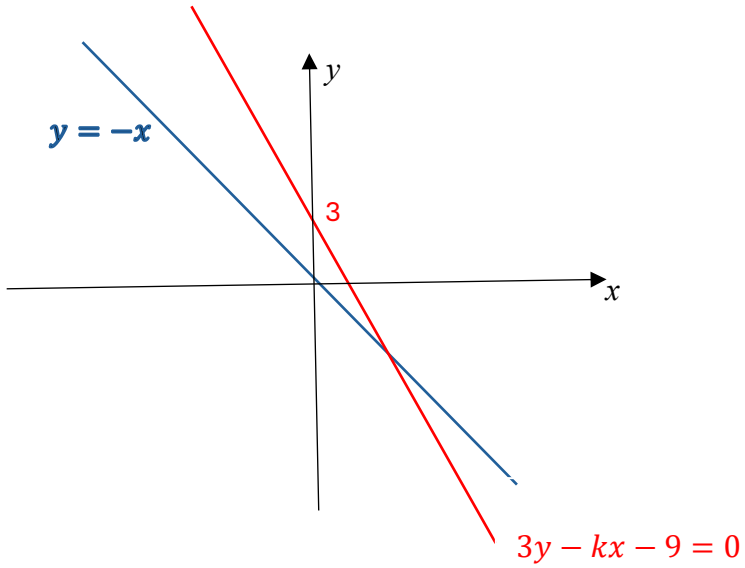
Nan Chiau High School

2024 Secondary 4 Mathematics

Mathematics Prelim Examination Paper 1 Solutions

Qn	Solution
1a	False
1b	False
2	$\frac{105b}{a} = \frac{(3\times5\times7)(2^2\times5\times7)}{2\times3^2\times5}$ $= \frac{2^2\times3\times5^2\times7^2}{2\times3^2\times5}$ $= \frac{2\times5\times7^2}{3}$ Numerator $2 \times 5 \times 7^2$ does not include the factor 3 to reduce $\frac{105b}{3}$ to a whole number.
3	Length of shorter portion of rope = $\frac{4}{11} \times 44$ = 16 cm Length of longer portion of rope = 28 cm Let x be the length rope cut. $\frac{16-x}{28-x} = \frac{2}{5}$ $5(16-x) = 2(28-x)$ $80 - 5x = 56 - 2x$ $24 = 3x$ $x = 8$
4	$7^{9-x^2} - 1 = 0$ $7^{9-x^2} = 1$ $7^{(3-x)(3+x)} = 7^0$ $x = 3 \quad or \quad x = -3$
5	$y^3 = \frac{k}{x^2}$ $\frac{k}{3^2} - \frac{k}{6^2} = 5$ $\frac{k}{9} - \frac{k}{36} = 5$ $\frac{k}{12} = 5$ $k = 60$ $y^3 = \frac{60}{x^2}$ $2^3 = \frac{60}{x^2}$ $x^2 = 7.5$ $x = \pm 2.74$

6	24 g → 12.044×10^{23} atoms 1 g → $0.5.01833 \times 10^{23}$ atoms 5000g → $0.5.01833 \times 10^{23} \times 5000$ $= 2509.167 \times 10^{23}$ $= 2.51 \times 10^{26}$
7a	Diagram A
7b	Diagram B
7c	Diagram D
8	$9x + 2.5y = 3y - 2x$ $11x = 0.5y$ $22x = y$ or $\frac{x}{y} = \frac{1}{22}$ Hence, $x = 22$ and $y = 1$ is not a solution, as $22(22) \neq (1)$ Or LHS = $9(22) + 2.5(1) = 200.5$ RHS = $3(1) - 2(22) = -41$ Since LHS ≠RHS, $x = 22$ and $y = 1$ is not a solution.
9	 $3y - kx - 9 = 0$ $3y = kx + 9$ $y = \frac{k}{3}x + 3$, where $k < -3$

10

Let the first digit be x and second digit be y

A number between 20 to 100	Let the number be $10x + y$
Add the two digits together	$x + y$
Subtract the sum of the two digits from your original number to get a new number	$(10x + y) - (x + y)$ $= 9x$
The new number is a multiple of 3	Since the new number is a multiple of 9 **and 3 is a factor** of 9, hence the new number is a multiple of 3.

11

$$2(mn)^2 - 2mn = 3mn + 3$$
$$2m^2n^2 - 5mn - 3 = 0$$
$$(mn - 3)(2mn + 1) = 0$$
$$mn = 3 \quad \text{or} \quad mn = -\frac{1}{2}$$
$$n = \frac{3}{m} \quad \text{or} \quad n = -\frac{1}{2m}$$

12

$$\frac{4}{3} + \frac{2}{9m} \div \sqrt[4]{81m^{16}} = \frac{4}{3} + \frac{2}{9m} \div 3m^4$$
$$= \frac{4}{3} + \frac{2}{9m} \times \frac{1}{3m^4}$$
$$= \frac{4}{3} + \frac{2}{27m^5}$$
$$= \frac{36m^5 + 2}{27m^5}$$

13

$$\frac{-9x^2 + 12x - 4}{3 - \frac{1}{x - \frac{x}{2}}} = \frac{-(3x-2)^2}{3 - \frac{1}{\frac{2x-x}{2}}}$$
$$= \frac{-(3x-2)^2}{3 - \frac{2}{x}}$$
$$= \frac{-(3x-2)^2}{\frac{3x-2}{x}}$$
$$= -x(3x - 2)$$

14

$y = \frac{1}{5}x^2 - 2x + 7$ ---------- [1]

$y = x$ ---------- [2]

Sub eqn[1] into eqn[2]

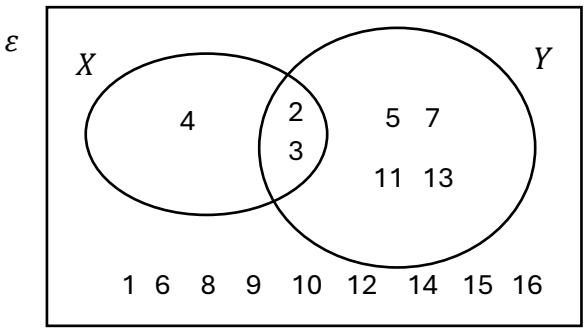
$$x = \frac{1}{5}x^2 - 2x + 7$$

$$\frac{1}{5}x^2 - 3x + 7 = 0$$

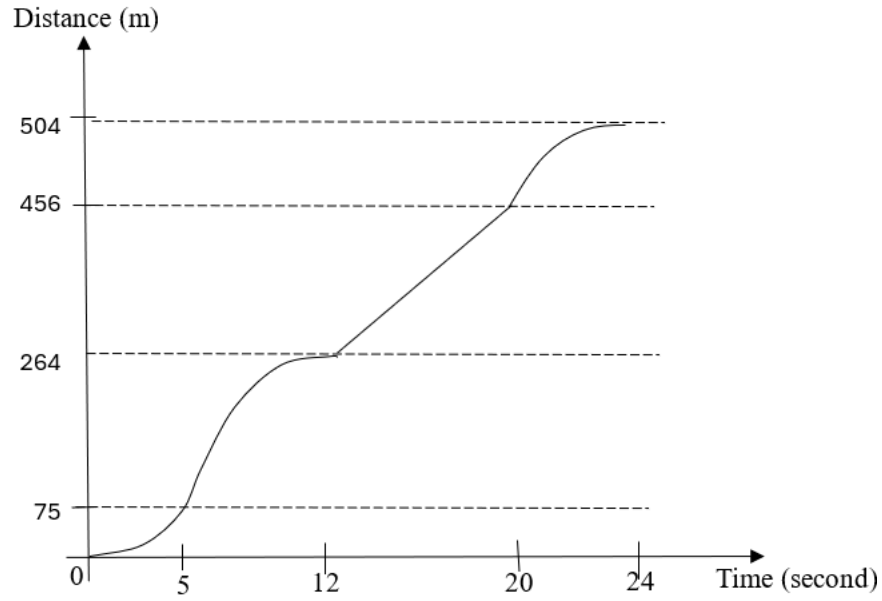
$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(\frac{1}{5})(7)}}{2(\frac{1}{5})}$$

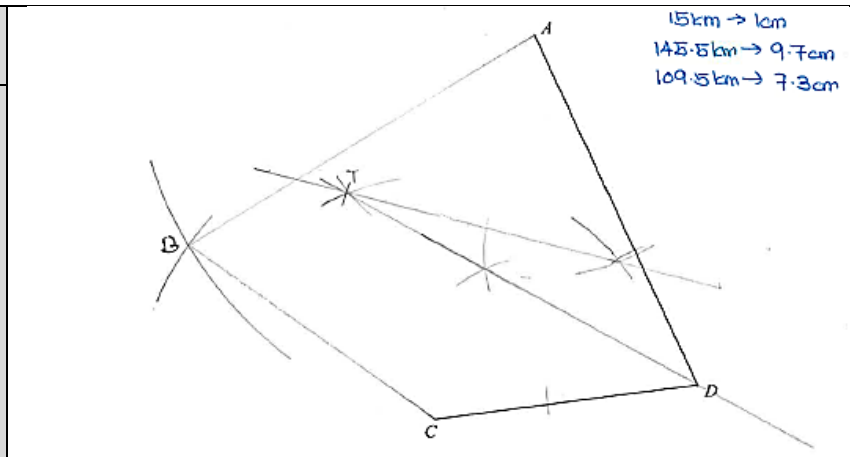
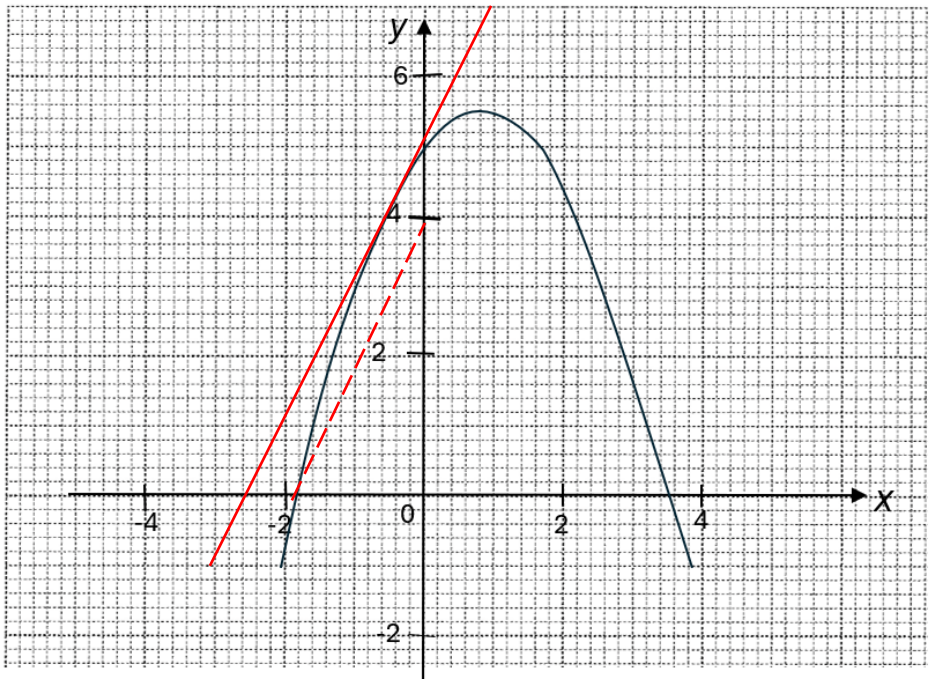
$x = 2.89$ or $x = 12.1$

	$y = 2.89$ or $y = 12.1$ (2.89, 2.89) and (12.1, 12.1)
15 a	Let the y-intercept (vertical distance) be d By Pythagoras' Thm $d = \sqrt{17^2 - 8^2}$ $d = 15$ $\therefore C(0, -15)$
15 b	Let the x-coordinate of point B be b $b = \frac{1}{8}(-15)$ $b = -\frac{15}{8}$ Equation of line of symmetry, $x = \frac{-\frac{15}{8}+(-8)}{2}$ $x = -\frac{79}{16}$ Since $y = -15$ when $x = 0$ Coefficient of $x^2 = -1$ Since curve is $y = -(x+8)(x+\frac{15}{8})$ or $y = -\frac{1}{8}(x+8)(8x+15)$ $\therefore y = -(-\frac{79}{16}+8)(-\frac{79}{16}+\frac{15}{8})$ $y = \frac{2401}{256}$ the coordinate of the maximum point is $(-\frac{79}{16}, \frac{2401}{256})$ or (- 4.94, 9.38)
16 a	$(\frac{8}{8+5+x})(\frac{7}{7+5+x}) = \frac{4}{33}$ $(\frac{8}{13+x})(\frac{7}{12+x}) = \frac{4}{33}$ $33(8)(7) = 4(13+x)(12+x)$ $462 = x^2 + 25x + 156$ $x^2 + 25x - 306 = 0$ $(x-9)(x+34) = 0$ $x = 9$ or $x = -34$ (rej)
16 b	P(second marble is green when three marbles are drawn) $= \left(\frac{17}{22}\right)\left(\frac{5}{21}\right)\left(\frac{16}{20}\right)$ $= \frac{34}{231}$

17 a	
17 bi	$\emptyset \ or \{\ \}$, $\{2\}, \{3\}, \{4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{2,3,4\}$
17 bii	Number of subsets $= 2^{n-1}$
18 a	$\text{Mean} = \frac{12(150)+10(200)+8(300)+x(400)+5(500)+2400}{12+10+8+x+5+1}$ $325 = \frac{11100+400x}{36+x}$ $11700 + 325x = 11100 + +400x$ $600 = 75x$ $x = 8$ $\therefore$ Total number of workers $= 12 + 10 + 8 + 8 + 5 + 1$ $= 44$
18 b	**Median** is a better gauge of workers' salary as there is salary that is **an extreme value of S2400** compared to the rest.
19 a	40% + 35% + 20% = 95% As pie charts are used to visualize parts of a whole, all the parts should always add up to 100%. [or] The title exaggerated the responses of the support for Candidate C.
19 b	To include in uncounted / spoiled votes (5%) to show the full result of votes [or] To re-calculate the votes according to the three candidates according to base of 95%. [or] To recraft the title to be less bias and allow readers to analysis themselves.

20 a	Area of rhombus $= 2 \times \frac{1}{2}(7.4)(7.4)\sin 60$ $= 47.424\text{cm}^2$ **[Or]** height of rhombus $= (7.4)(\sin 60) = 6.4086$ cm Area of rhombus $= (6.4086)(7.4) = 47.424\ \text{cm}^2$ $AD^2 = 7.4^2 + 7.4^2 - 2(7.4)(7.4)\cos 120$ $AD = 12.817$ cm Area of sector $= \frac{60}{360} \times \pi(12.817)^2$ $= 86.014\ \text{cm}^2$ Shaded area $= 86.014 - 47.424$ $= 38.6\ \text{cm}^2$
20 b	Arc length $= \frac{60}{360} \times 2\pi(12.817)$ $= 13.422$ Perimeter $= 13.422 + 2(12.817 - 7.4) + 2(7.4)$ $= 39.1$ cm
21 a	$10^2 + 4^2 = (10-4)^2 + 2(10)(4)$
21 b	$(2n)^2 + (n-1)^2 = (2n-n+1)^2 + 2(2n)(n-1)$ $or \quad = 4n^2 + n^2 - 2n + 1$ or $= 5n^2 - 2n + 1$
21 c	$p^2 + q^2 = 1249$ $5n^2 - 2n + 1 = 1249$ $5n^2 - 2n - 1248 = 0$ $(n-16)(5n+78) = 0$ $n = 16$ or $n = -\frac{78}{5}$ (rej) Hence, $p = 32$ and $q = 15$
22 a	$AB^2 + BC^2 = 6.4^2 + 4.8^2 = 64$ $AC^2 = 8^2 = 64$ Since $AB^2 + BC^2 = AC^2$, hence by **converse of Pythagoras' Theorem** , $\angle ABC = 90^o$ $\sin \angle ACB = \frac{6.4}{8}$ $= \frac{4}{5}$

22 b	$\cos\angle \mathrm{D}AC = -\frac{4}{5}$
23 a	$24 \text{ m/s} = \frac{24 \times \frac{1}{1000} km}{1 \times \frac{1}{3600} h}$ $= 86.4 \text{km/h}$
23 b	$504 = \left(\frac{1}{2} \times 5 \times v\right) + \left(\frac{1}{2}(v + 24)(7)\right) + \left(\frac{1}{2}(8 + 12)(24)\right)$ $504 = \frac{5}{2}v + \frac{7}{2}v + 84 + 240$ $180 = 6v$ $v = 30$ km/h
23 c	Distance from 0s to 5s = $\frac{1}{2} \times 5 \times 30 = 75\ m$ Distance from 0s to 12s = $75 + \frac{1}{2}(30 + 24)(7) = 264m$ Distance from 0s to 20s= $264 + (24)(8) = 456\ m$ 

24 a	
24 b	
24 c	5.9 x 15 = 88.5 km
25 a	-20
25 b	(0.8, 5.5)
25 c	 $k = 5.2$