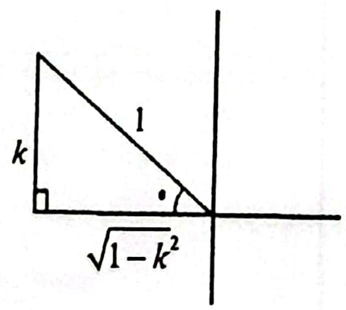
2022 PRELIM 4E5N AM P1 Marking Scheme

Q	Solution	Remarks
1	Solve the following simult equations, $y-2x=3$ --- (1) $\frac{y}{x}+y=10$ --- (2) (1): $y=2x+3$, Sub (1) into (2), (2): $\frac{2x+3}{x}+2x+3=10$ $2x+3+2x^2+3x=10x$ $2x^2-5x+3=0$ $(2x-3)(x-1)=0$ $x=\frac{3}{2}$ or $x=1$ Sub $x=1$ into (1), $y=5$ Sub $x=\frac{3}{2}$ into (1), $y=6$ Answer: $A(1,5)$ and $B\left(\frac{3}{2},6\right)$.	M1: Sub method used from linear to non-linear eqn M1: Attempt to solve quadratic eqn A1: correct x values A1: Final ans in coordinate form
2a	θ lies in 2[nd] quadrant.	B1
bi	$\text{cosec}\theta=\frac{1}{\sin\theta}$ $=\frac{1}{k}$	B1
bii	$\tan\theta=-\frac{k}{\sqrt{1-k^2}}$ 	M1: Apply Pythagoras theorem to derive adjacent side in terms of k. M1: Apply $\tan\theta=\frac{opp}{adj}$ A1: Correct final ans using ASTC.

3	$\frac{dy}{dx}=e^{x}-e^{-x}$ $=\frac{e^{2x}-1}{e^{x}}$ $=\frac{(e^{x}-1)(e^{x}+1)}{e^{x}}$ We have $e^{x}>0$ and $e^{x}+1>1>0$ for all real values of x. At the interval $x<0$, $e^{x}<e^{0}=1$ $e^{x}-1<0$ Since $\frac{dy}{dx}<0$ at the interval $x<0$, y is a decreasing function.	M1: find derivative M1: manipulate the expression to a suitable form M1: make an argument with $e^{x}>0$ A1: Conclusion from a reasonable argument
4a	$\frac{b+\sqrt{5}}{2+\sqrt{5}}\times\frac{2-\sqrt{5}}{2-\sqrt{5}}=\frac{(b+\sqrt{5})(2-\sqrt{5})}{2^{2}-5}$ $=5-2b+(b-2)\sqrt{5}$ $=a-3\sqrt{5}$ $b-2=-3$ $b=-1$ $\therefore a=5-2(-1)=7$	M1: Rationalising the denominator M1: simplify the expression M1: Deduce the values of unknowns from coeff of rational and irrational terms. A1
4b	Volume of cylinder $=(2\sqrt{3}+\sqrt{8})(7\sqrt{2}-\sqrt{3})$ $=2(\sqrt{3}+\sqrt{2})(7\sqrt{2}-\sqrt{3})$ $=2(7\sqrt{6}-3+14-\sqrt{6})$ $=22+12\sqrt{6}$	M1: Apply distributive law of multiplication over addition/subtraction M1: simplify surds A1
5ai	$\binom{10}{r}x^{20-3r}$, power of x is $20-3r$.	B1, B1
5aii	For term independent of x, let $20-3r=0$. However, $r=\frac{20}{3}$ is not an integer. So there is no term independent of x.	M1: deduce power of $x=0$ for term indep of x A1: correct conclusion stating value of r found is not an integer.

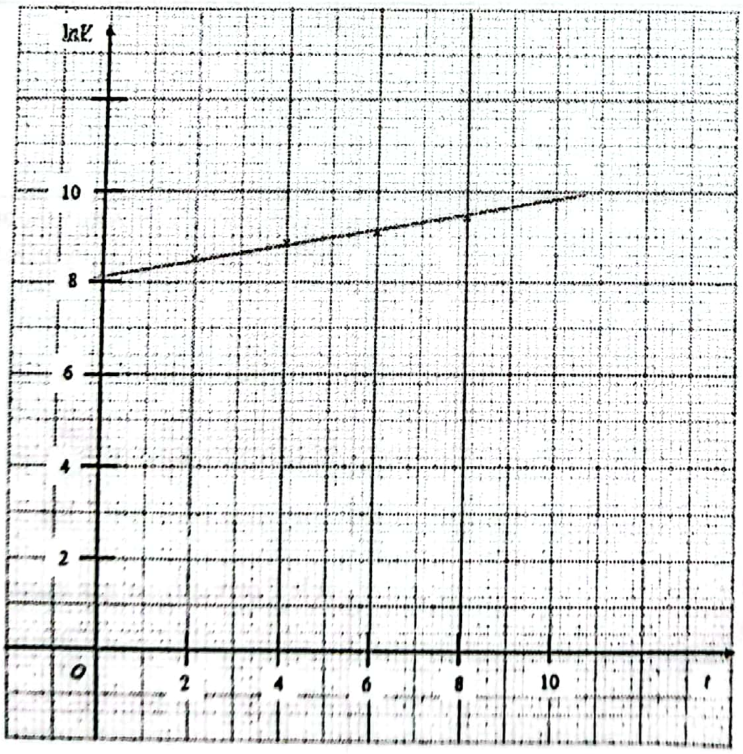
5b	For x^2 term in $\left(x^2+\frac{1}{x}\right)^{10}$, $20-3r=2$ $r=6$ Coefficient of x^2 term in $\left(x^2+\frac{1}{x}\right)^{10}+(a+x)^5$ is $\binom{10}{6}+\binom{5}{2}a^3=-60$ $210+10a^3=-60$ $a^3=-27$ $a=-3$	M1 M1 A1
6a	$\log_3(x+2)+\log_3(x-2)=\log_3(2x-1)$ $\log_3(x+2)(x-2)=\log_3(2x-1)$ $x^2-4=2x-1$ $x^2-2x-3=0$ $(x-3)(x+1)=0$ $x=3$ or $x=-1$ (N.A)	M1: correct use of product and quotient law of log M1: deduce quad eqn A1
6b	$\log_x 2^2=(\log_2 x)^2$ $2\log_x 2=(\log_2 x)^2$ $\frac{2}{\log_2 x}=(\log_2 x)^2$ $(\log_2 x)^3=2$ $\log_2 x=\sqrt[3]{2}$ $x=2^{\sqrt[3]{2}}$ $x=2.39$	M1: Correct use of power law of log M1: Correct use of change of base law of log M1: Convert log form to expo form A1
7	$\tan\theta=\frac{h}{1000}$ $h=1000\tan\theta$ $\frac{dh}{d\theta}=1000\sec^2\theta$ At $\theta=\frac{\pi}{6}$, $\frac{dh}{dt}=\frac{dh}{d\theta}\times\frac{d\theta}{dt}$ $=1000\sec^2\left(\frac{\pi}{6}\right)\times0.003$ $=4$ m/s	M1: convert angle of elevation into trigo ratio M1: Diff h in terms of θ M1: Apply chain rule for rate of change M1: Correct substitution to derive ans A1

<table>
<tr><td>8a</td><td>By long division,
$$\frac{4x^2+5}{2x^2-x-1}=2+\frac{2x+7}{(2x+1)(x-1)}$$
$$\frac{2x+7}{(2x+1)(x-1)}=\frac{A}{2x+1}+\frac{B}{x-1}$$
$$2x+7=A(x-1)+B(2x+1)$$
Using substitution method or comparing coefficient method, $A=-4$ and $B=3$

Ans: $\frac{4x^2+5}{2x^2-x-1}=2+\frac{3}{(x-1)}-\frac{4}{(2x+1)}$</td><td>M1: Attempt to make mixed fraction
A1: Correct mixed fraction

M1: Correct partial fraction form

M1: suitable method to find unknowns

A1</td></tr>
<tr><td>8b</td><td>$$\int\frac{4x^2+5}{2x^2-x-1}\,dx$$
$$=\int 2+\frac{3}{(x-1)}-\frac{4}{(2x+1)}\,dx$$
$=2x+3\ln(x-1)-2\ln(2x+1)+c$, where c is a constant.</td><td>B1B1B1: for each correct term integrated.</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>9a</td><td>$V=ae^{kt}$
$\ln V=kt+\ln a$
<table>
<tr><td>t</td><td>2</td><td>4</td><td>6</td><td>8</td></tr>
<tr><td>$\ln V$</td><td>8.517</td><td>8.825</td><td>9.127</td><td>9.441</td></tr>
</table>

</td><td>M1: apply log on both sides of eqn

M1: provide table of values for straight line graph

G1: correct plotting of points
G1: suitable line of best fit drawn</td></tr>
</table>

9b	gradient $= k = \frac{9.8-8.1}{10-0}$ $k = 0.17$ $\ln a = 8.1$ $a = e^{8.1} = 3294.468$ or 3290 (3 s.f.)	M1: Use two points on line drawn for gradient A1: correct k value (accept 0.15 – 0.19) B1: correct a value deduced from graph.
9c	From the graph, at $t = 10$, $\ln V = 9.8$. Value of one cryptocurrency in 2022 $= e^{9.8}$ $= \$18034$	M1: use graph to read off Y value when $x = 10$ A1
10a	Let the midpoint of AB be M. Coordinates of $M = (4, 3)$. Gradient of $AB = \frac{-3-9}{7-1} = -2$ Gradient of the perpendicular bisector $= \frac{1}{2}$ Equation of the perpendicular bisector is $y-3 = \frac{1}{2}(x-4)$ $y = \frac{1}{2}x+1$	M1 M1 M1 A1
10b	Sub $y = 0$ into $y = \frac{1}{2}x+1$, $x = -2$. The coordinates of D are $(-2,0)$. Length of $CD = \sqrt{(4-(-2))^2+6^2} = 6\sqrt{2}$ units	M1: length of line formula A1 (accept 8.49 units (3s.f.))
10c	Area of the quadrilateral $ABCD$ $= \frac{1}{2}\begin{vmatrix} -2 & 4 & 7 & 1 & -2 \\ 0 & -6 & -3 & 9 & 0 \end{vmatrix}$ $= \frac{1}{2}[12-12+63-(-42-3-18)]$ $= 63\text{units}^2$	M1 A1

11a	$\frac{d}{dx}\left(\frac{x}{(3x+1)^{\frac{1}{2}}}\right)$ $=\frac{(3x+1)^{\frac{1}{2}}\cdot 1-x\cdot\frac{3}{2}(3x+1)^{-\frac{1}{2}}}{(3x+1)}$ $=\frac{\frac{1}{2}(3x+1)^{-\frac{1}{2}}[6x+2-3x]}{(3x+1)}$ $=\frac{3x+2}{2(3x+1)^{\frac{3}{2}}}$	M1: quotient rule M1: Correct differentiation of $(3x+1)^{\frac{1}{2}}$ on the numerator M1: simplify fraction A1
11b	Since $\frac{d}{dx}\left(\frac{x}{(3x+1)^{\frac{1}{2}}}\right)=\frac{3x+2}{2(3x+2)^{\frac{3}{2}}}=\frac{3}{2}\left(\frac{x}{(3x+2)^{\frac{3}{2}}}\right)+\frac{1}{(3x+2)^{\frac{3}{2}}}$, $\left[\frac{x}{(3x+1)^{\frac{1}{2}}}\right]_0^5=\frac{3}{2}\int_0^5\frac{x}{(3x+1)^{\frac{3}{2}}}dx+\int_0^5\frac{1}{(3x+1)^{\frac{3}{2}}}dx$ $\left(\frac{5}{4}-0\right)=\frac{3}{2}\int_0^5\frac{x}{(3x+1)^{\frac{3}{2}}}dx+\left[-\frac{2}{3(3x+1)^{\frac{1}{2}}}\right]_0^5$ $\frac{3}{2}\int_0^5\frac{x}{(3x+1)^{\frac{3}{2}}}dx=\frac{5}{4}-\left(-\frac{1}{6}+\frac{2}{3}\right)$ $\int_0^5\frac{x}{(3x+1)^{\frac{3}{2}}}dx=\frac{1}{2}$	M1: Apply int step as reverse of diff M1: Split into two integrals on RHS M1: correct calculation of at least 1 integral. A1

12a	$LHS = \dfrac{\sin^3 x - \cos^3 x}{\sin x - \cos x}$ $= \dfrac{(\sin x - \cos x)(\sin^2 x + \sin x \cos x + \cos^2 x)}{(\sin x - \cos x)}$ $= \sin^2 x + \sin x \cos x + \cos^2 x$ $= 1 + \dfrac{1}{2}\sin 2x = RHS$	M1: apply diff of cube identity correctly M1: use $\sin^2 x + \cos^2 x = 1$ M1: $\sin 2x = 2\sin x \cos x$ A1
12b	Hence, solving $1 + \dfrac{\sin 2x}{2} = \dfrac{2}{3}$, $0° \le 2x \le 360°$ $\sin 2x = -\dfrac{2}{3}$ Basic angle $= 41.810°$ $2x$ lies in the 3rd and 4th quadrant. $\therefore 2x = 180° + 41.810°,\ 360° - 41.810°$ $x = 110.9°,\ 159.1°$ (1 d.p.)	M1: Find basic angle M1: ASTC to deduce quadrants that $2x$ lie in M1: correct two roots found using basic angle A1: correct x value
13a	$\dfrac{dy}{dx} = x \cdot 3(x-3)^2 + (x-3)^3$ $= (x-3)^2(4x-3)$ Let $\dfrac{dy}{dx} = 0$, $(x-3)^2(4x-3) = 0$ $x = 3$ or $x = \dfrac{3}{4}$ Sub $x = 3$ into $y = x(x-3)^3$, $y = 0$ Sub $x = \dfrac{3}{4}$ into $y = x(x-3)^3$, $y = -8.54$ (3s.f.) The coordinates of stationary points are $(3,0)$ and $\left(\dfrac{3}{4}, -8.54\right)$.	M1: Use product rule A1 M1: Let $\dfrac{dy}{dx} = 0$ to find stationary points. A1: correct x values A1: correct coordinates of stationary points.
13b	Note to teachers: $(3,0)$ is a point of inflexion so second derivative test is inconclusive. Using first derivative test,	M1: Use first derivative test. **No credit for second derivative test**

x	$\frac{3}{4}^-$	$\frac{3}{4}$	$\frac{3}{4}^+$	3^-	3	3^+
$\frac{dy}{dx}$	$-$	0	+	+	0	+
Sketch	\	—	/	/	—	/

	$\therefore (3,0)$ is a point of inflexion and $\left(\frac{3}{4},-8.54\right)$ is a minimum point.	A1A1: correct nature for each stationary point
13c	No, Ashley is wrong. Let $y=x^4$, Then $(0,0)$ is a minimum point but $\frac{d^2y}{dx^2}=12x^2=0$ at $x=0$. Hence, the second derivative test is inconclusive since two different curves produce different types of stationary points when $\frac{d^2y}{dx^2}=0$.	M1: Use of counter eg. to justify statement is incorrect. *Not enough to just state inconclusive when $\frac{d^2y}{dx^2}=0$ A1: deduce conclusion by comparing to the results of 13(b).