Name:	Register Number:	Class:

PRESBYTERIAN HIGH SCHOOL

MATHEMATICS **4045/02**
PAPER 2

1 August 2023 Tuesday 2 hours

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2023 SECONDARY FOUR NORMAL (ACADEMIC)
PRELIMINARY EXAMINATION

MARK SCHEME

Section A (62 marks)
Answer **all** the questions in this section.

1 **(a)** Work out

(i) $\sqrt{-12^2-(-9)^2(-6)}$, [1]

18.5	B1

(ii) $\dfrac{0.2031}{\sqrt[3]{17.95}+1.292}$. [1]

0.0519	B1

(b) **(i)** Write 29.951 correct to 3 significant figures. [1]

0.00201	B1

(ii) Write 4.523 million to the nearest ten thousand. [1]

4520000	B1

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2 **(a)** Given that $3^3 \times 81^{\frac{1}{4}} = 3^m$, find the value of m. [2]

$3^3 \times 81^{\frac{1}{4}} = 3^m$ $3^3 \times \left(3^4\right)^{\frac{1}{4}} = 3^m$ $3^3 \times 3^1 = 3^m$ $3^4 = 3^m$ $m = 4$	M1: $81 = 3^4$ A1

(b) Simplify $\sqrt{\dfrac{9a^7}{a^3}}$. [2]

$\sqrt{\dfrac{9a^7}{a^3}} = \left(\dfrac{9a^7}{a^3}\right)^{\frac{1}{2}}$ $= \left(9a^4\right)^{\frac{1}{2}}$ $= 3a^2$	M1: a^4 /fractional indices A1

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3 **(a)** p is directly proportional to q^3.
Given that $p = 24$ when $q = 2$, find

(i) the formula connecting p and q. [2]

$p = kq^3$ $24 = k(2)^3$ $k = 3$ Eqn is $p = 3q^3$	M1 for finding constant A1

(ii) the value of q when $p = 192$. [1]

$192 = 3q^3$ $64 = 3q^3$ $q = 4$	 B1

(b) 5 men can paint a house in 6 days. The house was painted in n days.
Write down an expression, in terms of n, for the number of men needed to paint the house. [2]

1 man takes 30 days n days requires $\frac{30}{n}$ men	M1 A1

4 **(a)** Find 130% of 2 litres in millilitres. [1]

$2600ml$	B1

(b) A map is drawn to a scale of 1 : 50000.

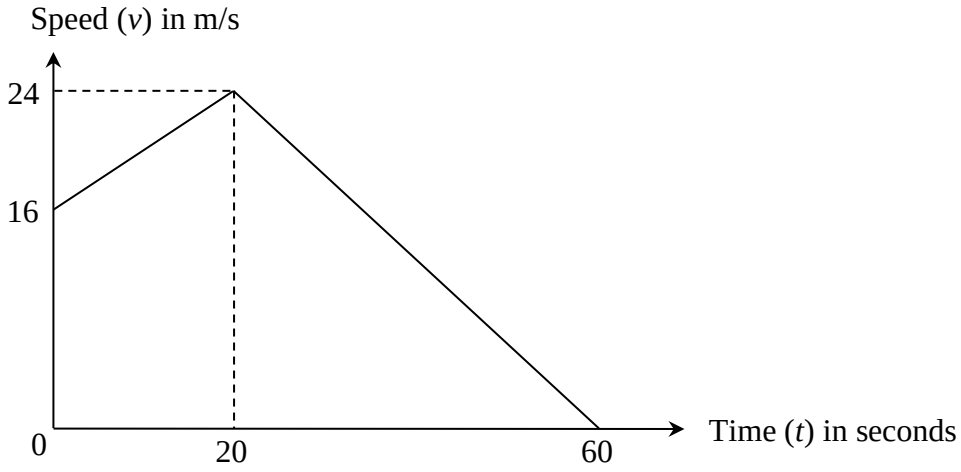
(i) The perimeter of a reservoir on the map is 22.8 cm.
Find the actual perimeter, in kilometres, of the reservoir. [2]

$1\,\text{cm} : 50000\,\text{cm}$ $1\,\text{cm} : 0.5\,\text{km}$ $\text{actual perimeter} = 0.5 \times 22.8$ $= 11.4\,\text{km}$	 M1 A1

(ii) The actual area of a plantation is 46 km^2.
Calculate the area, in square centimetres, of the plantation on the map. [2]

$(1\,\text{cm})^2 : (0.5\,\text{km})^2$ $1\,\text{cm}^2 : 0.25\,\text{km}^2$ $\text{map area} = \frac{46}{0.25}$ $= 184\,\text{cm}^2$	M1: area scale A1

5 The diagram shows the speed-time graph of a particle over a period of 60 seconds.



(a) Describe what is happening between $t = 20$ and $t = 60$. [1]

The particle is decelerating.	B1

(b) Calculate the acceleration of the particle in the first 20 seconds. [1]

$\text{acceleration} = \dfrac{24-16}{20}$ $= 0.4\,\text{m/s}^2$	B1

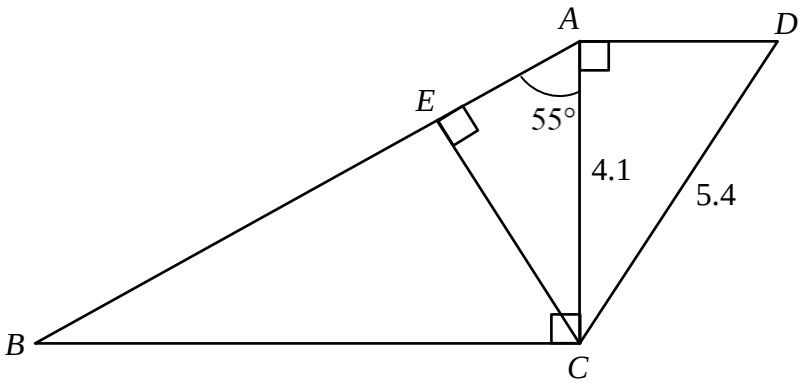
(c) Calculate the speed of the particle at $t = 45$. [2]

$\dfrac{v}{24} = \dfrac{15}{40}$	M1
$v = \dfrac{15}{40} \times 24$ $= 9\,\text{m/s}$	A1

(d) The area under the graph represents the total distance travelled. Calculate the total distance travelled by the particle. [2]

$\text{total distance} = \dfrac{1}{2}(16+24)(20) + \dfrac{1}{2}(40)(24)$ $= 880\,\text{m}$	M1: area of triangle/ trapezium A1

6 In the diagram, angle AEC = angle ACB = angle CAD = $90°$.
Angle $EAC = 55°$, AC = 4.1 cm and CD = 5.4 cm.



Calculate

(a) the length of AD, [2]

$AD = \sqrt{5.4^2 - 4.1^2}$	M1
$= 3.51425$	
$= 3.51$ cm	A1

(b) the length of AB, [2]

$\cos 55° = \frac{4.1}{AB}$	M1
$AB = \frac{4.1}{\cos 55°}$	
$= 7.1481$	
$= 7.15$ cm	A1

(c) the angle ADC. [2]

$\sin \angle ADC = \frac{4.1}{5.4}$	M1
$\angle ADC = \sin^{-1}\left(\frac{4.1}{5.4}\right)$	
$= 49.3989$	
$= 49.4°$	A1

7 The table shows the number of times students of a class were late for school in a month.

Number of times	0	1	2	3	4
Number of students	6	12	3	8	p

(a) State the largest value of p if the mode is 1. [1]

11	B1

(b) Find the value of p if the probability of choosing a student who was late more than 2 times is 0.5. [2]

$\frac{8+p}{29+p} = \frac{1}{2}$ $2(8+p) = 29+p$ $16+2p = 29+p$ $p = 13$	M1 A1

(c) It is given that $p = 11$.

(i) Find the mean. [2]

$\text{mean} = \frac{0(6)+1(12)+2(3)+3(8)+4(11)}{40}$ $= \frac{86}{40}$ $= 2.15$	M1 A1

(ii) A student's record was left out of the table. This student was late 3 times in a month. The table is now updated to include this student's record. Without calculation, explain how the mean will be affected. [1]

The mean will **<u>increase</u>** as the student's record of 3 times is **<u>more than the mean</u>**.	B1

8 **(a)** Lightning strikes the earth approximately 1.4 billion times per year.

(i) Write 1.4 billion in standard form. [1]

1.4×10^9	B1

(ii) Calculate the average number of times the lightning strikes the earth in a day, assuming there are 365 days in a year.
Write your answer in standard form, correct to 3 significant figures. [2]

$\dfrac{1.4\times10^9}{365}$ $=3.84\times10^6$	M1 A1

(b) Solve the simultaneous equations. [3]

$$5x-2y=16$$
$$x+3y=-7$$

$(2):\ x=-7-3y\cdots(3)$ subs (3) into (1) $5(-7-3y)-2y=16$ $-35-15y-2y=16$ $-17y=51$ $y=-3$ $x=-7-3(-3)=2$	 M1 A1 A1

(c) Solve $3x^2-5x-1=0$.
Give your answers correct to 2 decimal places. [3]

$x=\dfrac{-(-5)\pm\sqrt{(-5)^2-4(3)(-1)}}{2(3)}$ $x=1.8471$ or -0.18046 $=1.85$ or -0.18	M1 A1, A1

9 **(a)** Complete the table of values for $y = x^3 - 9x$. [2]

x	–3	–2	–1.5	–1	0	1	1.5	2	3	4
y	0	10		8	0	–8	–10.125		0	28

10.125, –10	B1, B1

(b) On the grid provided, plot the graph of $y = x^3 - 9x$ for $-3 \le x \le 4$. [3]
Answer on next page.

All points plotted correctly. 4 points plotted correctly. Smooth curve	B2 B1 B1

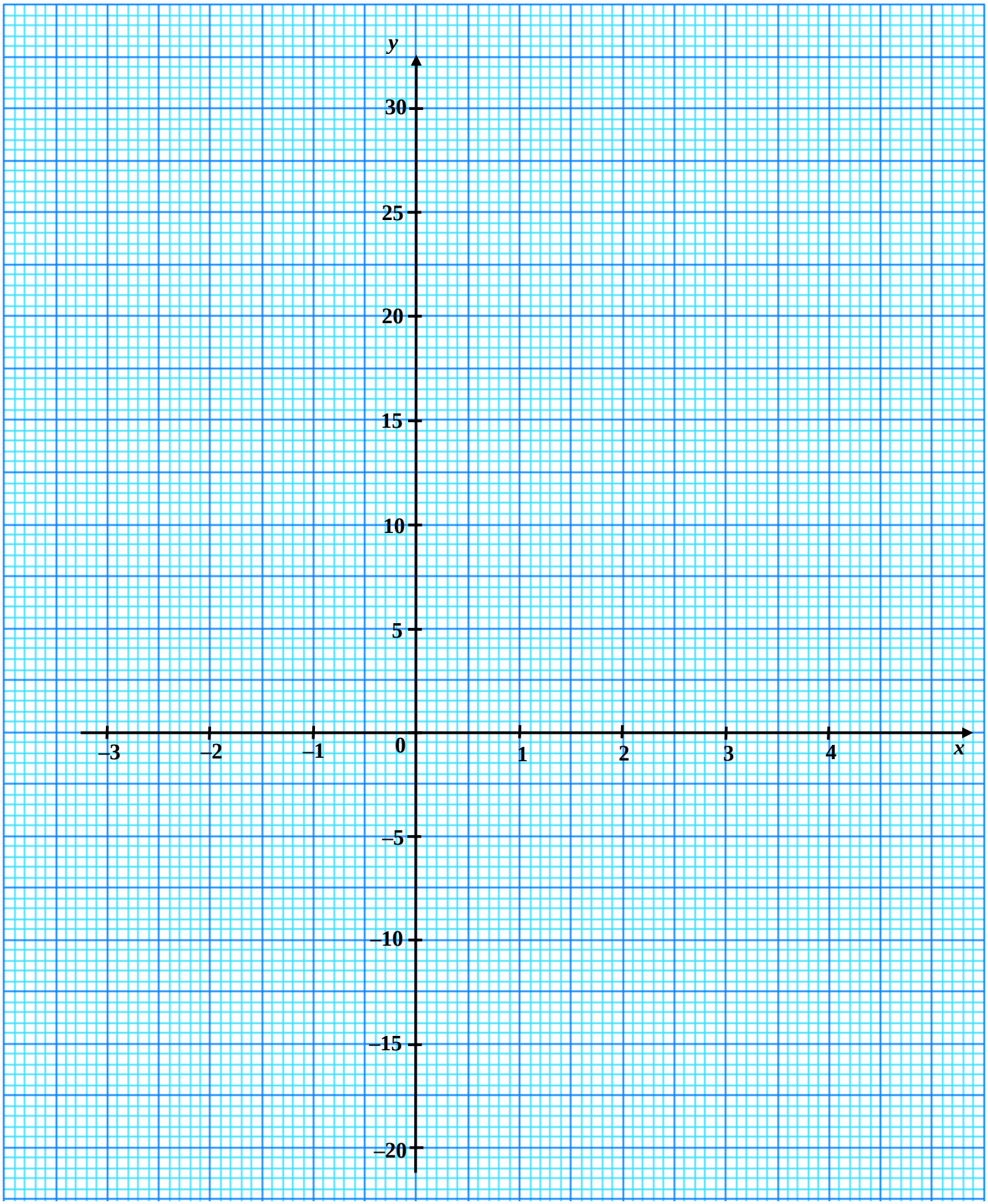
(c) Use your graph to find the values of x when $y = 5$. [2]

–2.65, –0.6, 3.25 (range ± 0.1)	B1 for any two correct B2 for 3 correct

(d) By drawing a tangent, find the gradient of the curve $y = x^3 - 9x$ when $x = 2.5$. [2]

$(1.5, -16.5), \quad (4, 7.5)$ $m = \dfrac{7.5 - (-16.5)}{4 - 1.5}$ $= 9.6$	B1 for tangent at $x = 2.5$ B1 for gradient Accept range: 9 – 10.5

Answer **9(b)**



10 The rescue base detected that a group of mountain climbers were stranded on a mountainous area, 230 km away from the base.

A rescue helicopter is sent out for the rescue mission. The rescue helicopter travels at a speed of 306 km/h.

(a) Find the time, in minutes, the helicopter takes to travel 230 km to reach the mountain climbers. [2]

$\text{time} = \frac{230}{306}$ $= \frac{115}{153}\text{h}$ $= 45.098$ $= 45.1 \text{ min}$	M1 A1

Useful information	
Fuel Tank Capacity	760 gallons
Fuel Consumption	576 litres/ hour
Estimated time taken for lift off	2 minutes
Estimated time taken to land	3 minutes
Estimated time taken to rescue one person	10 minutes
Source: Sikorsky-S92-multi-mission-helicopter-brochure.	

(b) Given that 1 gallon = 3.785 litres, calculate the fuel tank capacity, in litres, of the helicopter. [1]

$760 \times 3.785 = 2876.6$ litres	B1

10 **(c)** Calculate the time taken, in hours, for the helicopter to run out of 760 gallons of fuel.

time for the fuel to run out $= \frac{2876.6}{576}$ $= 4.99409$ $= 4.99\text{h}$	 B1

(d) The helicopter is unable to land on the mountainous terrain and has to hover for the rescue mission. It can carry 8 passengers in one trip.

Assume the helicopter leaves the rescue base with a full fuel tank, does the helicopter have sufficient fuel for the rescue mission?
Show working to support your answer. [4]

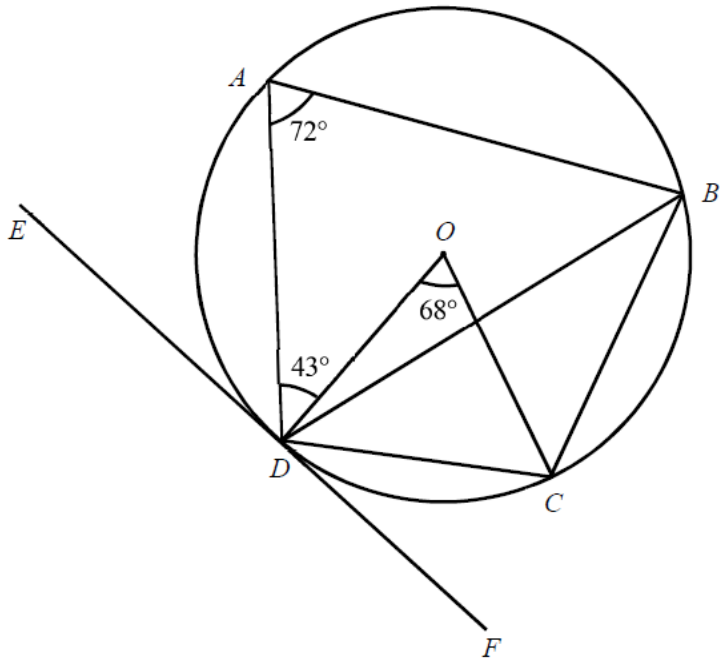
Answer

time for the fuel to run out $= 4.99409\text{ h}$ $= 299.6454\,\text{min}$	
time to lift off, to and fro and land for 1 trip $= 2 + 45.098 + 45.098 + 3$ $= 95.196\text{ min}$	M1
time to lift off, to and fro and land for 2 trips $= 95.196 \times 2$ $= 190.392\text{ min}$	M1
time to rescue 12 persons $= 10 \times 12$ $= 120\,\text{min}$	
total time required $= 190.392 + 120$ $= 310.392\,\text{min}$	M1
Since the time required for the two trips is **more** than the time for the fuel to run out, the helicopter does not have sufficient fuel.	A1

Section B (8 marks)

Answer **one** question from this section. Each question carries 8 marks.

11 **(a)** *A*, *B*, *C* and *D* are points on the circle, centre *O*. *EF* is a tangent to the circle at *D*. Angle *ODA* = 43°, angle *BAD* = 72° and angle *COD* = 68°.



Stating the reasons clearly, find

(i) angle *CBD*, [1]

$\angle CBD = 68° \div 2 \quad (\angle \text{ at centre}=2\ \angle \text{at circumference})$ $= 34°$	B1

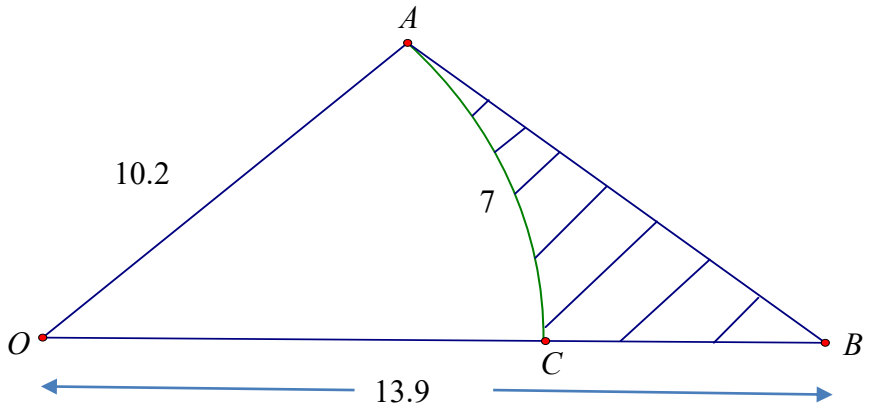
(ii) angle *ABC*, [2]

$\angle ODC = \dfrac{180° - 68°}{2} \quad (\angle \text{sum of isos } \Delta)$ $= 56°$ $\angle ABC = 180° - 56° - 43° \ (\angle \text{in opp segment})$ $= 81°$	M A1

(iii) angle *ADE*. [1]

$\angle ODE = 90° \quad (\tan \perp \text{rad})$ $\angle ADE = 90° - 43°$ $= 47°$	 B1

(b) The figure shows a triangle *OAB*.
AC is the arc of a circle with centre *O* and radius *OA*.
OA = 10.2 cm and *OB* = 13.9 cm. The length of the arc *AC* is 7 cm.



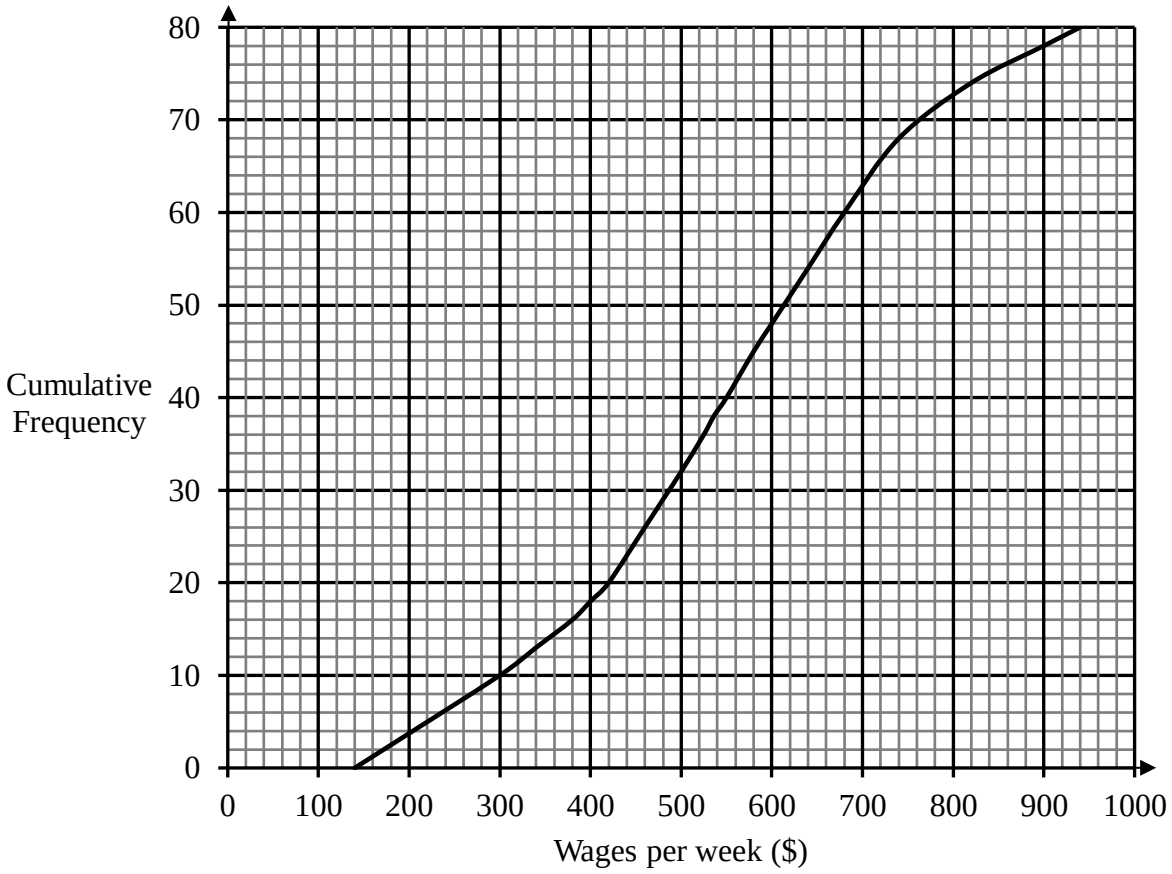
(i) Show that angle *AOB* is 0.686 radian. [1]

$7 = 10.2(\angle AOB)$ $\angle AOB = 0.68627$ $= 0.686$ radian	B1

(ii) Calculate the area of the shaded region. [3]

area of sector $= \frac{1}{2}(10.2)^2(0.68627)$ $= 35.6997$	M1
area of $\Delta = \frac{1}{2}(10.2)(13.9)\sin(0.68627)$ $= 44.9198$	M1
shaded area $= 44.9198 - 35.6997$ $= 9.22$ cm^2	A1

12 **(a)** The cumulative frequency graph shows the wages per week of 80 workers in a shipyard.



Use the graph to

(i) estimate the median wage per week, [1]

$550	B1

(ii) estimate the interquartile range, [1]

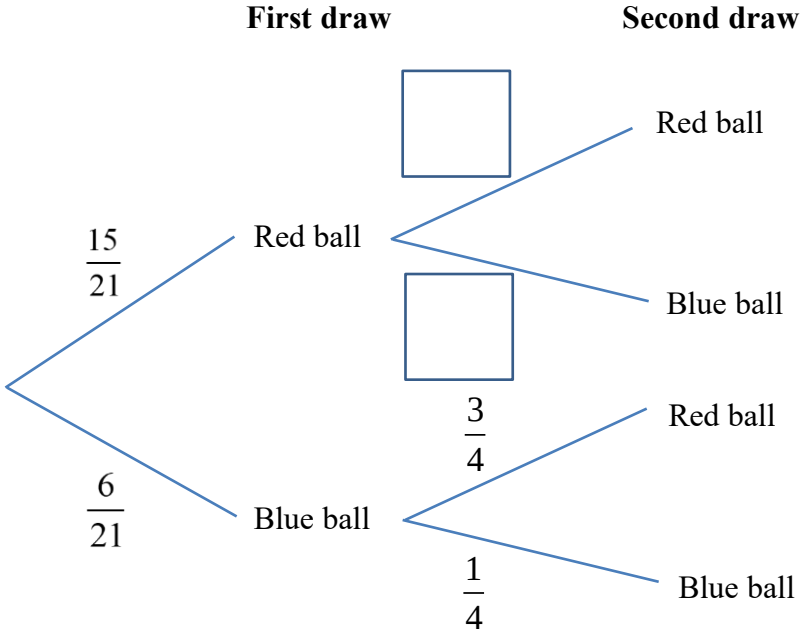
680 – 420 =$260	B1

(iii) find the percentage of workers who earn more than $460 a week. [1]

54 workers who earn more than $460. $\frac{54}{80} \times 100 = 67.5\%$	B1

12 **(b)** A box contains 15 red balls and 6 blue balls. Two balls are drawn at random from the box **without** replacement.

(i) Complete the tree diagram.



[1]

Red ball: $\frac{14}{20}=\frac{7}{10}$ Blue ball: $\frac{6}{20}=\frac{3}{10}$	B1 for both answers

(ii) Find the probability that the second ball drawn is blue. [2]

$\frac{2}{7}\times\frac{1}{4}+\frac{5}{7}\times\frac{3}{10}$ $=\frac{2}{7}$	M1

(iii) Find the probability that both balls drawn are different in colour. [2]

$\mathrm{P(R, B)}+\mathrm{P(B, R)}$ $=\frac{5}{7}\times\frac{3}{10}+\frac{2}{7}\times\frac{3}{4}$ $=\frac{3}{7}$	M1

End of Paper