Problemset Solutions

Exercises

E1(a)

Work done by Denise = $Fx \cos \theta$

$= (30)\ (5.0)\ (\cos 60^{o})$

$= 75$ J

E1(b)

Hilda does not do work on the book because the book's displacement is zero.

E2. (a) Can work done be positive or negative? State the condition for it.

Yes, work done can be positive or negative.

When force and displacement are in same direction, workdone is positive.

When force and displacement are in opposite direction, workdone is negative.

(b) Can work done be zero? State the condition(s) for it.

Yes. Conditions: - force is zero

- Displacement is zero
- Force and displacement is perpendicular to each other.

E3 The elastic potential energy stored in the bow just before the release of an arrow is 95 J. When the arrow of mass 170 g is fired, 90% of the elastic potential energy is transferred to the arrow. Show that the speed of the arrow as it leaves the bow is 32 ms^{-1}.

Amount of energy transferred to the arrow $= \frac{90}{100} \times 95 = 85.5$ J

$$\Rightarrow \frac{1}{2} mv^2 = 85.5$$

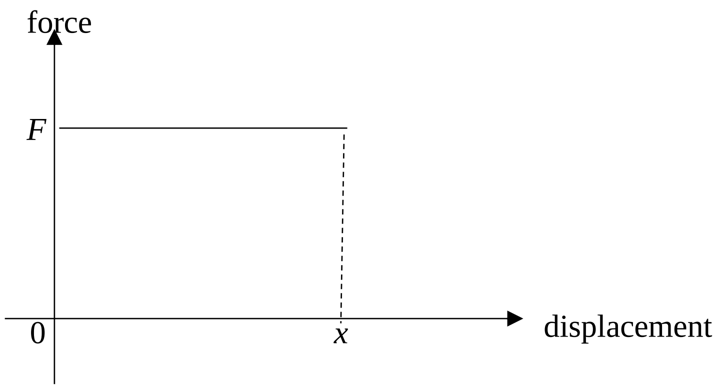
$$v = 32 \text{ m s}^{-1}$$

E4. An object at rest is pulled by a constant horizontal force, *F* on a smooth floor to a displacement of *x.*

(i) Sketch a graph of force applied on object against the displacement. Label *F* and *x* in the graph.

(ii) What does the area under force against displacement graph in **(i)** represent? Hint: work done.

(i)



(ii) Area under force against displacement graph represent the work done by force on object.
Work done = $F(x)$

E5. An object at rest is pulled by a varying horizontal force, F_x on a smooth floor to a displacement of *x.*

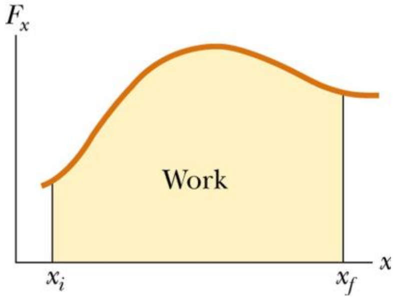
(i) Can we still use this formula to calculate the work done by *Fx, i.e workdone* = $F_x(x)$?

No. The formula work done = $F_x(x)$ is only for a constant force.

(ii) How can the work done by F_x be calculated?

To calculate the work done by a varying force acting on an object that

Work done by a varying force is equal to the area under the force–displacement graph.



E6(a)

Gain of KE of the sack = Work done on the sack = $2.0 \times 0.35 = 0.70$ J

E6(b)

$$\frac{1}{2}mv^2 = 0.7 \Rightarrow v = 0.37 \text{ ms}^{-1}$$

E7(a)

By Principle of COE,

Loss of GPE = Gain in KE

$m(9.81)(30) – 0 = ½ m (2.80)^2 - 0$

$\therefore v = 24.4 \text{ m s}^{-1}$ (3 s.f.)

OR

By Principle of COE,

GPE at A + KE at A = GPE at B + KE at B

$m(9.81)(30) + ½ m (2.80)^2 = 0 + ½ m v^2$

$\therefore v = 24.4 \text{ m s}^{-1}$ (3 s.f.)

E7(b)

By Principle of COE,

Loss of GPE = Gain in KE

$m(9.81)(30) – m(9.81)(25) = ½ m (2.80)^2 - 0$

$\therefore v = 10.3 \text{ m s}^{-1}$ (3 s.f.)

OR

By Principle of COE,

GPE at A + KE at A = GPE at C + KE at C

m(9.81)(30) + ½ m $(2.80)^2$ = m(9.81)(25) + ½ m v^2

$\therefore$ v = 10.3 m s^{-1} (3 s.f.)

E8(a)

Work done in compressing spring = Elastic PE stored in spring

$$= \frac{1}{2} kx^2$$

$$= \frac{1}{2}(500)(0.10)^2$$

$$= 2.5 \text{ J}$$

E8(b)

Assume all the elastic PE stored in the spring is converted to kinetic energy.

$$\Rightarrow \frac{1}{2} mv^2 = 2.5$$

$$\Rightarrow v = \sqrt{\frac{2(2.5)}{2.0}} = 1.6 \text{ ms}^{-1}$$

E9

Power of engine P = F_e(v) where Fe is driving force from engine.

At maximum speed, acceleration = 0

$\Rightarrow$ D = F_e

P = F_e v

P = D v, D α v^2

P α v^3

72 k α 12^3

36 k α $v_{one}^{\ 3}$

V_{one} = 9.5 m s^{-1} (Ans D)

Problems

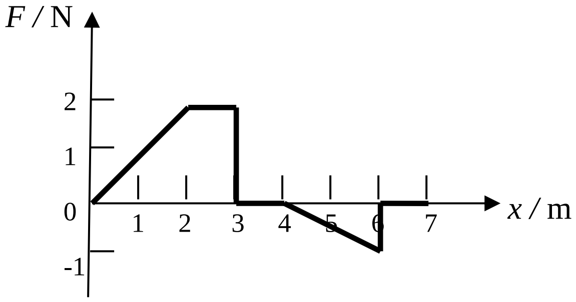
Work

P1. 2011/P1/Q11, TYS Topic 5: MCQ 12

Gain of kinetic energy = work done by *F*

= *Fs* (Ans C)

P2. A force F that is parallel to the x-axis acts on a block of ice. The magnitude of the force varies with the x-coordinate of the block as shown. Calculate the work done on the block of ice by the force F when the block moves from x = 0 m to x = 7.0 m.



Work done on block $= \text{Area under graph}$

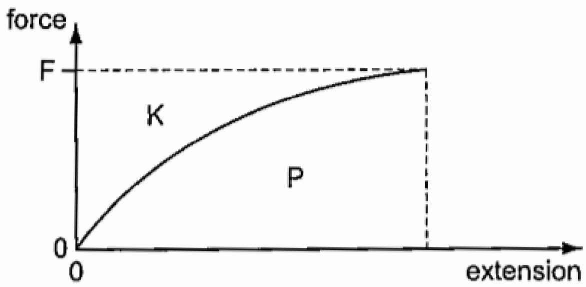
$$= \frac{1}{2}(2)(1+3) - \frac{1}{2}(1)(2)$$

$$= 3 \text{ J}$$

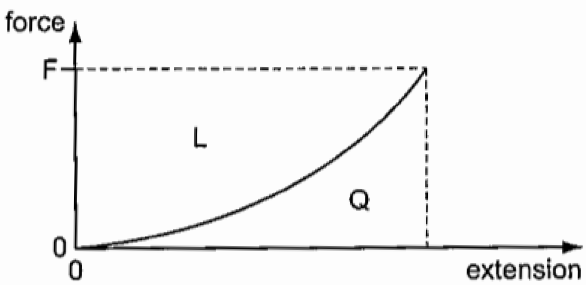
Work done: Force-extension graph

P3. N14/P1/Q17, TYS: Topic 5: MCQ 5

A long nylon fibre is stretched by a force that is increased from zero to a final value F. The force-extension graph obtained for this process is shown below.



When the force is subsequently reduced from F to zero, the force-extension graph obtained is shown below.



Which combination of areas from these graphs gives the net work done on the fibre?

A K – L **B** K + L **C** P – Q **D** P + Q

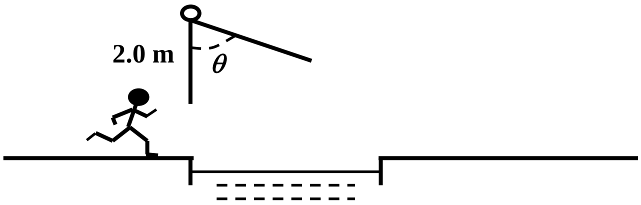
Area under force-extension graph is the work done
P is work done on fibre by stretching force

Q is work done by fibre
Hence net work done on fibre = P – Q (Ans: C)

Conservation of energy

P4.

In the process of crossing an obstacle course, a 65 kg student running at 5.0 m s^{-1} grabs a hanging rope of length 2.0 m, and swings out over a pit of water. He releases the rope when his speed is 2.0 m s^{-1}. What is the angle θ when he releases the rope?



Using the Principle of Conservation of Energy,

Loss of KE = Gain in GPE

$$\frac{1}{2}(65)(5.0)^2 - \frac{1}{2}(65)(2.0)^2 = (65)(9.81)h - 0$$

$$h = 1.07 \text{ m}$$

OR

Using the Principle of Conservation of Energy,

student's initial KE = student's final KE + student's increase in GPE

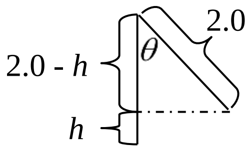
$$\Rightarrow \frac{1}{2}(65)(5.0)^2 = \frac{1}{2}(65)(2.0)^2 + (65)(9.81)h$$

$$\Rightarrow h = 1.07 \text{ m}$$

Now,

$$\cos\theta = \frac{2.0 - h}{2.0}$$

$$\Rightarrow \theta = 62°$$



P5.
The top end of a spring is attached to a fixed point and a mass of 4.2 kg is attached to its lower end. The mass is released and after bouncing up and down several times it comes to rest at a distance 0.29 m below its starting point. Which row gives the gain in the gravitational potential energy of the mass E_p and the gain in the elastic potential energy of the spring E_s?

	E_p / J	E_s / J
A	–12	+12
B	–12	+6
C	+12	+12
D	+12	+6

The GPE obviously decreased as the final position is lower. Hence either choice A or B.

We know that there must be stored energy in the spring, so it's a matter of how much. Since the spring came a rest after some time due to resistive forces, there must be some loss of energy to the surrounding (and also heat in the spring). The gain in EPE must hence be less than the loss in GPE. Hence, choice B. (Common mistake is choosing A, thinking that mechanical energy is conserved. It is suitable to bring in an initial discussion of SHM)

However if we want to verify, we can do so too. At equilibrium position,
Spring force = weight = mg
EPE = ½ Fx = ½ (mg)(x) = ½ loss in GPE
Ans: B

P6. N13/P1/Q10, TYS: Topic 5: MCQ 8

10 Which row in the table gives the gravitational potential energy, the elastic potential energy and the kinetic energy of a bungee jumper during the first fall? Air resistance is negligible.

		gravitational potential energy / kJ	elastic potential energy / kJ	kinetic energy / kJ
A	top middle bottom	120 60 0	0 10 120	0 50 0
B	top middle bottom	120 60 0	0 30 60	0 30 60
C	top middle bottom	120 60 0	0 30 120	0 60 0
D	top middle bottom	120 60 0	0 60 120	0 0 0

Ans: A
B: KE must be 0 at bottom

C: Total energy =120 J but total energy at middle = 150 J
D: KE cant be 0 at middle.

P7.
An 80.0 kg sky diver jumps out of a balloon at an altitude of 1000 m and opens the parachute at an altitude of 200 m. The total retarding force on the diver is constant at 50.0 N with the parachute closed and constant at 3600 N with the parachute open.

(a) What is the speed of the diver when he lands on the ground?

(b) At what height should the parachute be opened so that the final speed of the sky diver when he hits the ground is 5.00 m s^{-1}?

(a) Gravitational Potential energy is converted into work done by total retarding force and kinetic energy
Loss in GPE = workdone by retarding force + gain in kinetic
$80.0 \times 9.81 \times 1000 = 50.0 \times 800 + 3600 \times 200 + ½ \times 80.0 \times v^2 \Rightarrow v = 24.9$ m s^{-1}

(b) Gravitational Potential energy is converted into work done by total retarding force and kinetic energy
$80.0 \times 9.81 \times 1000 = 50.0 \times s_1 + 3600 \times s_2 + ½ \times 80.0 \times 5.00^2$, where $s_1 + s_2 = 1000$
Solving for s_2 gives that the parachute should be opened at a height of 207 m.

Notice what a difference a mere 7 m makes!

8. 2009 H1 P2 Q7

7 Fig. 7.1 shows a man doing a bungee jump.

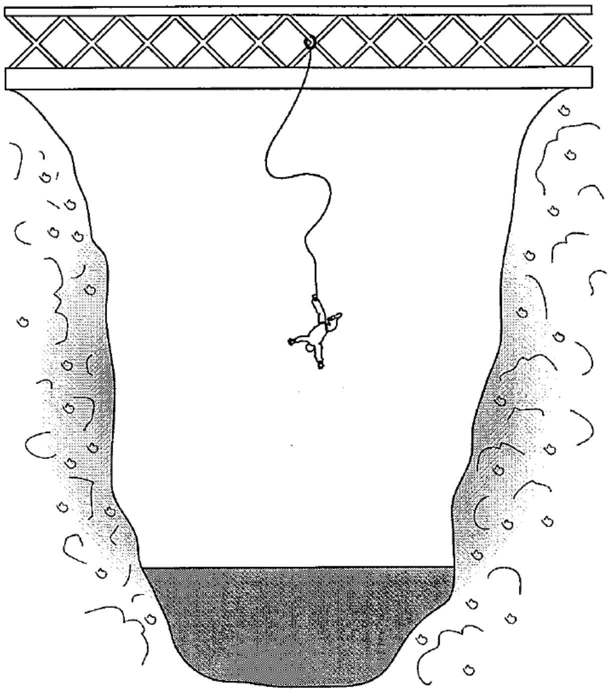


Fig. 7.1

The man has a mass of 75 kg and falls a distance of 41 m before the elastic rope attached to him starts to exert any force on him.

The man has a mass of 75 kg and falls a distance of 41 m before the elastic rope attached to him starts to exert any force on him.

(a) **(i)** Calculate the theoretical time taken for a fall of this distance.

time = ...$\boxed{2.89\text{ s}}$.... s [2]

(ii) The actual time taken for this fall is 2.9 s. State the deduction you can make by comparing the actual time with your answer to **(i)**.

....

....[1]

(b) A force-extension graph for the elastic rope used for the bungee jump is shown in Fig. 7.2.

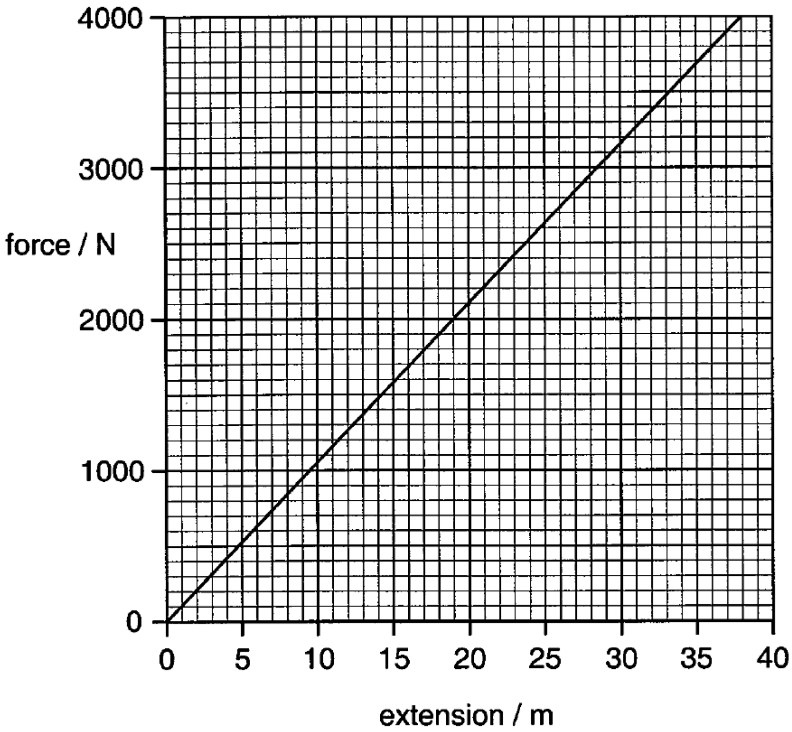


Fig. 7.2

The total distance of fall for the man before he stops for the first time is 73 m. Deduce

(i) the extension of the rope when the man stops for the first time,

32 m

extension = ... m [1]

(ii) the elastic potential energy stored in the rope at this time.

5.4×10^4 J

(c) **(i)** Complete Fig. 7.3 to show the gravitational potential energy and the kinetic energy of the man at the three points stated, together with the elastic potential energy stored in the rope. The gravitational potential energy at the bottom of the fall is taken to be zero.

	at the top	after falling 41 m	after falling 73 m (i.e. when stopped)
gravitational potential energy /J			0
elastic potential energy /J			
kinetic energy /J			

[5]

Fig. 7.3

(ii) Calculate

1. how far the man has fallen from the top when he has maximum kinetic energy,

2. the maximum kinetic energy of the man during the fall.

[48 m, 3.3 x 10^4 J]

(iii) Use your values to sketch **three** graphs on Fig. 7.4 showing how the three different types of energy vary with distance fallen. Label each graph.

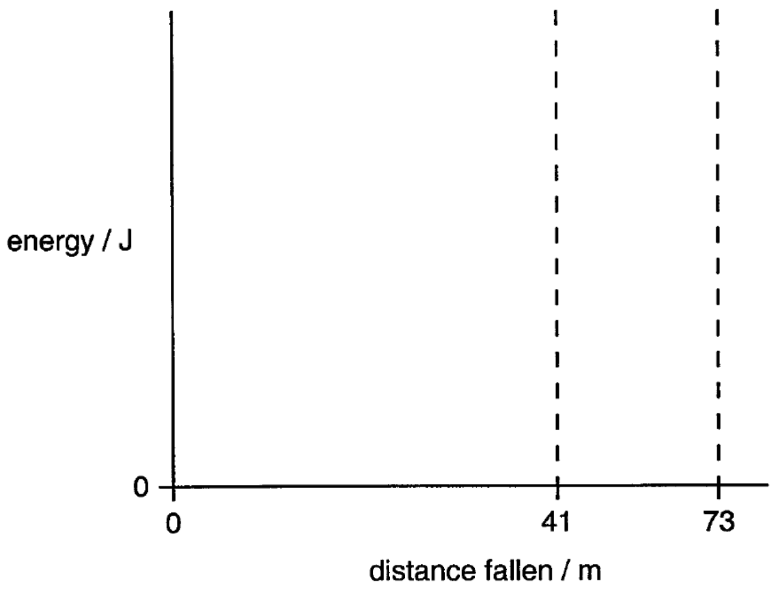


Fig. 7.4

[4]

P8

ai) $s = ut + 1/2at^2$

$t = 2.89$ s = 2.9 s(2 sf)

ii) Since the difference is negligible, the effect of air resistance is negligible on the fall of the man.

bi) 73 – 41 = 32 m

ii) From Fig 7.2, F = 3400 N,

Hence, EPE = 5.4×10^4 J

ci)

5.4×10^4 J	mg(73-41) =2.4×10^4 J	0
0	0	5.4×10^4 J
0	TE- GPE = 3.0×10^4 J	0

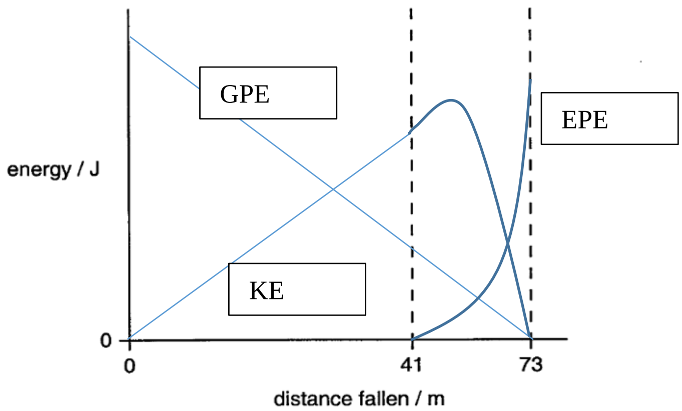
cii) 1. At max ke and hence max speed, the acceleration is 0. Hence T = mg = 735.75 N. From Fig 7.2, extension is 7 m. Hence, distance = 41 +7 = 48 m

2. From Fig 7.2, area under graph when extension = 7m, EPE = ½(7)(750)= 2625 J, GPE = mg(25) = 18 393 J

By COE, max KE = Total Energy – GPE – EPE

= 5.4×10^4 - 18393 - 2625

= 3.3×10^4 J



- EPE curve is a parabola.

Power

P9.

A car of mass 1200 kg starts from rest at the foot of a hill inclined at an angle θ to the horizontal, where $\sin\theta = 1/5$. 3.0 minutes later, the car has a speed of 20.0 m s^{-1} and has travelled a distance of 1.6 km. The frictional forces resisting the motion are constant and of magnitude 210 N. Calculate, in this time, the average power of the engine of the car.

Increase in vertical height of the car = $1600 \times \sin\theta = 320$ m

$\therefore$ Increase in GPE of the car $= (1200)(9.81)(320) = 3.767 \times 10^6$ J

Increase in KE of the car $= \frac{1}{2}(1200)(20.0)^2 = 2.400 \times 10^5$ J

Work done by frictional force = $-210 \times 1600 = -3.360 \times 10^5$ J

Work done by engine + Work done by friction = Gain in KE + Gain in GPE

$$\begin{aligned}\text{Average power of the car's engine} &= \text{Work done by enegine} \\ &= \text{Gain in KE of car + Gain in GPE of car - work done by friction} \\ &= \frac{3.767 \times 10^6 + 2.400 \times 10^5 - (-3.360 \times 10^5)}{3.0 \times 60} \\ &= 2.41 \times 10^4 \text{ W}\end{aligned}$$

P10. N15/1/8, TYS Topic 5: MCQ 1

A car travels along a road at a constant speed of 20 m s^{-1}. Its power output is 23 kW. The total resistive force on the car is proportional to the square of its speed.

What power will be required for the car to travel at a constant speed of 40 m s^{-1}?

A 46 kW **B** 92 kW **C** 184 kW **D** 368 kW ( )

P = Fv where F is the driving force. As car is travelling at constant speed, F = total resistive force, R

R = 23000/20 = 1150 N

R α v^2

1150 α 20^2 ---------- (1)

R α 40^2------------- (2)

(2)/(1), R=4600 N

The power required = 4600(40) = 184 kW (Ans: C)

P11.

A tugboat, moving by itself in calm water, requires 80 kW at the propeller to move with a steady speed of 5.0 m s^{-1}. When towing a large ship at the same steady speed, 600 kW must be delivered to the propeller. Calculate the force in the towline.

Use $Power = Force \times Velocity$.

When moving by itself in calm water,

$80000 = F \times 5.0 \Rightarrow F = 16000$ N. This force is due to the water resistance on the tugboat.

When towing a large ship,
$600000 = F \times 5.0 \Rightarrow F = 120000\ \text{N}$. This force is due to the water resistance on the tugboat, plus the tension in the towline.

$\therefore$ the force in the towline $= 120000 - 16000 = 1.04 \times 10^5\ \text{N}$

P12. N12/P1/19, TYS Topic 5: MCQ 9

A railway locomotive pulling a train delivers a constant power of 2.0×10^6 W to the wheels. The resistive forces are constant at all speeds. The maximum speed that the train can achieve on a level track is $40\,\text{m s}^{-1}$.

What is the resultant force accelerating the train when it is travelling at $10\,\text{m s}^{-1}$?

A 5.0×10^4 N **B** 1.5×10^5 N **C** 2.0×10^5 N **D** 2.5×10^5 N

P= Fv where F= driving force.
At constant speed, F = total resistive force, R
$2.0 \times 10^6 = R(40)$
$R = 5.0 \times 10^4$ N
At 10 ms^{-1}, $2.0 \times 10^6 = F_1(10)$
$F_1 = 20$ kN
Resultant force = 20k – 5.0×10^4 = 1.5×10^5 N (Ans: B)

Efficiency

P13. N16/P1/11, TYS Topic 5: MCQ 3

A small car of mass 800 kg travels at a constant speed of $20\,\text{m s}^{-1}$ along a level road against resistive forces of 400 N. The efficiency of the car in converting the energy of the fuel into work done against the resistive forces is 16%.

One kilogram of fuel has an energy value of 48 MJ.

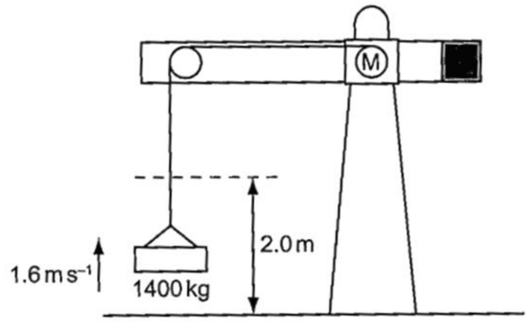
How much fuel does the car use to travel one kilometre?

A 52 g **B** 85 g **C** 220 g **D** 260 g

Power required = (400)20 = 8000 W
Time taken to travel 1 km = 1000/20 = 50 s
Amount of fuel energy required = 8000(50)/0.16 = 2.5×10^6 J
Amt of fuel required = 2.5M/48M = 0.052 kg (Ans: A)

P14. N13/P1/11, TYS Topic 5: MCQ 7

The motor M in a crane is used to lift a total mass of 1400 kg through a height of 2.0 m at a constant speed of 1.6 m s^{-1}. The motor is 20% efficient.



What is the minimum input power to motor M?

A 11 kW **B** 22 kW **C** 110 kW **D** 140 kW

Output power = mgv = 1400(9.81)(1.6) = 2197.44 W
Minimum input power = 2197.44/0.2 = 109.872 kW = 110 kW (Ans: C)