational field strength and (b) gravitational potential at point **X** which is located at the midpoint of **AC**?



(1.78×10^{-11} N kg^{-1}, towards B; - 6.9×10^{-8} J kg^{-1})

D10 A satellite of mass m orbits a planet of mass M and radius R_p. The radius of the orbit is R. The satellite and the planet may be considered to be point masses with their masses concentrated at their centres. They may be assumed to be isolated in space.

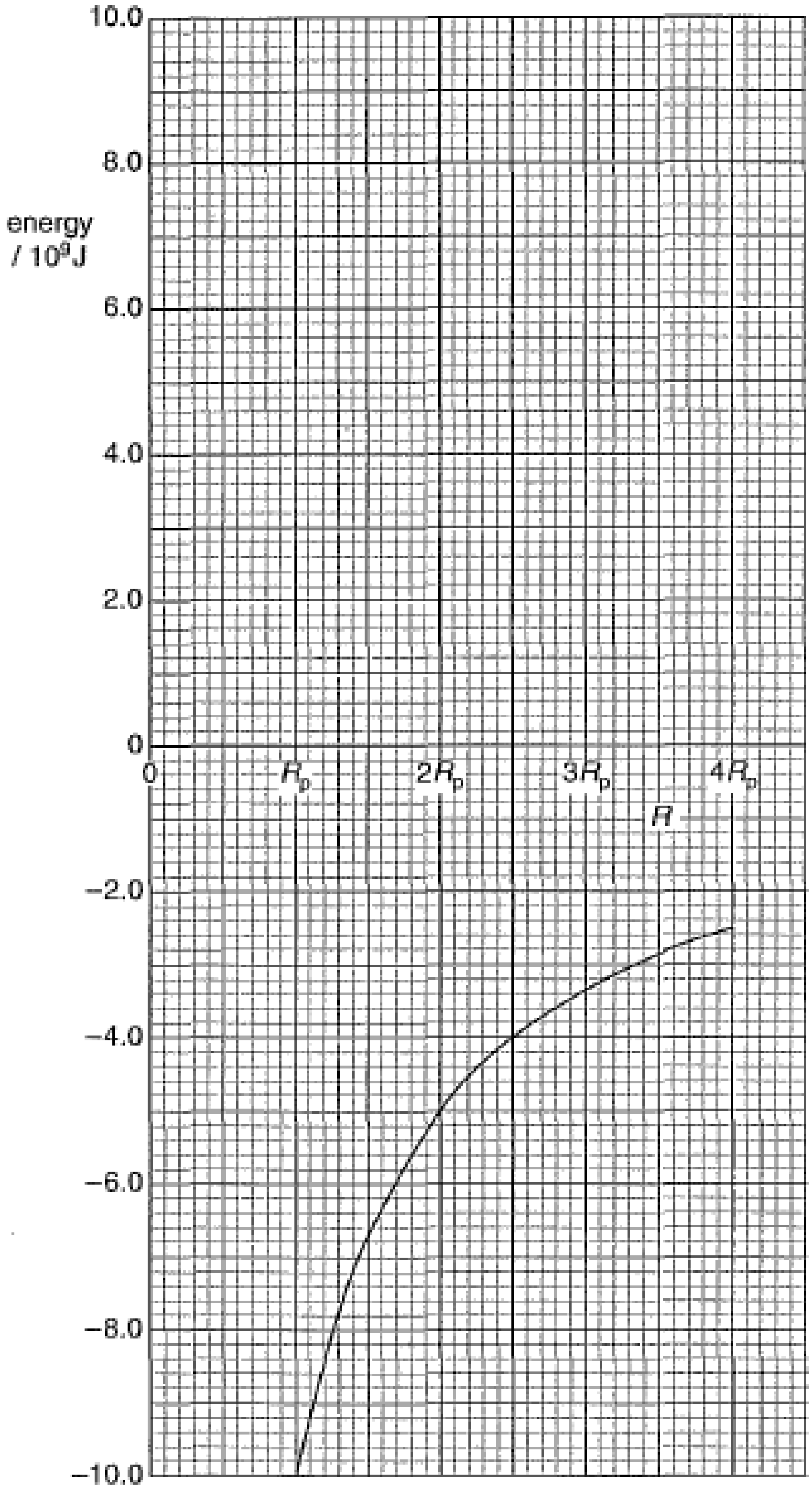
(a) **(i)** Derive an expression, in terms of M, m and R, for the kinetic energy of the satellite. Explain your working. [2]

(ii) Show that, for the satellite in orbit, the ratio

$$\frac{\text{gravitational potential energy of satellite}}{\text{kinetic energy of satellite}}$$

Is equal to –2. [1]

(b) The variation with orbital radius R of the gravitational potential energy of the satellite is shown in the figure.

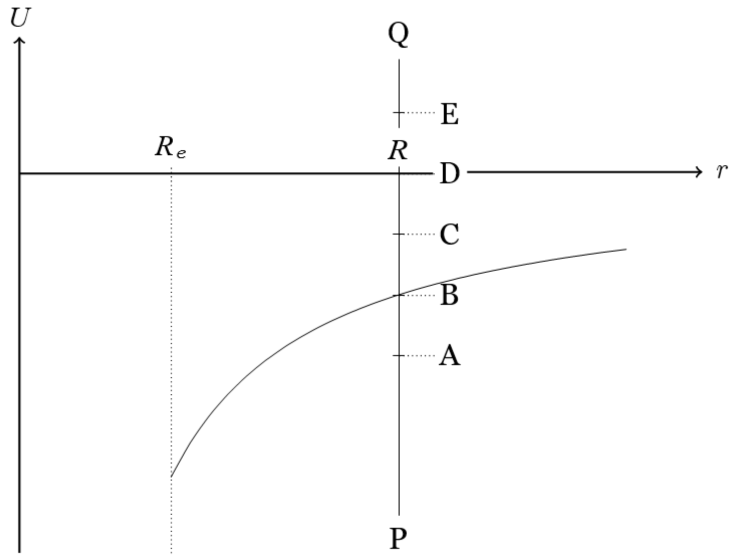


(i) On the figure, draw the variation with orbital radius of the kinetic energy of the satellite. Your line should extend from $R = 1.5R_p$ to $R = 4R_p$. [2]

(ii) The mass m of the satellite is 1600 kg.

The radius of the orbit of the satellite is changed from $R = 4R_p$ to $R = 2R_p$. Use the figure to determine the change in orbital speed of the satellite. [5]

D11 The graph below shows the variation of the gravitational potential energy of a rocket with its distance from the centre of the Earth.



At a distance *R* from the centre of the Earth, the total energy of the rocket may be represented by a point on the line PQ. Which point (or points) could represent the total energy of the rocket if at this distance *R* the rocket

(a) is momentarily at rest?

(b) is falling towards the Earth?

(c) is moving away from Earth, with sufficient energy to reach infinity?

In each case, explain briefly how you arrive at your answer.

D12 In the diagram provided below, sketch for the region between the two dotted lines

(a) the variation of gravitational field strength due to the Earth with distance,

(b) the variation of gravitational field strength due to the Moon with distance and

(c) the resultant field strength of all points on the straight line between the surface of the Earth and the surface of the Moon.

Assume both the Earth and the Moon to be uniform spheres.

D13 In the diagram provided below, sketch for the region between the two dotted lines

(a) the variation of gravitational potential due to the Earth with distance,

(b) the variation of gravitational potential due to the Moon with distance and

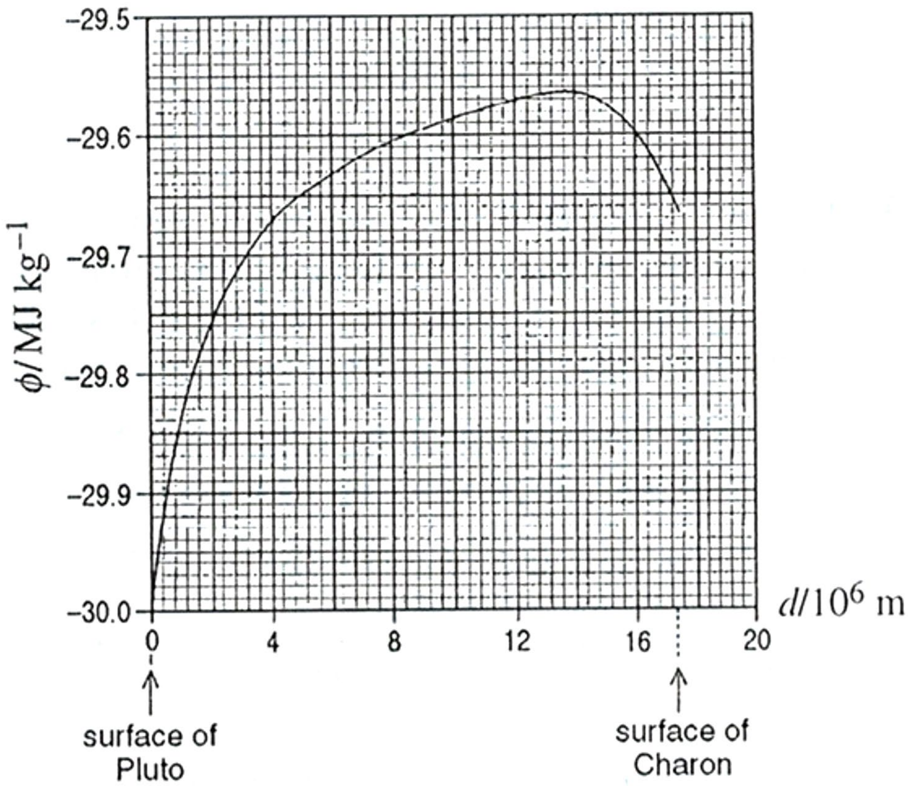
(c) the resultant potential of all points on the straight line between the surface of the Earth and the surface of the Moon.

Assume both the Earth and the Moon to be uniform spheres.

Data Analysis Question(s)

D14 [J01/P3/2 (part)] (modified)

The moon Charon (discovered in 1978) orbits the planet Pluto. Figure below shows the variation of the gravitational potential ϕ with distance d above the surface of Pluto along a line joining the centres of Pluto and Charon.



The gravitational potential is taken as being zero at infinity.

(a) **(i)** State the equation relating the gravitational field strength g and the gravitational potential ϕ. [1]

(ii) With reference to your answer in (a)(i), suggest why the gradient at a point on the graph above gives the magnitude of the acceleration of free fall at that point. [2]

(iii) Use the figure above to determine, giving an explanation of your working,

1. the distance from the surface of Pluto at which the acceleration of free fall is zero, (13.6×10^6 m) [2]

2. the acceleration of free fall on the surface of Charon. (0.0562 m s^{-2}) [3]

(b) A lump of rock of mass 2.5 kg is ejected from the surface of Charon such that it travels towards Pluto.

(i) Using data from the above figure, determine the minimum speed with which the rock hits the surface of Pluto. (938 m s^{-1}) [4]

(ii) Suggest why, if the rock travels from Pluto to Charon, the minimum speed on reaching Charon is different from that calculated in (i). [1]

~ THE END ~