Statistics 4 Tutorial: Special DRV – Binomial Distribution

Additional Practice Questions

1. [Modified 2010 HCI CT/11]
[For this question, give your answers correct to 4 decimal places.]
At a college canteen drinks stall, the probabilities of a student buying a cup of tea and a cup of coffee are 0.6 and 0.05 respectively. Sale of drinks takes place independently at random times.

(i) Ten students queue up to buy a cup of drink each. State a suitable distribution for the number of students who do not order coffee.

B(10, 0.95)

(ii) Three students queue up to buy a cup of drink each. Find the probability that exactly one of them buys a cup of tea and exactly one of them buys a cup of coffee.

Probability = (0.6)(0.05)(0.35) 3!
= 0.0630

(iii) Five students queue up to buy a cup of drink each. Show that the probability that none of these 5 students bought tea is 0.0102.

Let X denote no. of students buying tea.

$X \sim \mathrm{B}(5, 0.6)$

$P(X = 0) = 0.01024 = 0.0102$

There are 80 classes in the college and 5 students from each class each buys a drink from the stall. Find the probability that there are more than 75 classes with at least 1 student from the class buying tea.

Let W denote the no. of classes, out of 80, with no student buying tea.

$W \sim \mathrm{B}(80, 0.01024)$

$P(W \leq 4)$

$= 0.9986 \quad \text{(4 d.p.)}$

2. [Modified 2009 JJC/I/9]
In a large batch of items from a production line, it was found that the probability that an item is faulty is p, where $0 < p < 0.5$.
A random sample of 10 items is taken and the random variable X is the number of faulty items. Given that P(X = 7) = 0.0000221, find the value of p, giving your answer correct to 3 decimal place.

$X \sim B(10, p)$

$P(X = 7) = \binom{10}{7} p^7 (1-p)^3 = 0.0000221$

From GC, $p = 0.115$

Assume that $p = 0.345$.

Find the probability that in a random sample of 60 items, there are more than 20 items that are faulty.

Let Y denote the number of faulty items in the sample. Then $Y \sim B(60, 0.345)$

$P(Y > 20) = 0.516$ (3sf) (from GC)

3. [Modified 2009 MJC/I/9]

On average, 70% of teenagers in Singapore have myopia. A junior college in Singapore has 1200 students, with 20 students in each class.

(i) State the expected number of students who do not have myopia in a randomly selected class.

Expected value = 20 * 0.3 = 6

(ii) Find the probability that there are less than 5 students who do not have myopia in a randomly selected class.

Let X denote the number of students in a class who do not have myopia.

Then $X \sim B(20, 0.3)$

$P(X < 5) = 0.238$ (3sf) (from GC)

(iii) Find the least number of classes to be selected such that the probability of at least one class with less than 5 students who do not have myopia exceeds 0.8.

Let Y denote the number of classes that have less than 5 students with myopia, and let $y \le 60$ denote the number of classes sampled.

Then $Y \sim B(y, 0.23751)$

$P(Y \geq 1) > 0.8$

$1 - P(Y = 0) > 0.8$

$P(Y = 0) < 0.2$

$$\binom{y}{0}(0.23751)^0(1-0.23751)^y < 0.2$$

$y \ln(0.76249) < \ln(0.2)$

$$y > \frac{\ln(0.2)}{\ln(0.76249)} = 5.935$$

Therefore, at least 6 classes need to be sampled.

4. [2011 RJC/I/10]

In a certain country, 27% of the adults are myopic .

Eight adults are chosen at random. Let X denotes the number of adults who are myopic.

(i) Show that $P(X \geq 3) = 0.372$

Let X denote the number of adults out of 8 who are myopic.

Then $X \sim B(8, 0.27)$

$P(X \geq 3) = 1 - P(X \leq 2) = 0.371826 = 0.372.$

Ten such samples of eight adults are taken.

(ii) Find the probability that four of these samples each have less than three adults who are myopic.

Let Y be the number of samples out of 10 that has less than 3 adults who wear glasses.

Then $Y \sim B(10, 0.628)$

$P(Y = 4) = 0.0866$

Nine adults are randomly selected and asked if they are myopic. Find the probability that

(iii) the 9th adult is the 5th adult who is myopic.

P(the 9^{th} adult is the 5^{th} adult who is myopic) $= (0.27)(P(X = 4)) = 0.0285$

(iv) the 9th adult is the first adult who is myopic

P(the 9^{th} adult is the first adult who is myopic) $= (0.73)^8(0.27) = 0.0218$

5. The probability that a certain type of cactus seed will germinate is p. In a long-term study, 1500 of these seeds were planted, of which 600 germinated.

(i) Write down an estimate of p.

$$p \approx \frac{600}{1500} = \frac{2}{5}$$

Suzy plants 24 such seeds at the beginning of the growing season in three batches, each batch consisting of 8 seeds.

(ii) Assuming that the seeds germinate independently, use the value of p found in (i) to find

(a) the probability that at least 12 seeds germinate,

Let X rep the no. of seeds that will germinate out of 24, $X \sim B(24, \frac{2}{5})$

$P(X \geq 12) = 1 - P(X \leq 11) = 0.213$

(b) the most probable number of seeds that will germinate in a batch of 8,

$P(X = 2) = 0.209$
$P(X = 3) = \underline{0.279}$
$P(X = 4) = 0.232$
$\therefore$ Most probable no. that will germinate in a batch of eight is 3.

(c) the probability that only one batch produces more than 4 germinating seeds.

Let Y rep the no. of seeds that will germinate in a batch of 8, $Y \sim B(8, \frac{2}{5})$

$P(Y > 4) = 1 - P(Y \leq 4) = 0.1736704.$

P(only one batch produces more than 4 germinating seeds)
$= 3\,P(Y > 4)\,[P(Y \leq 4)]^2$
$= 3(0.1736704)\,[1 - 0.1736704]^2$
$= 0.356$

6*. An urn contains n white balls and m black balls. Suppose k balls are drawn at random with replacement after each ball is drawn. Let the random variable X denote the number of black balls drawn.

(i) State the distribution of X, together with its parameter(s).

Let X be the rv of number of black balls drawn from a total of k draws, $X \sim \mathrm{B}\left(k, \frac{m}{n+m}\right)$

(ii) Find the probability that at least one black ball is drawn.

$$P(X \geq 1) = 1 - \mathrm{P}(X = 0) = 1 - \left(\frac{n}{n+m}\right)^k$$

(iii) Given that the number of white balls is 9 times that of the black balls, find the least number of balls one must draw, so that the probability that at least one black ball is drawn exceeds 0.5.

Since n = 9m, $X \sim \mathrm{B}\left(k, \frac{1}{10}\right)$.

$P(X \geq 1) > 0.5$

$1 - \left(\frac{9}{10}\right)^k > 0.5$

$(0.9)^k < 0.5$

$k > 6.58$

Least value of k = 7.

In a new sampling scheme, an urn contains a very large number (of the order of 10^6) of white and black balls in the proportion of 99 white balls to 1 black ball. Suppose 120 balls are randomly drawn without replacement after each ball is drawn. Let Y denote the number of black balls drawn.

(iv) What would you consider to be the distribution that best fits Y? Justify your conclusion.

$Y \sim \mathrm{B}\left(120, \frac{1}{100}\right)$. Since the urn contains large number of balls (at least 10^6), the probability of black ball will remain almost constant even if 120 balls are taken out without replacement. Thus a binomial distribution is a suitable model.

(v) The above new sampling scheme is repeated 10 times. Find the probability that at least 3 black balls are drawn exactly twice.

Let W be the rv of the number of times out of 10 when at least 3 black balls are drawn, W~B(10, 0.1196365407).

P(W= 2) = 0.232.

7. [Modified 2010 IJC/I/8]
At a particular university, the probability that a student wears glasses is 0.7.

(i) A random sample of 8 students is taken. Find the probability that as many students wear glasses as do not wear glasses.

Let X be the number of students in the sample who wear glasses. Then $X \sim B(8, 0.7)$

$P(X = 4) = \binom{8}{4}(0.7)^4(0.3)^4 = 0.136$

(ii) Three random samples of 8 students each are taken. Find the probability that each of these samples has more than 2 students who wear glasses.

$P(X > 2) = 1 - P(X \le 2) = 1 - (0.00006561 + 0.00122472 + 0.01000188) = 0.98870779$

Required probability = $(0.98870779)^3 = 0.967$ (3sf)

(iii) At another university, p% of the students wear glasses, where $p > 80$. A random sample of 12 students is taken and the number of students that wear glasses is denoted by G. Given that $P(G = 9) = 0.2$, write down an equation for the value of p, and find this value numerically.

Note $G \sim B(12, p/100)$

Then, $P(G = 9) = \binom{12}{9}\left(\frac{p}{100}\right)^9\left(1-\frac{p}{100}\right)^3 = 0.2$

From GC, $p = 65.4$ (rej) or $p = 83.1$ (3sf)

8. [Modified 2011 CJC/2/7]
At an international airport, 3% of the suitcases are slightly damaged and 0.2% are badly damaged.

(i) A random sample of 10 suitcases is taken. Find the probability that at least two suitcases are slightly damaged.

Let X be the random variable denoting the number of suitcases that are slightly damaged out of 10 suitcases.

$X \sim \mathrm{B}(10, 0.03)$

$\mathrm{P}(X \geq 2) = 1 - \mathrm{P}(X \leq 1) \quad = 0.0345$

(ii) N suitcases are taken at random. Find the least value of N so that the probability that at most two suitcases are damaged (either slightly or badly) is less than 0.85.

Let W be the random variable denoting the number of suitcases that are damaged out of N suitcases.

$W \sim \mathrm{B}(N, 0.032)$

$\mathrm{P}(W \leq 2) < 0.85$

Using GC,

N	$\mathrm{P}(W \leq 2)$
41	0.85698
42	0.84942
45	0.84174

$\therefore$ The least value of N is 42.

9. [Modified 2011 JJC/2/9]
A large batch of beans contains only red beans and green beans. The proportion of red beans is p.
A sample of ten beans is taken from this batch at random.

(i) Explain why the probabilities of obtaining various numbers of red beans in the sample can be well approximated by a binomial probability distribution.

Though the probability of picking a red bean (p) changes each time a bean is picked, this change is very insignificant as the batch is **large**. So p can be assumed to be the same for each trial. Hence the probabilities of obtaining various numbers of red beans in the sample can be well approximated by a binomial probability distribution.

(ii) Given that the probability of obtaining at most one red bean in the sample is 0.96, show that p satisfies the equation

$$25(1-p)^9(1+9p) - 24 = 0$$

and hence find the value of p.

Let X be the number of red beans in the sample, Then $X \sim \text{B}(10,\ p)$

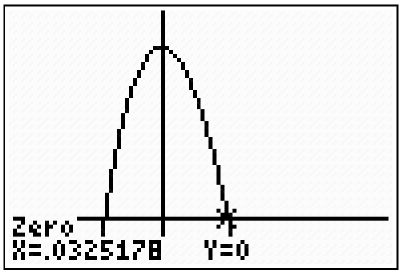
$\text{P}(X \leq 1) = 0.96$

$\Rightarrow \text{P}(X = 0) + \text{P}(X = 1) = 0.96$

$$\Rightarrow (1-p)^{10} + 10p(1-p)^9 = 0.96$$

$$\Rightarrow (1-p)^9[(1-p)+10p] = \frac{24}{25}$$

$$\Rightarrow 25(1-p)^9(1+9p) - 24 = 0 \text{ (shown)}$$



Using GC, $p = 0.0325$ (3 s.f.).

Eighty random samples of ten beans are taken.

(iii) Find the probability that the number of these samples that contain at most one red bean is more than 77.

Let Y be the number of samples containing at most one red bean.
Then $Y \sim \text{B}(80,\ 0.96)$.

P(more than 77 samples containing at most one red bean)

$= \text{P}(Y > 77)$

$= 1 - P(Y \leq 77)$

$= 0.375$

10 [CJC 2018 J2 MYE/Q10]

A retailer sells 5 grades of washing detergent, Grades A, B, C, D and E, with Grade A being the premium grade. The price of a bottle of detergent sold is denoted by X. The price and the probability at which each grade of detergent is sold are as follows:

Grade	A	B	C	D	E
Price per bottle, $\$x$	22	19	16	13	10
$\mathrm{P}(X=x)$	$\frac{1}{12}$	$\frac{1}{4}$	a	$\frac{1}{3}$	$\frac{1}{6}$

Assuming that the bottles of detergent are sold independently, find

(i) a. [1]

(ii) the expectation and variance of the price of one bottle of detergent. [2]

In order to improve sales, the retailer decides to sell every bottle of detergent with a free towel which comes in different colours. The probability that a bottle of detergent is sold with a pink towel is p.

(iii) It is given that $p = 0.2$.

(a) If a housewife buys 19 bottles of detergent, find the most likely number of bottles with a pink towel. [2]

(b) If N bottles of detergent are packed in a box, find the least value of N such that the probability of getting at least two bottles with a pink towel in a box is more than 0.5. [3]

(iv) For an unknown value of p, it is given that if a housewife buys 20 bottles of detergent, the probability that she gets exactly 10 pink towels is 0.003237, correct to 6 decimal places. By forming an equation in terms of p, find the possible values of p. [3]

(i)

$$\frac{1}{6}+\frac{1}{3}+a+\frac{1}{4}+\frac{1}{12}=1$$

$$a=1-\frac{5}{6}=\frac{1}{6}$$

(ii)

Grade	A	B	C	D	E
Price $\$X$	10	13	16	19	22
$\mathrm{P}(X=x)$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{12}$

Expected revenue,

$$\mathrm{E}(X) = 10\times\frac{1}{6}+13\times\frac{1}{3}+16\times\frac{1}{6}+19\times\frac{1}{4}+22\times\frac{1}{12}$$

$$= \$15.25 \text{ (exact value)}$$

$\mathrm{Var}(X) = \mathrm{E}(X^2) - [\mathrm{E}(X)]^2$

	$=\frac{10^2}{6}+\frac{13^2}{3}+\frac{16^2}{6}+\frac{19^2}{4}+\frac{22^2}{12}-[15.25^2]$ $=\frac{219}{16}$ $=13.6875$ (exact value)
(iiia)	Let Y be the random variable denoting the number of bottles of detergent with a pink towel out of 19 bottles. $X \sim \mathrm{B}(19, 0.2)$ Using GC: y / $\mathrm{P}(Y=y)$: 2 / 0.1540; 3 / 0.2182; 4 / 0.2182; 5 / 0.1636 Hence, the most likely number of bottles is 3 and 4.
(iiib)	Let W be the random variable denoting the number of bottles of detergent with a pink towel out of N bottles. $W \sim \mathrm{B}(N, 0.2)$ $\mathrm{P}(W \geq 2) > 0.5$ $1-\mathrm{P}(W \leq 1) > 0.5$ Using GC: w / $1-\mathrm{P}(W \leq 1)$: 8 / 0.4967; 9 / 0.5638; 10 / 0.6242 Hence least value of N is 9.
(iv)	Let V be the random variable denoting the number of bottles of detergent with a pink towel out of 20 bottles. Then $V \sim \mathrm{B}(20, p)$ Given $\mathrm{P}(V=10)=0.003237$ ${}^{20}C_{10}p^{10}(1-p)^{10}=0.003237$ $p(1-p)=\left(\frac{0.003237}{184756}\right)^{\frac{1}{10}}$ $p^2-p+0.1676310305=0$ Hence, $p=0.2130000531$ or 0.7869999469 $p \approx 0.213$ or 0.787

Table for (iiia):

y	$\mathrm{P}(Y=y)$
2	0.1540
3	0.2182
4	0.2182
5	0.1636

Table for (iiib):

w	$1-\mathrm{P}(W \leq 1)$
8	0.4967
9	0.5638
10	0.6242

11. **[2020 JC2 EJC/MYE_P2/9]**

Bob works in a telemarketing company 6 days a week. Bob makes 40 calls each day. The probability of him making a sale on a customer call is 0.16.

(i) Find the probability that Bob makes more than 7 sales in a day. [2]

Bob is said to have a productive day if he makes more than 7 sales in the day.

(ii) Find the probability that Bob has exactly 2 productive days in a week. [2]

Bob decides to increase his productivity such that he can expect to make at least 10 sales each day.

(iii) Find the minimum number of calls Bob needs to make each day to meet this requirement. [2]

(iv) Find the minimum number of calls Bob needs to make each day for the probability of making at least 10 sales to exceed 90%. [2]

Suggested Solution
(i) Let X be the 'number of sales out of 40 calls. $X \sim \mathrm{B}(40, 0.16)$ Required Prob $= \mathrm{P}(X > 7) = 1 - \mathrm{P}(X \leq 7)$ $= 0.30436 = 0.304$ (3 s.f.)
(ii) Let Y be the 'number of productive days out of 6 days. $Y \sim \mathrm{B}(6, 0.30436)$ Required Prob $= \mathrm{P}(Y = 2) = 0.32539 = 0.325$ (3sf)
(iii)[to change] Assume Bob made N calls each day. Let W be the 'number of sales out of N calls. $W \sim \mathrm{B}(N, 0.16)$ $E(W) \geq 10 \Rightarrow N(0.16) \geq 10 \Rightarrow N \geq 62.5$ Hence Bob needs to make at least 63 calls each day such that he can expect to make at least 10 sales each day.
(iv) Considering W in (iii) We have $\mathrm{P}(W \geq 10) > 0.9$ $\Rightarrow 1 - \mathrm{P}(W \leq 9) > 0.9 \Rightarrow \mathrm{P}(W \leq 9) < 0.1$ By GC

n	$\mathrm{P}(W \leq 9)$
86	0.1007
87	0.0933

Least N is 87

12. **[2020 JC2 TJC/MYE_P2/8]**

In a large population of Bichon Frises (a breed of dog), on average, one in three of them have blood type *Dal*–. Specimens of blood from the first five of this breed of dog attending a local veterinary clinic are to be tested. It can be assumed that these five Bichon Frises are a random sample from the population.

(i) State, in context, two assumptions needed for the number of Bichon Frises in the sample who are found to have blood type *Dal*– to be well modelled by a binomial distribution. [2]

Assume now that the number of Bichon Frises in the sample who are found to have blood type *Dal*– has a binomial distribution.

(ii) Find the probability that more than two of the Bichon Frises in the sample are found to have blood type *Dal*–. [2]

(iii) Three such samples of five Bichon Frises are taken. Find the probability that one of these three samples has exactly one Bichon Frise with blood type *Dal*–, another has exactly two Bichon Frises with blood type *Dal*–, and the remaining sample has more than two Bichon Frises with blood type *Dal*–. [3]

(iv) N such samples of five Bichon Frises are taken. Find the least value of N such that the probability that the number of these samples that contain two or fewer Bichon Frises with blood type *Dal*– will be at least 15 is more than 90%. [3]

Suggested Solution

The **probability** of a randomly chosen Bichon Frises having blood type *Dal*– is **constant**.
The **event of** a Bichon Frises having blood type *Dal*– is **independent** of the event of another Bichon Frises having blood type *Dal*–.

Let X be the number of Bichon Frises (out of 5) who has blood type *Dal*–.

$X \sim \mathrm{B}(5, \frac{1}{3})$

$\mathrm{P}(X > 2) = 1 - \mathrm{P}(X \le 2) = 0.20988 \approx 0.210$ (3 sf)

Required probability
$= \mathrm{P}(X = 1)\,\mathrm{P}(X = 2)\,\mathrm{P}(X > 2) \times 3! \quad = 0.136$ (3 sf)

Let W be the number of these samples (out of N) that contain two or fewer Bichon Frises with blood type *Dal*–.

$W \sim \mathrm{B}(N, \mathrm{P}(X \le 2))$ i.e $W \sim \mathrm{B}(N, 0.790123)$

$\mathrm{P}(W \ge 15) = 1 - \mathrm{P}(W \le 14) > 0.90$
or $\mathrm{P}(W \le 14) < 0.01$
Using GC:

N	$\mathrm{P}(W \ge 15)$
21	0.8675 < 0.90
22	0.9284 > 0.90

Therefore, the least value of N is 22.