H

ANGLO-CHINESE JUNIOR COLLEGE
JC2 MID-YEAR LEARNING CHECKPOINT

Higher 1

/65

CANDIDATE NAME

TUTORIAL/ FORM CLASS

INDEX NUMBER

MATHEMATICS 8865/01

Paper 1 **24 June 2024**

2 hours

Candidates answer on the Question Paper.
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your index number, class and name on all the work you hand in.
Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 65.

Question	Marks
1	/6
2	/6
3	/10
4	/4
5	/7
6	/9
7	/11
8	/12

This document consists of **16** printed pages.

Section A: Pure Mathematics [22 marks]

1 **(a)** Differentiate $\ln\left(5+x^2\right)$ with respect to x. [1]

(b) Differentiate $\dfrac{7x-x^2}{\sqrt{x}}$ with respect to x. [2]

(c) Find $\int\left(\dfrac{1}{4x}+x\right)^2 \mathrm{d}x$. [3]

2 A curve C has equation $y = 4\mathrm{e}^{3-2x} + \mathrm{e}^{-x}$.

(a) Sketch C, stating the exact value of the y-coordinate of its point of intersection with the y-axis and the equation of the asymptote. [2]

(b) Without using a calculator, find the equation of the tangent to C at the point where $x = 3$, giving your answer in the form $y = mx + c$, where m and c are exact constants. [4]

3 A manufacturing company makes doors. The manufacturer makes x doors per order, where x is a positive integer, to prevent overstocking of inventory. He estimates that the total production cost for each order consists of the following components:

- raw material,
- wages for the workers,
- fixed cost and maintenance cost.

The manufacturing process per order requires 7 workers. The cost of raw material for every 5 doors manufactured is \$1200, the wages for each worker is \$1200 and the total fixed cost and maintenance cost is $\$\dfrac{1875000}{x^2}$.

(a) Show that the estimated total cost for one order, \$$T$, is given by

$$T = 240x + 8400 + \frac{1875000}{x^2}.$$ [1]

(b) Use differentiation to find the minimum value of T, justifying that this value is a minimum. [5]

The annual profit from the sale of these doors is P thousand dollars per year. The accounts manager of the company believes that the relationship between P and the time, t years, is given by $P = 1 + 13\ln(7t + 2)$, for $0 \le t \le 8$.

(c) Find the value of t for which $P = 45$. [1]

(d) Sketch the graph of P against t, indicating the endpoints of the graph clearly. [1]

(e) Use your calculator to find the value of $\int_0^5 P \, \mathrm{d}t$, leaving your answer to the nearest integer. In the context of this question, what does this value represent? [2]

Section B: Probability and Statistics [43 marks]

4 A website states that the mean length of a particular species of plant is 40 cm. A sample of 80 plants are taken and the lengths of the plants, x cm, are summarised by $\sum(x - 40) = 24$ and $\sum(x - 40)^2 = 83.2$.

(a) Find unbiased estimates of population mean and variance. [3]

(b) What do you understand by the term 'unbiased estimate'? [1]

5 **In this question you should state clearly all the distributions that you use, together with the values of the appropriate parameters.**

During an annual health check, the heights of a large group of girls, aged 17 to 19, have been recorded. It is found that the heights of girls in TI Junior College are normally distributed with mean μ cm and standard deviation σ cm. 40% of the girls have height that differs from μ cm by at most L cm.

(a) Show that $L = 0.524\sigma$. [2]

(b) Hence find the probability that the height of a randomly chosen girl exceeds μ cm by at least $2L$ cm. [3]

(c) Find the probability that the total height of 2 randomly chosen girls is at least twice the height of a randomly chosen girl. [2]

6 It is known that the population variance for the number of days taken by the customer service department to reply to an enquiry is 0.303. A manager claims that the mean duration taken by the customer service department to reply an enquiry is 3.4 days. A random sample of 70 enquiries is selected and the mean number of days taken to reply an enquiry is 3.3 days.

(a) Test, at the 10% significance level, whether the manager's claim should be rejected. You should state your hypotheses and define any symbols you use. [5]

(b) State, with a reason, whether it is necessary to assume that the number of days taken to reply an enquiry follows a normal distribution in order for this test to be valid. [1]

A new process was introduced by the manager and the new population variance for the number of days taken to reply an enquiry is 0.8. The manager took a random sample of 50 enquiries and found that the mean number of days taken to reply an enquiry is 3.28 days. Hence the manager now claims that the population mean number of days taken to reply an enquiry is less than 3.4 days.

(c) It is given that there is insufficient evidence to reject the null hypothesis at the $\alpha\%$ level of significance. Find the set of possible values of α. [3]

7 **In this question you should state clearly all the distributions that you use, together with the values of the appropriate parameters.**

A manufacturer produces a large number of coloured glowing light sticks, which are packed in packets of 8. Each packet consists of randomly chosen coloured glowing light sticks. On average, 12% of the glowing light sticks are green and 15% of the glowing light sticks are yellow.

(a) **(i)** Show that the probability that a randomly chosen packet of glowing sticks contains at least 3 green or yellow glowing sticks is 0.372. [2]

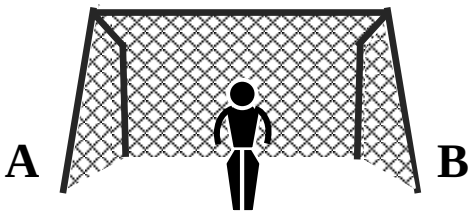
(ii) Find the expected number of light sticks that are not green or yellow. [2]

Kate buys 80 randomly chosen packets of glowing sticks for a concert.

(b) **(i)** Find the probability that fewer than 30 of these packets contain at least 3 green or yellow glowing sticks. [3]

(ii) Find the probability that the mean number of light sticks that are not green or yellow in each packet is at most 5.8. [4]

8



(a) The diagram above shows the front view of a goal post labelled *A* on one side and *B* on the other side. The diagram also shows Benjamin, a goalkeeper of a football club, standing in front of the goal post. Based on Benjamin's past records as a goalkeeper during penalty shoot-outs, the probability that he dives towards side *B* is 0.7.

In a particular match, Benjamin's team went into a penalty shoot-out. The probability that a penalty kicker kicks the ball to side *B* is *p*, where $0 < p < 1$. Assume that Benjamin's choice of direction to dive is independent of the penalty kicker's choice of direction to kick the ball.

(i) Show that the probability Benjamin dives in the same direction as the ball is kicked is $0.4p + 0.3$. [1]

(ii) If Benjamin dives in the same direction as the ball is kicked, the probability that he comes into contact with the ball is 0.35. Assume that he will not be able to be in contact with the ball if he goes in opposite direction as the ball. Find, in terms of *p*, the probability that Benjamin fails to be in contact with the ball. [2]

(b) Benjamin can either be a goalkeeper or a forward in a match, depending on the strategy that the team uses. The rest of the team members consist of another goalkeeper, 6 defenders, 5 midfielders and 5 forwards. Among the 5 forwards, 3 of them are siblings. Altogether, there are 18 members in the team.

(i) Find the number of ways to select 4 defenders, 4 midfielders, 2 forwards and a goalkeeper from the team, given that

(a) Benjamin is selected, [3]

(b) Benjamin is not selected and not more than one of the siblings is selected. [3]

After a match, the team stands in two rows to take a photo.

(ii) The first row consists of the all the 5 forwards and the 5 midfielders. The 5 midfielders are standing together and the 3 siblings are all separated. In the second row, Benjamin and the other goalkeeper will stand in the middle, with 3 defenders on each end. Find the number of ways for this to be done. [3]

Summary of Areas for Improvement			
Knowledge (K)	**Careless Mistakes (C)**	**Read/Interpret Qn wrongly (R)**	**Presentation (P)**