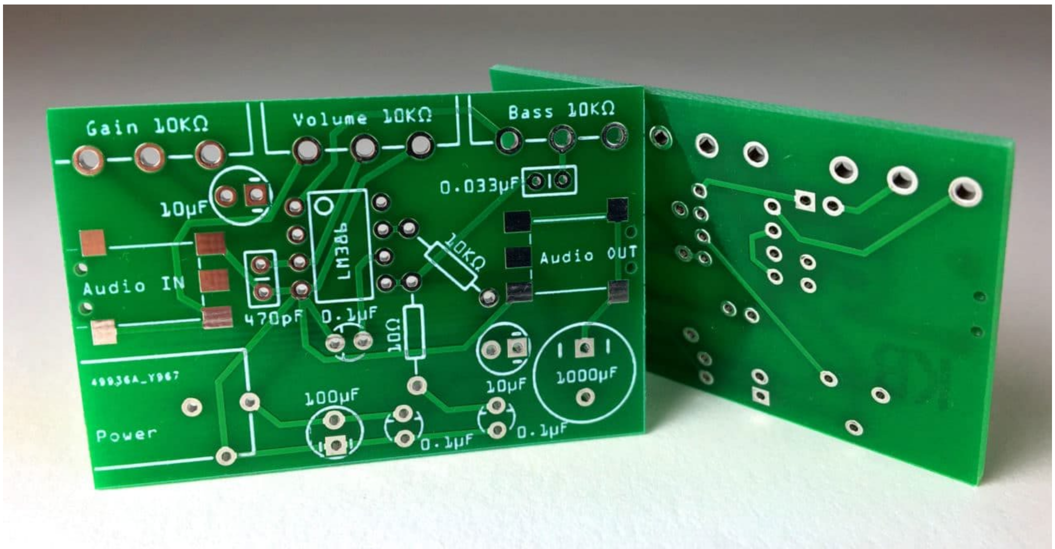
H2 Topic 15 – D.C. Circuits



A simple printed circuit board (PCB). *A PCB is a mechanical support for the layout and electrical connection between electronic components. A PCB can have many layers sandwiched together, which allow many different connections between components within a small physical area.*

Content

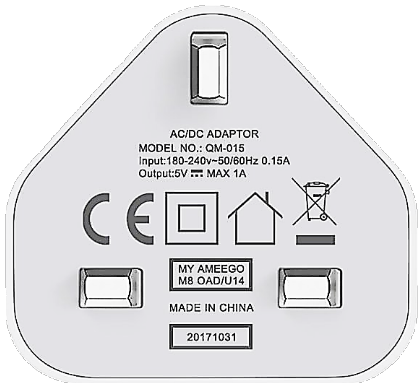
- Circuit symbols and diagrams
- Series and parallel arrangements
- Potential divider
- Balanced potentials

Learning Outcomes
Candidates should be able to:

(a) recall and use appropriate circuit symbols as set out in the ASE publication *Signs, Symbols and Systematics (The ASE Companion to 16–19 Science, 2000)*
(b) draw and interpret circuit diagrams containing sources, switches, resistors, ammeters, voltmeters, and/or any other type of component referred to in the syllabus
(c) solve problems using the formula for the combined resistance of two or more resistors in series
(d) solve problems using the formula for the combined resistance of two or more resistors in parallel
(e) solve problems involving series and parallel circuits for one source of e.m.f.
(f) show an understanding of the use of a potential divider circuit as a source of variable p.d.
(g) explain the use of thermistors and light-dependent resistors in potential divider circuits to provide a potential difference which is dependent on temperature and illumination respectively
(h) recall and solve problems by using the principle of the potentiometer as a means of comparing potential differences

15.0 Introduction

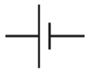
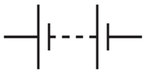
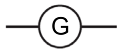
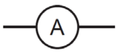
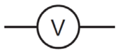
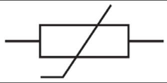
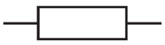
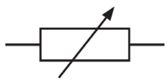
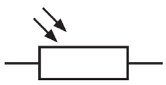
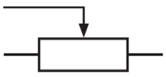
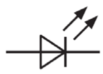
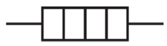
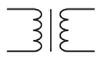
A direct current (D.C.) circuit is one where the direction of flow of current is maintained in the same direction. Such circuits are the basis for most (if not) all digital devices – the "power brick" connected to our laptops and the "mobile phone chargers" convert household alternating current (A.C.) electricity into D.C. for use by the devices.



A typical AC/DC adaptor. *This particular "handphone charger" converts household electricity into a 5V D.C. source and supplies a maximum of 1 A to the device.*

Here, we learn how to represent real circuits using symbols and diagrams, and to apply conservation of (i) electric charge and (ii) energy when analysing circuits.

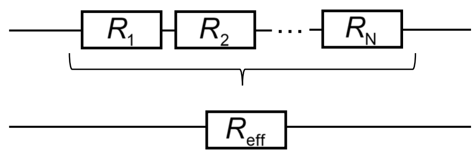
15.1 Circuit Symbols

cell		switch		junction of conductors	
battery of cells	 or 	earth		galvanometer (a sensitive ammeter)	 or 
ammeter		voltmeter		oscilloscope	
power supply		a.c. power supply		thermistor	
fixed resistor		variable resistor		light-dependent resistor (LDR)	
potentiometer		diode		light-emitting diode (LED)	
lamp		heater		transformer	

15.2.1 Resistors in Series

When 2 or more resistors are connected in series, the effective resistance R_{eff} is the sum of the resistances.

The electric current only has 1 possible path to flow. By conservation of charge, the same current flows through each of the resistors.

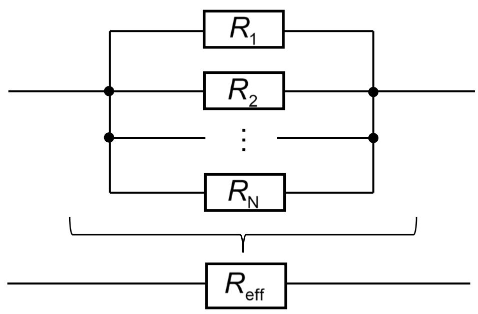


$$R_{eff} = R_1 + R_2 + \ldots + R_N$$

15.2.2 Resistors in Parallel

The potential difference (p.d.) across each resistor that is connected in parallel is the same.

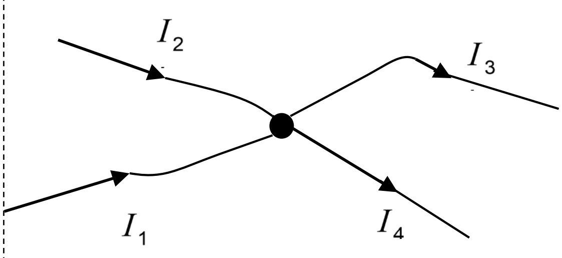
Resistors that are connected in parallel will result in *wire junctions* at which the current splits up and re-combines where the branches meet again.



$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N}$$

15.3 Junction Law

Total current flowing into a junction must be equal to the total current flowing out of it. It is a consequence of charge conservation.
It is also known as Kirchhoff's 1st Law.

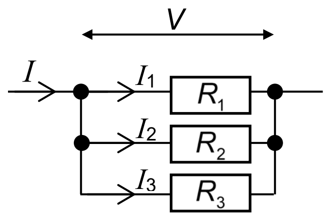


In the figure on the right, $I_1 + I_2 = I_3 + I_4$

Example 1

The total current in the combination of resistors is I and the p.d. across the combination is V. Show that the total resistance R of the combination shown is given by the equation

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$$



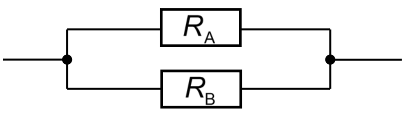
Solution

By conservation of charge, $I = I_1 + I_2 + I_3$

so $\frac{V}{R}=\frac{V}{R_1}+\frac{V}{R_2}+\frac{V}{R_3}$

since p.d. is same across parallel branches, $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$

Example 2



Resistors A and B are connected in parallel.
Express the effective resistance in terms of R_A and R_B.

Solution

$$\frac{1}{R_{eff}}=\frac{1}{R_A}+\frac{1}{R_B}=\frac{R_B+R_A}{R_AR_B}$$

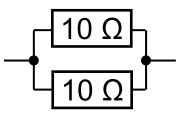
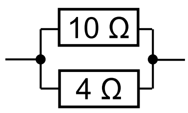
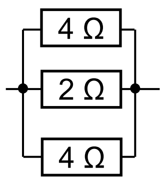
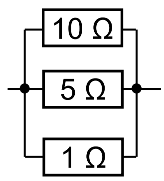
$$R_{eff}=\left(\frac{R_B+R_A}{R_AR_B}\right)^{-1}=\frac{R_BR_A}{R_A+R_B}$$

Note: Only for a **pair** of resistors in parallel, R_{eff} is equal to $\frac{\text{product of resistances}}{\text{sum of resistances}}$.

Example 3

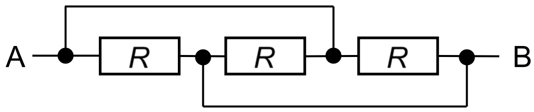
Find the effective resistance of the following sets of resistors connected in parallel.

Solution

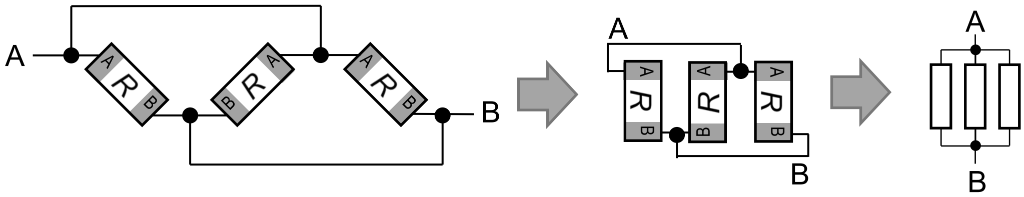
<table>
<tr><td>Layout</td><td></td><td></td><td></td><td></td></tr>
<tr><td>R_{eff}</td><td>5 Ω</td><td>2.86 Ω</td><td>1 Ω</td><td>0.769 Ω</td></tr>
<tr><td rowspan="2">Notes</td><td rowspan="2">Identical resistors paired in parallel: R_{eff} is halved
$R_{eff}=\frac{R^2}{2R}=\frac{R}{2}$</td><td>$R_{eff}=\frac{R_BR_A}{R_A+R_B}$</td><td>if convenient, use $R_{eff}=\frac{R_BR_A}{R_A+R_B}$ multiple times</td><td>neater to apply $\frac{1}{R_{eff}}=\frac{1}{R_1}+...+\frac{1}{R_N}$</td></tr>
<tr><td colspan="3">R_{eff} is lower than lowest individual resistor in parallel branch</td></tr>
</table>

Example 4

What is the effective resistance between A and B?



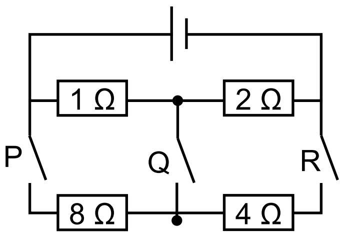
Solution



The 3 identical resistors are in parallel so $\dfrac{1}{R_{\text{eff}}} = \dfrac{1}{R} + \dfrac{1}{R} + \dfrac{1}{R} = \dfrac{3}{R}$, $R_{\text{eff}} = \dfrac{R}{3}$

Note: For N number of identical resistors R, $R_{\text{eff}} = \dfrac{R}{N}$

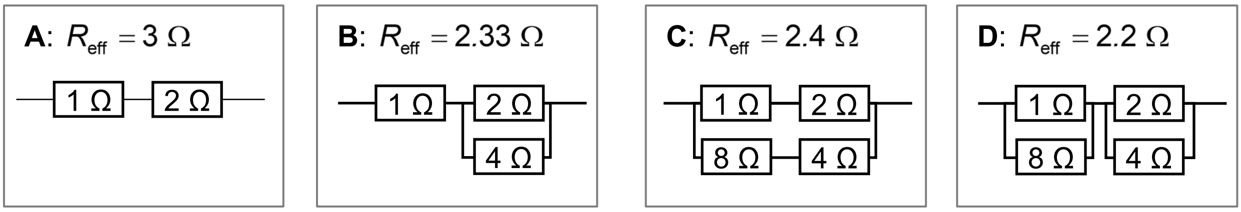
Example 5



Which of the following switch settings will result in the largest current in the circuit?

	P	Q	R
A	open	open	open
B	open	closed	closed
C	closed	open	closed
D	closed	closed	closed

Solution



Option **D** has the lowest $R_{\text{eff}} = 2.2\ \Omega$, so largest current in circuit.

15.4 Circuit Analysis

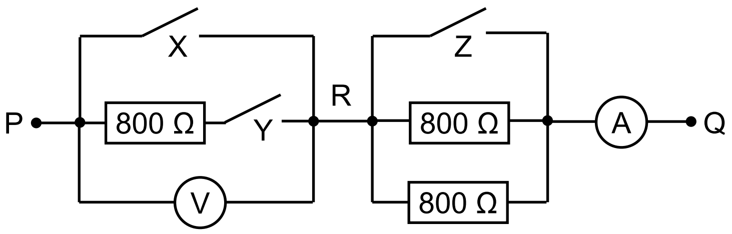
Circuits can look complex. There are several guidelines we can refer to when analysing circuits:

a. Check if it is a "closed circuit" or a circuit-segment where (conventional) current can flow from high electric potential to low electric potential.
b. Identify segments of the circuit where combinations of resistors can be simplified as effective resistances.
c. When comparing power output across components, useful to determine which quantity is kept constant (e.g. potential difference across resistors in parallel or current value through resistors in series)
d. Where it is useful to think about potential differences across different parts the circuit, <u>one</u> single earth point can be added to the circuit to arbitrarily assign a zero potential value.

Example 6

The potential at P is +10 V. The potential at Q is +4 V.

Which option <u>inaccurately</u> describes the reading on the voltmeter and ammeter?



	X	Y	Z	V / V	A / mA
A	closed	open	open	0	15
B	open	open	open	6	0
C	open	closed	closed	6	7.5
D	closed	closed	open	0	5

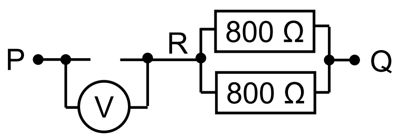
Solution

Option **D** is inaccurate.

The potential *difference* between P and Q, $V_{PQ} = V_P - V_Q = (+10) - (+4) = 6$ V

Since P is at a higher potential, (conventional) current flows from P to Q *if* there is uninterrupted path

Option **B** is accurate:

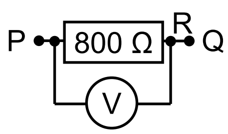


An ideal voltmeter provides infinite resistance, PQ is a broken path so zero current.

No current so the potential *drop* across the parallel resistors is zero ($V_{//} = IR_{//} = 0$). So potential *at* R is same as potential *at* Q.

Voltmeter reads the potential *difference* between P and R, 6 V.

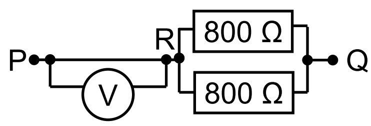
Option **C** is accurate:



R has the same potential as Q since Z is closed. Hence voltmeter reads the potential difference between P and R, V_{PR}: 10 – 4 = 6 V

$$V_{PR} = IR$$
$$6 = I(800)$$
$$I = 0.0075 \text{ A} = 7.5 \text{ mA}$$

In both Options **A** and **D**, the LHS resistor is shorted by X:



Voltmeter reads zero.

$$V_{PQ} = IR \text{ so } I = \frac{6}{400}$$
$$= 0.015 \text{ A} = 15 \text{ mA}$$

Therefore **D** is inaccurate.

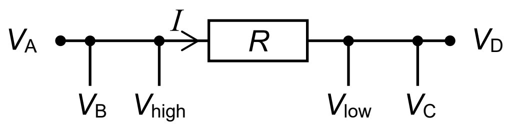
Notes: To *short* a component is to provide a path of zero resistance across it, usually via a wire.

A *shorted* resistor can be ignored and treated as a connected wire segment.

A *shorted* battery is dangerous because all available e.m.f. is applied as p.d. across the internal resistance, which causes rapid Joule heating within the battery physically, risking a fire hazard.

15.5 Potential, Potential Difference and Earth Points

A potential *difference* (p.d.) can only be established using 2 points in a circuit. When a component of some resistance is part of a working circuit, (conventional) current flows into component from a higher potential and flow out of component at lower potential, hence "p.d. *across* the component". All points along a wire segment have the same potential as a connecting wire is assumed to have no resistance.



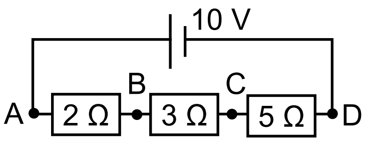
$V_A = V_B = V_{high}$

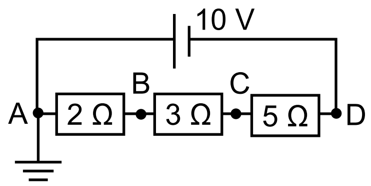
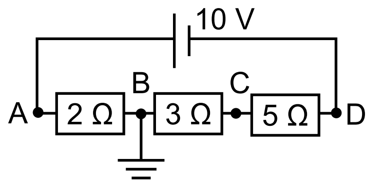
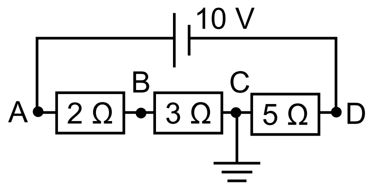
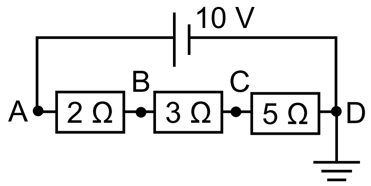
$V_C = V_D = V_{low}$

$V_{high} > V_{low}$

p.d. across $R = \Delta V = IR = \left(V_{high} - V_{low}\right) = \left(V_A - V_D\right)$

In closed circuits, the p.d.'s are fixed by the e.m.f. source and the resistances of components. Often, an earth point (0 V) can *arbitrarily be* assigned to simplify the analysis of potential differences.

Circuit		$V_{2\Omega} = IR_{2\Omega}$	$V_{3\Omega} = IR_{3\Omega}$	$V_{5\Omega} = IR_{5\Omega}$	E
	$R_{total} = 2 + 3 + 5 = 10\ \Omega$ $I = \dfrac{E}{R_{total}} = 1\text{ A}$	2 V	3 V	5 V	10V

	V_A	V_B	V_C	V_D	$V_{2\Omega} = V_A - V_B$	$V_{3\Omega} = V_B - V_C$	$V_{5\Omega} = V_C - V_D$	$E = V_A - V_D$
	earth 0 V	-2 V	-5 V	-10 V	0 – (-2) = 2 V	(-2)-(-5) = 3V	(-5)-(-10) = 5 V	0 – (-10) = 10 V
	2 V	earth 0 V	-3 V	- 8V	2 – 0 = 2 V	0-(-3) = 3V	(-3)-(-8) = 5 V	2 – (-8) = 10 V
	5 V	3 V	earth 0 V	- 5 V	5 – 3 = 2 V	3 – 0 = 3V	0 – (-5) = 5 V	5 – (-5) = 10 V
	10 V	8 V	5 V	earth 0 V	10 – 8 = 2 V	8 – 5 = 3V	5 – 0 = 5 V	10 – 0 = 10 V

Notes:

(i) The potential *values* are arbitrary but the potential *differences* are invariant.

(ii) The net e.m.f around a closed circuit loop equals the sum of potential *drops* around the loop. It is a consequence of energy conservation. It is also known as Kirchhoff's 2nd Law.

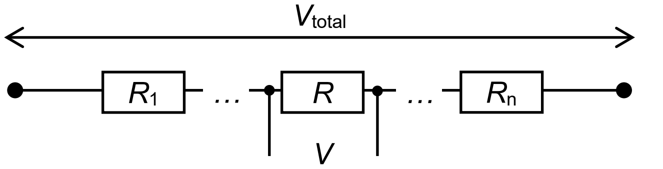
(iii) Along direction of current flow, potential drops across a resistance

(iv) The positive terminal of a source of e.m.f. is higher in potential than the negative end.

15.6 Potential Divider

A potential divider circuit provides a source of variable potential difference.

The ratio of resistances determines the proportion of the total p.d. that is output.

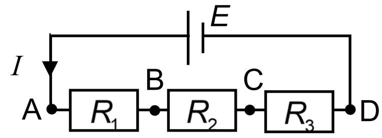


By potential divider rule,

$$\frac{V}{V_{total}} = \frac{R}{R_{total}}$$

Example 7

For the circuit shown,



(a) express the current I in terms of the e.m.f. E and the resistances,

(b) express the p.d. between points A and B in terms of I, E and the resistances,

(c) determine the p.d. in (b) if $E = 9$ V, $R_1 = 2\ \Omega$, $R_2 = 1\ \Omega$ and $R_3 = 3\ \Omega$.

Solution

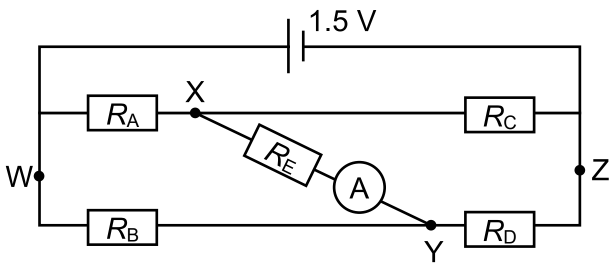
(a) $I = \dfrac{E}{R_1 + R_2 + R_3}$ (b) $V_{AB} = I(R_1) = E\left(\dfrac{R_1}{R_1 + R_2 + R_3}\right)$ (c) $E\left(\dfrac{R_1}{R_1 + R_2 + R_3}\right) = 9\left(\dfrac{2}{2+1+3}\right) = 3$ V

Note: Answer to (b) is a form of potential divider rule $V_{AB} = E\left(\dfrac{R_1}{R_1 + R_2 + R_3}\right) \to \dfrac{V_{AB}}{E} = \dfrac{R_1}{R_1 + R_2 + R_3}$

Example 8

(a) For R_A = 2 Ω, R_B = 8 Ω, R_C = 1 Ω, R_D = 4 Ω and R_E = 3 Ω, find ammeter reading.

(b) Find the effective resistance across WZ.



Solution

e.m.f = $V_W - V_Z = 1.5$ V

We *choose* to earth point Z, $V_Z = 0$ V

(a) By potential divider rule,

$$\frac{V_{XZ}}{1.5} = \frac{R_C}{R_A + R_C} = \frac{1}{3}$$

$$V_{XZ} = 0.5 \text{ V}$$

so potential at X, $V_X = V_Z + \Delta V = 0 + V_{XZ}$

$= 0 + 0.5 = 0.5$ V

By potential divider rule,

$$\frac{V_{YZ}}{1.5} = \frac{R_D}{R_B + R_D} = \frac{4}{12}$$

$$V_{YZ} = V_Y = 0.5 \text{ V}$$

p.d. between X and Y is zero so zero current.

Tip: If $\dfrac{R_A}{R_C} = \dfrac{R_B}{R_D}$, then $V_X = V_Y$

=> $V_{XY} = 0$ and $I_{XY} = 0$

(b) From part (a), since $I_{XY} = 0$, we can consider a circuit where branch XY does not exist.
i.e. R_E is inconsequential in this circuit.

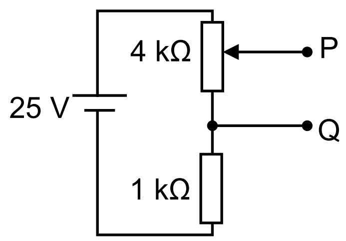
Effective $R_{WZ} = \dfrac{3 \times 12}{3 + 12}$

$= 2.4\ \Omega$

Note: Practise alternative solutions by earthing other points in the circuit.

Example 9

Find the range of potential difference between points P and Q.



Solution

By potential divider rule,

$$\frac{V_{\text{PQ}}}{25}=\frac{R_{\text{var}}}{R_{\text{total}}}$$

R_{var} set to zero,

$$\frac{V_{\text{PQ}}}{25}=\frac{0}{5}$$
$$V_{\text{PQ}}=0\text{ V}$$

R_{var} set to 4 kΩ,

$$\frac{V_{\text{PQ}}}{25}=\frac{4}{4+1}$$
$$V_{\text{PQ}}=20\text{ V}$$

p.d. between P and Q ranges from 0 V to 20 V

Note: Using variable resistors allow a *continuous* range of p.d. output from potential divider circuits. Else, the output p.d. will be *discrete fixed* values determined by resistance values of resistors used.

Substituting the variable resistor with thermistors (recall NTC (negative temperature coefficient) thermistors) and light-dependent resistors (LDRs) allows potential divider circuits to respond to changes in temperature and/or light. A reminder from H214 Current of Electricity:

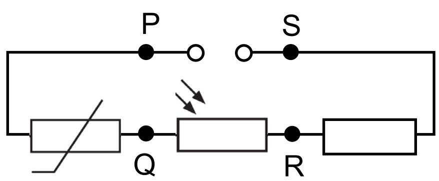
- A NTC thermistor has resistance which *decreases* as temperature *increases*.
- A LDR has resistance which typically *decreases* as the intensity of light incident on it *increases*.

Example 10

The circuit below controls an air-con cooler unit. The air-con accepts and compares 3 p.d. signals namely V_{signal}, V_{ref1} and V_{ref2} . The air-con starts operating when V_{signal} is highest of the 3 signals, and does not operate otherwise. How best should V_{signal}, V_{ref1} and V_{ref2} be connected to P, Q, R and S if

(i) an admin staff wishes for the air-con in the staff room to be operating whenever it is warm only,

(ii) a teacher wishes for the air-con in a classroom to operate only when it is warm and bright.



	V_{signal}	V_{ref1}	V_{ref2}
A	RS	PR	QS
B	RS	PQ	QR
C	QS	PR	RS
D	QS	PQ	QR

Solution

(i) Option **D** is most suitable.

	V_{signal}	V_{ref1}	V_{ref2}
D	QS	PQ	QR

$V_{\text{signal}}>V_{\text{ref2}}$ under all conditions as $V_{\text{QS}}=V_{\text{QR}}+V_{\text{RS}}$
As a thermistor has low resistance when warm, $V_{\text{ref1}}=V_{\text{PQ}}$ is small. Alternatively can be viewed as $V_{\text{signal}}=V_{\text{QS}}=E-V_{\text{PQ}}$ so V_{signal} is likely largest

(ii) Option **B** is most suitable.

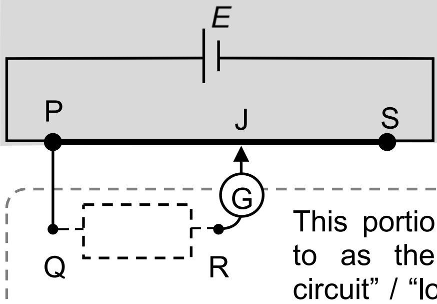
	V_{signal}	V_{ref1}	V_{ref2}
B	RS	PQ	QR

Both thermistor and LDR has low resistance as conditions are warm and bright.

Note: The circuit arrangements may fail if resistance of fixed resistor is not significantly larger than that of the thermistor under "warm" conditions and the LDR under "bright" conditions.

15.7 Potentiometer Circuits

Potentiometer circuits *compare* potential differences. It allows potential differences to be more accurately measured than by directly using a voltmeter, as real voltmeters have finite resistances.



This portion is referred to as the "secondary circuit" / "lower circuit" / "slave circuit".

As we are typically interested in the p.d. V_{QR}, it is necessary that $V_{\text{QR}} \leq V_{\text{PS}}$.

A potentiometer circuit consists of a "primary circuit" / "driver circuit" / "master circuit".

The primary circuit usually consists of an e.m.f. source providing p.d. across a long resistance wire of length L. The resistance wire is usually denoted by thicker lines, as per PS in the diagram on the left.

V_{QR} is usually the p.d. we are interested in accurately measuring. By potential divider rule

$$\frac{V_{\text{PJ}}}{V_{\text{PS}}} = \frac{R_{\text{PJ}}}{R_{\text{PS}}}$$

15.7.1 The Balance Length

If resistance wire PS has uniform cross-sectional area, R_{PJ} will be directly proportional to length L_{PJ}:

$$R_{\text{PJ}} = \frac{\rho L_{\text{PJ}}}{A}$$

Then the ratio of length of the resistance wire determines the p.d. output via V_{PJ}:

$$\frac{V_{\text{PJ}}}{V_{\text{PS}}} = \frac{R_{\text{PJ}}}{R_{\text{PS}}} = \frac{\left(\frac{\rho}{A}\right) L_{\text{PJ}}}{\left(\frac{\rho}{A}\right) L_{\text{PS}}} = \frac{L_{\text{PJ}}}{L_{\text{PS}}}$$

For the example circuit above, $V_{\text{PJ}} = \dfrac{L_{\text{PJ}}}{L_{\text{PS}}}(E)$.

We sometimes call the p.d. output via PJ as the "tapped voltage".

For potentiometer circuits,

$$\frac{V_{\text{tapped}}}{V_{\text{total}}} = \frac{L_{\text{tapped}}}{L_{\text{total}}} = \frac{R_{\text{tapped}}}{R_{\text{total}}}$$

"J" is a *wire jockey.* The sharp edge is tapped perpendicularly and gently onto a wire mounted on a ruler so that the length is easily read. Even though sometimes referred to as a "sliding contact", we should not slide/scrap it along the wire to avoid spoiling the uniformity of the wire.

Under balanced conditions,

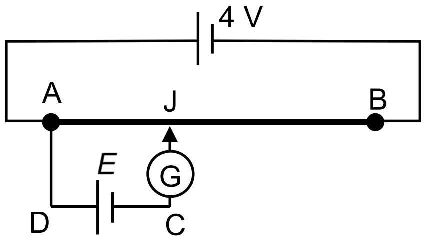
- $V_{\text{PJ}} = V_{\text{QR}}$
- $0 = I_{\text{PQ}} = I_{\text{RJ}}$
- Galvanometer shows null deflection (no current)

L_{PJ} is known as the "**balance length**"

A wire jockey and resistance wire on a ruler.

Example 11

AB is a 1.0 m long uniform wire of 8.0 Ω resistance.



a) The galvanometer shows null deflection when length $L_{AJ} = 0.4$ m. Determine the unknown e.m.f. E.

By potential divider rule in primary circuit, $\dfrac{V_{AJ}}{V_{AB}} = \dfrac{L_{AJ}}{L_{AB}}$

$$V_{AJ} = \frac{0.4}{1.0}(4) = 1.6 \text{ V}$$

under balance conditions, $E = V_{DC} = V_{AJ} = 1.6$ V

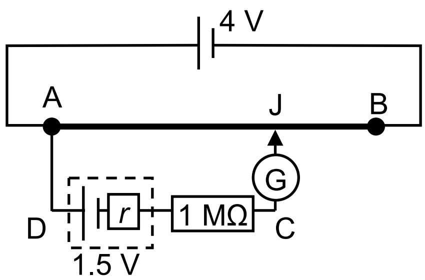
b) The unknown e.m.f. source has an internal resistance of 0.1 Ω.
Explain why the balance length will be the same in the circuit regardless of the internal resistance.

Under balance conditions, potential difference between AJ and DC is the same.
No current flows along ADCJ so no potential drop across internal resistance.
Terminal p.d. across is equal to the e.m.f., and is equal to p.d. across AJ.
Same balance length is obtained.

c) Determine the direction of current flowing through the unknown e.m.f. source if $L_{AJ} = 0.45$ m.

p.d. across AJ is greater than p.d. across DC so current flows from D to C

d) A protective resistor of 1 MΩ is connected in series with the galvanometer. The unknown e.m.f. source is replaced by a 1.5 V battery. Determine the new balance length.



Under balance conditions, no current flow along ADCJ so no potential drop across protective resistor.

By potential divider rule in primary circuit, $\dfrac{V_{AJ}}{V_{AB}} = \dfrac{L_{AJ}}{L_{AB}}$

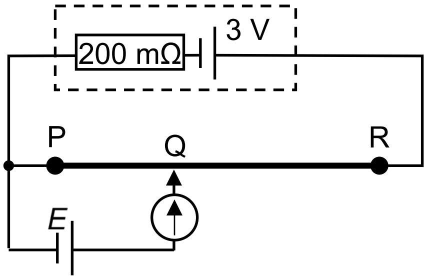
$$V_{AJ} = E = 1.5 = \frac{L_{AJ}}{1.0}(4)$$

$$L_{AJ} = 0.375 \text{ m}$$

Note: A very sensitive galvanometer is needed if precise locations of the jockey is to be made. A very large (typically in the order of MΩ) resistor is usually connected in series with the galvanometer to protect it from relatively-high currents under non-balance conditions.

Example 12

An iron rod PR has length 120 cm and a uniform cross-sectional area of 4.6×10^{-7} m^2. The resistivity of iron is 9.71×10^{-8} Ω m and the balance length is found to be 55 cm.



(a) Determine the current flowing through the rod.

(b) Determine the unknown e.m.f. E

(c) Find the new balance length if polarity of E is reversed.

(a) Resistance of PR:

$$R_{PR} = \frac{\rho L_{PR}}{A} = \frac{(9.71\times10^{-8})(1.2)}{4.6\times10^{-7}} = 0.25\ \Omega$$

$$R_{total} = r + R_{PR} = 0.200 + 0.25 = 0.45\ \Omega$$

$$I = \frac{V}{R} = \frac{E}{R_{total}} = \frac{3}{0.45} = 6.6\ \text{A}$$

(b) By potential divider rule in primary circuit:

$$\frac{V_{PR}}{3} = \frac{R_{PR}}{R_{total}}$$

$$V_{PR} = \frac{0.25}{0.45}(3) = 1.67\ \text{V}$$

By potential divider rule in resistance wire:

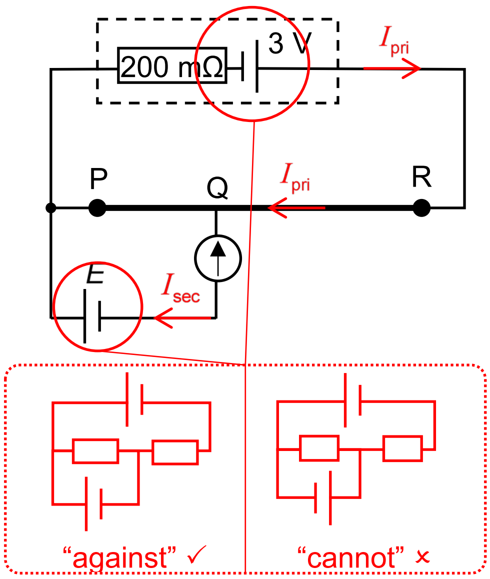
$$\frac{V_{PQ}}{V_{PR}} = \frac{E}{V_{PR}} = \frac{L_{PQ}}{L_{PR}}$$

$$E = \frac{L_{PQ}}{L_{PR}}V_{PR} = \frac{55}{120}(1.6667) = 0.76\ \text{V}$$

Note: (c) is impossible. In potentiometer circuits, polarities of the 2 e.m.f. sources must be "against" each other so that the differences in p.d. can be compared.

Current in primary circuit moves from R (higher potential) to P (lower potential). When E is reversed, current due to E moves from Q (higher potential) through the galvanometer,

The currents due to both e.m.f.'s will always flow in the same direction with respect to the galvanometer, which will never have null deflection.



It is practically advantageous to reduce percentage uncertainties in the measurement of the balance length as this increases the precision of the balance length measurement.

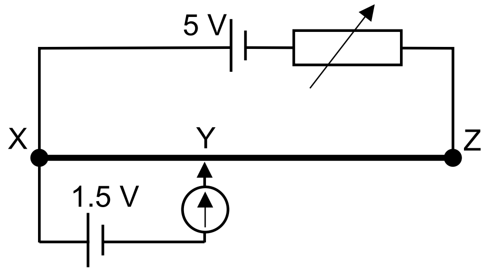
To increase the recorded value of balance length, reduce the p.d. across the resistance wire.

Doing so decreases the p.d. per unit length, hence a longer balance length is required to provide a given p.d.

A variable resistor can be connected in series with the resistance wire within the primary circuit to achieve this.

Example 13

XZ is a uniform resistance wire of length 1.0 m and of resistance 4.0 Ω. XZ is mounted on a metre rule.
A student estimates his jockey position reading to have an uncertainty of 0.5 cm. The variable resistor is initially set to a zero resistance.



(a) Express the balance length with its associated uncertainty.
(b) Determine the percentage uncertainty of the balance length measurement.
(c) Determine the maximum resistance of the variable resistor applicable in this set up.
(d) Determine a suitable resistance of the variable resistor in order for a new balance length measurement to have a percentage uncertainty of 0.60%.
(e) Suggest one other way of reducing the percentage uncertainty if a variable resistor is a fixed resistor instead.

Solution

(a) By potential divider rule

$$\frac{V_{XY}}{V_{XZ}} = \frac{L_{XY}}{L_{XZ}}$$

$$L_{XY} = \frac{V_{XY}}{V_{XZ}} L_{XZ} = \frac{1.5}{5}(1) = 0.3 \text{ m}$$

Balance length = 0.300 ± 0.005 m

(b) Percentage uncertainty:

$$\frac{\Delta L}{L_{XY}} \times 100\% = \frac{0.005}{0.300} \times 100\%$$

$$= 1.7\% \text{ (2 s.f.)}$$

(c) When variable resistance is at maximum allowable, Y is at Z.
P.d. across XZ, $V_{XY} = 1.5$ V
By potential divider rule in primary circuit:

$$\frac{V_{XZ}}{\varepsilon} = \frac{R_{XZ}}{R_{XZ} + R_{var}}$$

$$\frac{1.5}{5} = \frac{4}{4 + R_{var}}$$

$$R_{var} = 9.33 \ \Omega$$

(d)

new balance length L_{new}

$$0.60\% = \frac{0.005}{L_{new}} \times 100\%$$

$$L_{new} = 0.833 \text{ m}$$

by potential divider rule in wire

$$\frac{V_{XY}}{V_{XZ}} = \frac{L_{new}}{L_{XZ}}$$

$$V_{XZ} = V_{XY} \frac{L_{XZ}}{L_{new}}$$

$$= (1.5)\frac{1}{0.83333}$$

$$= 1.8 \text{ V}$$

by potential divider rule in primary circuit

$$\frac{V_{XZ}}{\varepsilon} = \frac{R_{XZ}}{R_{XZ} + R_{var}}$$

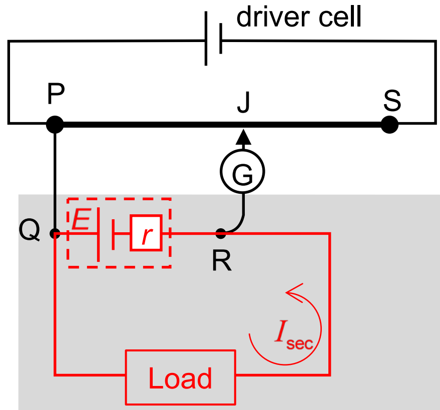
$$\frac{1.8}{5} = \frac{4}{4 + R_{var}}$$

$$R_{var} = 7.11 \ \Omega$$

(e) increase the length of the resistance wire (drawback: limited by length of ruler)
replace the resistance wire with another of lower *resistance* (either larger cross-sectional area of same material or lower resistivity with same cross-sectional area)

Note: To increase sensitivity of potentiometer circuit is to decrease potential gradient per unit length of the resistance wire. The potential gradient of XZ is 5 V m^{-1} when $R_{var} = 0$, and 1.5 V m^{-1} when $R_{var} = 9.33 \ \Omega$.

Besides *comparing* p.d.'s, potentiometer circuits can be used to find the internal resistance of an e.m.f. in the secondary circuit.



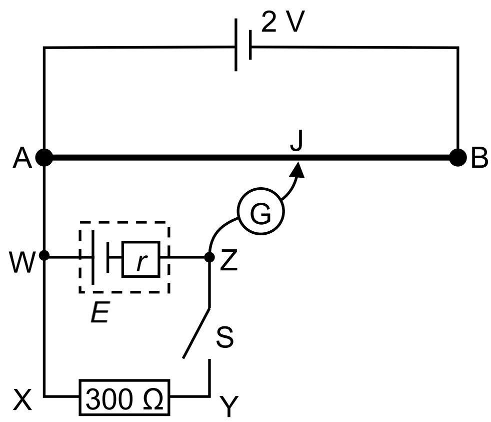
To do so,
1) find the e.m.f. in the secondary circuit
2) set up a current flow through the e.m.f. in the secondary circuit
3) apply the relationship $V_{terminal} = E - Ir$

The balance conditions still apply:

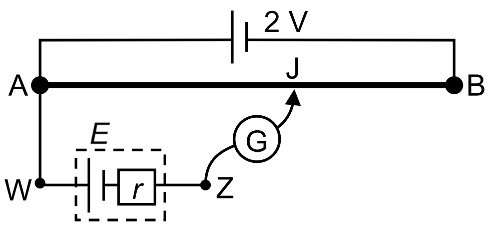
- $V_{PJ} = V_{QR}$
- $0 = I_{PQ} = I_{RJ}$
- Galvanometer shows null deflection

Example 14

AB is a uniform resistance wire of length 1.5 m and total resistance 8.0 Ω. When switch S is open, the balance length is found to be 1.05 m. When switch S is closed, the balance length is found to be 97.5 cm. Determine the internal resistance *r* of the unknown e.m.f. source in the secondary circuit.



switch S is open, WXYZ is not connected:



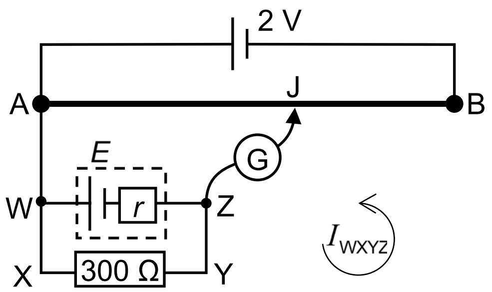
By potential divider rule:

$$\frac{V_{WZ}}{V_{AB}} = \frac{V_{AJ}}{V_{AB}} = \frac{L_{AJ}}{L_{AB}}$$

$$\frac{E}{2} = \frac{1.05}{1.5}$$

$$E = 2\left(\frac{1.05}{1.5}\right) = 1.4 \text{ V}$$

When switch S is closed



(cont'd) By potential divider rule:

$$\frac{V_{WZ}}{V_{AB}} = \frac{L_{WZ}}{L_{AB}}$$

$$\frac{V_{WZ}}{2} = \frac{0.975}{1.5}$$

$$V_{WZ} = 2\left(\frac{0.975}{1.5}\right) = 1.3 \text{ V}$$

the p.d. across $300\ \Omega$ resistor, $V_{XY} = V_{WZ}$

so $I = \dfrac{V_{XY}}{R} = \dfrac{1.3}{300} = 0.00433$ A

Considering secondary circuit WXZY:

$$E = I(r + R)$$

$$r = \frac{E}{I} - R$$

$$= \frac{1.4}{0.00433} - 300$$

$$= 23.1\ \Omega$$

15.8 Summary

15.8.1 Big Ideas

Direct Current (D.C.)	direction of flow of current is maintained in same direction
resistors in series	same current flows through resistors $$R_{\text{eff}} = R_1 + R_2 + ... + R_{\text{N}}$$
energy conservation	e.m.f. is equal to sum of potential drops within circuit loop
resistors in parallel	same p.d. across resistors current splits up when entering resistors and re-combine when exiting $$\frac{1}{R_{\text{eff}}} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_{\text{N}}}$$ R_{eff} is lower than lowest component resistor in parallel branch
charge conservation	total current flowing into a junction equals total current flowing out of it
pair of resistors in parallel	effective resistance is product-over-sum $R_{\text{eff}} = \frac{R_{\text{B}}R_{\text{A}}}{R_{\text{A}} + R_{\text{B}}}$
identical resistors in parallel	effective resistance is divided by number of resistors $R_{\text{eff}} = \frac{R}{N}$
short	a path of zero resistance
earth point	a point arbitrarily assigned as zero potential
potential divider rule	$$\frac{V_{\text{out}}}{V_{\text{total}}} = \frac{R}{R_{\text{total}}}$$
potentiometer	$$\frac{V_{\text{tapped}}}{V_{\text{total}}} = \frac{L_{\text{tapped}}}{L_{\text{total}}} = \frac{R_{\text{tapped}}}{R_{\text{total}}}$$
balance length	length of resistance wire tapped from primary circuit resulting in null deflection in galvanometer between primary and secondary circuit

15.8.2 Potentiometer Circuits

pros	• accurate comparison and/or measurement of p.d. in secondary circuit • (when balanced) can draw no current from p.d. source in secondary circuit, shows e.m.f. • depends on length-measurements and values of standard resistances/e.m.f. sources
cons	• slow, time needed to find balance length, cannot monitor rapidly changing p.d. • need for uniform resistance wire

15.8.2 Potentiometer Circuits Practical Details

e.m.f. polarity	driver cell "against" secondary e.m.f. source i.e. positive ends connected together
percentage uncertainty	increase balance length to reduce percentage uncertainty in measurement adjust resistances in primary circuit such that p.d. across resistance wire lowers
time of usage	switch off circuit when not in use. Avoid - resistance wire increasing resistance due to heating - e.m.f. of driver cell decreasing ("weakening")
jockey	tap gently and perpendicular to resistance wire. Do not scrape: keep wire uniform.
galvanometer	~MΩ resistor in series with it to protect against high current when unbalanced

H2 Topic 15 – D.C. Circuits

Annex to Kirchoff's Laws (Out of syllabus)

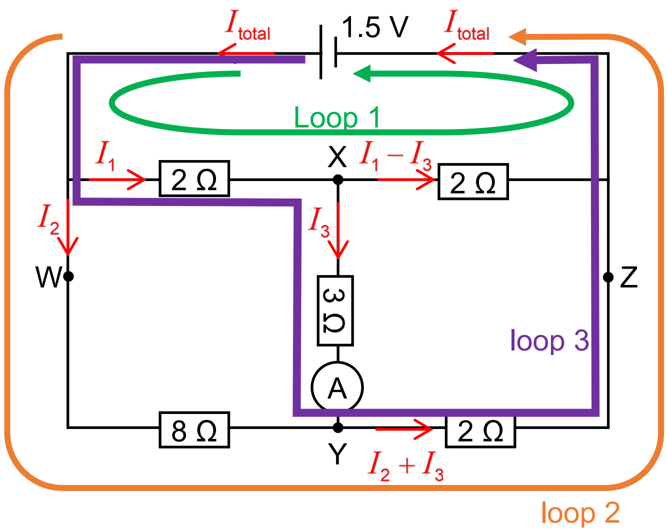
For leisure reading only

Kirchhoff's 1st Law: Total current flowing into a junction equals total current flowing out of it

Kirchhoff's 2nd Law: Around a closed circuit loop, net e.m.f. equals sum of potential drops

Example

Determine the ammeter reading when
$R_A = R_B = 2\ \Omega$



By K1L:
$I_{total} = I_1 + I_2$

by K2L around Loop 1:
$1.5 = 2(I_1) + (I_1 - I_3)(2)$

by K2L around Loop 2:
$1.5 = I_2(8) + (I_2 + I_3)(2)$

by K2L around Loop 3:
$1.5 = I_1(2) + I_3(3) + (I_2 + I_3)(2)$

Solving 3 simultaneous equations using the 3 loops:

from Loop 1:

$$\begin{aligned} 1.5 &= 2(I_1) + (I_1 - I_3)(2) \\ &= 2I_1 + 2I_1 - 2I_3 \\ &= 4I_1 - 2I_3 \\ I_1 &= \frac{1.5 + 2I_3}{4} \text{①} \end{aligned}$$

from Loop 2:

$$\begin{aligned} 1.5 &= I_2(8) + (I_2 + I_3)(2) \\ &= 8I_2 + 2I_2 + 2I_3 \\ &= 10I_2 + 2I_3 \\ I_2 &= \frac{1.5 - 2I_3}{10} \text{②} \end{aligned}$$

(cont'd) from Loop 3:

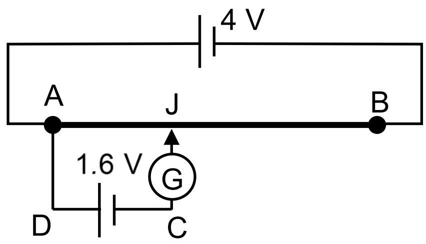
$$\begin{aligned} 1.5 &= I_1(2) + I_3(3) + (I_2 + I_3)(2) \\ &= 2I_1 + 3I_3 + 2I_2 + 2I_3 \\ &= 2I_1 + 2I_2 + 5I_3 \text{③} \end{aligned}$$

sub ① and ② into ③:

$$1.5 = 2\left(\frac{1.5 + 2I_3}{4}\right) + 2\left(\frac{1.5 - 2I_3}{10}\right) + 5I_3$$

$$I_3 = 0.0804 \text{ A}$$

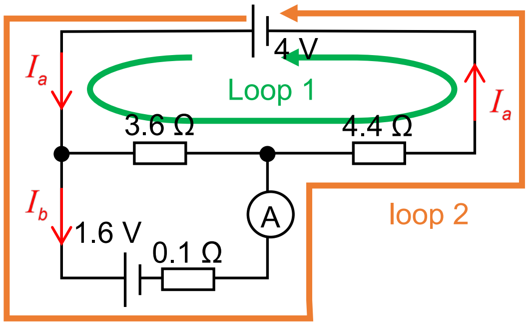
Example



AB is a 1.0 m long uniform wire of 8.0 Ω resistance.
The secondary e.m.f. has internal resistance of 0.1 Ω.

c) Determine the direction of current flowing through the unknown e.m.f. source if $L_{AJ} = 0.45$ m.

$V_{AJ} > E$ so current flows from D to C



To find the current through the secondary circuit:
By K1L, current through 3.6 Ω resistor = $I_a - I_b$

By K2L in Loop 1:

$$4 = (I_a - I_b)(3.6) + I_a(4.4)$$
$$= 8I_a - 3.6I_b$$
$$I_a = \frac{4 + 3.6I_b}{8} \quad ①$$

By K2L in Loop 2:

$$4 - 1.6 = I_b(0.1) + I_a(4.4)$$
$$2.4 = 4.4I_a + 0.1I_b$$

sub ① into above:

$$2.4 = 4.4\left(\frac{4 + 3.6I_b}{8}\right) + 0.1I_b$$
$$I_b = 0.0962 \text{ A}$$