- Differentiate each of the following with respect to x. (a) $(1+4x)^{10} \cos x$ 1

[2]

$$(\mathbf{b}) \quad \frac{4x-5}{\tan x}$$

[2]

2 The function f is defined by $f(x) = \frac{1}{3x+2} + x^2$ for x < -1. Determine whether f is an increasing or a decreasing function. [3]

3 The point *A* lies on the curve $y = 3x^2 - 7x + 11$. The line 5y + x = k is a normal to the curve at point *A*. Find the coordinates of *A* and the value of *k*. [4]

4 A balloon in the shape of a sphere is being inflated by a pump. The volume of the balloon is increasing at a constant rate of 500 cm³ per second. The balloon was empty at the start of the pumping.

[The volume of sphere is $\frac{4}{3}\pi r^3$.]

(i) Find the radius of the balloon after 20 seconds.

(ii) Find the rate of increase of the radius after 20 seconds.

[3]

[2]

- 5 The equation of a curve is $y = (1 + \sin 3x)^4$, where $0 \le x \le \frac{\pi}{2}$.
 - (i) Express $\frac{dy}{dx}$ in the form $k \cos 3x (1 + \sin 3x)^3$, where k is a constant to be found. [1]

(ii) Find the exact *x*-coordinate(s) of the point(s) at which the tangent to the curve is parallel to the *x*-axis.

[5]



The diagram shows a pond *OAB* that is designed in a shape of a sector of a circle, where its radius is r m and angle *AOB* is θ radians. The area of the pond is 10 m².

[The arc length of a sector is $r\theta$ and area of a sector is $\frac{1}{2}r^2\theta$.]

(i) Given that the perimeter of the pond is P m, show that $P = 2r + \frac{20}{r}$. [3]

Given that *r* and θ can vary,

6

(ii) find the value of r for which P has a stationary value, [3]

(iii) determine whether this value of *P* is a maximum or a minimum.

[2]

Answer Key

1a	$-(1+4x)^{10}\sin x + 40(1+4x)^9\cos x$
1b	$4 \tan x - (4x - 5)(\sec^2 x)$
	$\tan^2 x$
2	$f'(x) = \frac{-3}{(3x+2)^2} + 2x$
	For $x < -1$,
	$(3x+2)^2 > 0$ then $\frac{-3}{(3x+2)^2} < 0$
	For $x < -1$,
	2x < -2 < 0
	\therefore f'(x) < 0
	Hence, f is a decreasing function.
3	A(2, 9); k = 47
4i	13.4 cm
4ii	0.223 cm/s
5i	$12\cos 3x \left(1+\sin 3x\right)^3$
5ii	$\frac{\pi}{6}$ and $\frac{\pi}{2}$
6ii	r = 3.16
6iii	<i>P</i> is maximum

