Name:		Index Number:		Class:	

DUNMAN HIGH SCHOOL
Preliminary Examination
Year 6

MATHEMATICS (Higher 2) 9740/01

Paper 1

11 September 2012

3 hours

Additional Materials: Answer Paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Name, Index Number and Class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

At the end of the examination, **attach the question paper to the front of your answer script.**

The total number of marks for this paper is 100.

For teachers' use:

Qn	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Total
Score													
Max Score	**3**	**4**	**5**	**7**	**8**	**9**	**9**	**10**	**11**	**11**	**11**	**12**	**100**

1 By using the substitution $y=|x|$, solve the inequality $\dfrac{|x|+3}{x^2+1}>1$. [3]

2 Obtain a formula for $\displaystyle\int_{\frac{1}{2}}^{n}\frac{\left(\tan^{-1}2x\right)^2}{1+4x^2}\,dx$ in terms of n, where $n>0$.

Hence evaluate $\displaystyle\int_{\frac{1}{2}}^{\infty}\frac{\left(\tan^{-1}2x\right)^2}{1+4x^2}\,dx$ exactly. [4]

3 Express $\dfrac{2+x}{\sqrt{9-x}}$ as a series in ascending powers of x, up to and including the term in x^2. [3]

By using only the first two terms in the series expansion above and substituting $x=\dfrac{1}{9}$, find an approximation for $\sqrt{5}$ in the form $\dfrac{p}{q}$ expressed in its lowest terms, where p and q are integers to be determined. [2]

4 Given $\mathrm{f}(r)=\dfrac{r}{(r-1)!}$, show that $\mathrm{f}(r-1)-\mathrm{f}(r)=\dfrac{r^2-3r+1}{(r-1)!}$. [2]

(i) Hence find $\displaystyle\sum_{r=2}^{n}\frac{r^2-3r+1}{(r-1)!}$. [3]

(ii) State the value of $\displaystyle\sum_{r=2}^{\infty}\frac{r^2-3r+1}{(r-1)!}$, justifying your answer. [2]

5 A function f is defined as $\mathrm{f}(x)=\ln(5-x),\ x<5$.

(a) Find the volume of revolution when the region bounded by the curve of $y=\mathrm{f}(x)$ and the x- and y-axes is rotated completely about the y-axis. Give your answer correct to 2 decimal places. [2]

(b) **(i)** State the set of values of x for which $\mathrm{f}(|x|)=|\mathrm{f}(x)|$. [1]

(ii) Evaluate the value of $\displaystyle\int_{0}^{\frac{9}{2}}|\mathrm{f}(x)|\,dx-\int_{-\frac{9}{2}}^{0}\mathrm{f}(|x|)\,dx$, giving your answer in the form $a+b\ln 2$, where a and b are constants to be determined. [5]

[Turn over

6 It is given that $y = \mathrm{e}^{\cos^{-1} x}$.

(i) Show that $\left(1-x^2\right)\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} - x\dfrac{\mathrm{d}y}{\mathrm{d}x} = y$. [3]

(ii) Find the Maclaurin's series for y with exact coefficients, up to and including the term in x^3. [3]

(iii) Hence find the expansion of $-\dfrac{\mathrm{e}^{\cos^{-1} x}}{\sqrt{1-x^2}}$ up to and including the term in x^2 and estimate the gradient of the tangent of $y = \mathrm{e}^{\cos^{-1} x}$ at $x = 0.5$. [3]

7 Referred to the origin O, the points A and B are such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The point C lies on OB such that $\overrightarrow{OC} = p\overrightarrow{OB}$, where p is a constant. D is on AC such that $AD : DC = 2 : 3$ and E is on AB such that $AE : EB = 1 : 3$.

(i) Find $\overrightarrow{OD}$ and $\overrightarrow{OE}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and p. [2]

(ii) Given that O, D and E are collinear, find p. [3]

(iii) If $OB = 5$, show that the shortest distance from E to OB can be expressed as $k\left|\mathbf{a}\times\mathbf{b}\right|$, where k is a constant to be found. [3]

(iv) Give a geometrical interpretation of $\left|\mathbf{a}\cdot\hat{\mathbf{b}}\right|$. [1]

8 The curve C has equation

$$y = \frac{x^2 + (\lambda - 1)x}{x-1}, \quad x \neq 1,$$

where λ is a constant. Find

(i) the equations of the asymptotes of C, [2]

(ii) the range of values for λ such that C has 2 stationary points for $x > 0$. [4]

For the range of values for λ obtained in **(ii)**, sketch, on separate diagrams, the graphs of

(iii) C, [2]

(iv) $y = \mathrm{f}'(x)$, where $\mathrm{f}(x) = \dfrac{x^2 + (\lambda - 1)x}{x-1}$, $x \neq 1$, [2]

indicating the coordinates of the points where the graphs cut the x-axis and the equations of asymptotes, if any.

[Turn over

9 **(a)** Solve $z^4 = -4-4\sqrt{3}\,\mathrm{i}$, expressing your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r>0$ and $-\pi<\theta\le\pi$. (You do not need to list out the roots.) [3]

Hence solve $w^4 = -1+\sqrt{3}\,\mathrm{i}$, expressing your answers in a similar form. [3]

(b) Given $|p|=2$, $\arg(p)=\dfrac{\pi}{3}$, $|q|=7$ and $\arg(q)=-\dfrac{2\pi}{3}$, determine the modulus and argument of $\dfrac{p^7}{q^3}$. Hence express $\dfrac{p^7}{q^3}$ in the form $x+\mathrm{i}y$, where $x, y\in\mathbb{R}$. [4]

State the smallest positive integer n such that $\left(\dfrac{p^7}{q^3}\right)^n$ is real and negative. [1]

10 The functions f and g are defined by

$$\mathrm{f}: x\mapsto x^2+2x+4,\ \ x\le -1 \quad\text{and}\quad \mathrm{g}: x\mapsto \ln(2x-1),\ \ x>\frac{1}{2}.$$

(i) Find an expression for $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [3]

(ii) Sketch the graphs for $y=\mathrm{f}(x)$ and $y=\mathrm{f}^{-1}(x)$ on the same diagram. What can you say about the solution of the equation $\mathrm{f}(x)=\mathrm{f}^{-1}(x)$? State your reason clearly. [3]

(iii) Determine if the composite function gf exists. If so, find gf(x) and the exact range of gf. [5]

[Turn over

11 **(a)** Given that $z = e^{2x}\frac{dy}{dx}$, express $\frac{dz}{dx}$ in terms of x, $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

Hence show that the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = e^{1-4x}$ can be reduced to

$$\frac{dz}{dx} = e^{1-2x}.$$

Solve this differential equation, expressing y in terms of x. [5]

(b) An aviary keeper started an insect breeding programme with an initial number of 5000 insects in a controlled environment to feed the birds in the aviary. The rate of birth of the insects is $\frac{1}{25}$ of the number I (in thousands) of insects, at time t days after the start of the breeding programme. The insects are also being fed to the birds at a constant rate of 0.25 (in thousands) per day. Show that the population of insects can be modelled with the differential equation

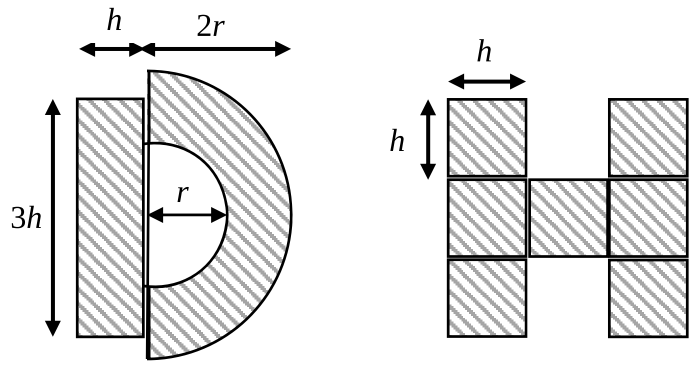
$$\frac{dI}{dt} = 0.04(I - 6.25).$$

Hence solve the differential equation to obtain the particular solution of I in terms of t. [4]

By sketching the solution curve, show that it is possible for the insect breeding programme to be depleted of insects after some time. After how many complete days will this happen? [2]

[Turn over

12 **(a)** A Dunman High alumnus plans to donate money to build two koi ponds in the middle of Zheng Xin Yuan. The ponds are in the shape of "**D**" and "**H**" as shown below (all dimensions are in metres and the diagrams are not drawn to scale).



The shaded areas represent the pond surface. The "**D**" pond is made up of a 3*h* by *h* rectangle and a curved area formed from two concentric semi-circles of radii 2*r* and *r* respectively. The "**H**" pond is made up of seven identical squares of side *h*.

(i) Show that the total surface area of the two ponds, A m^2, is given by $A = 10h^2 + \frac{3}{2}\pi r^2$. [2]

(ii) The landscape designer proposes that h and r be related by the equation $rh^2 = k$, where k is a constant. As r and h vary, find the exact value of $\frac{r}{h}$ at the stationary value of A. Determine the nature of this stationary point and use a calculator to evaluate the stationary value of A if $k = 1$. [6]

(b) The equation of a graph C is given by $y = 2x + 5 + \frac{4}{x}$.

By differentiation, find the exact set of values of y for which there are no points on C. [4]

END OF PAPER