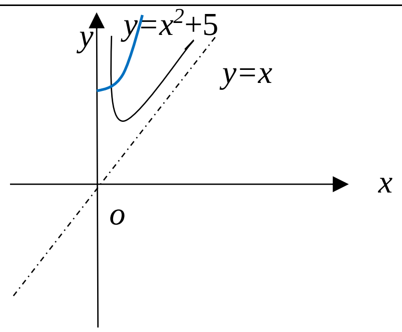
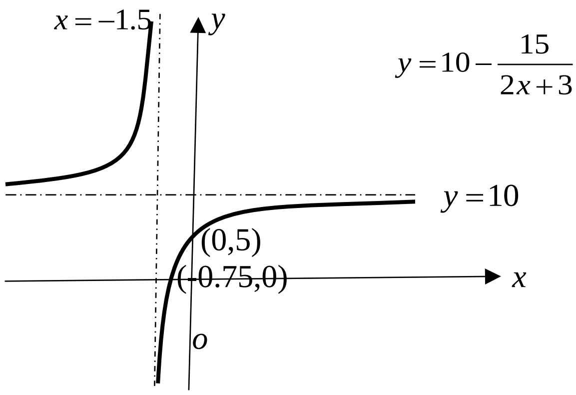
2022 JC2 H1 Math Prelim Marking Scheme

<table>
<tr><th>Qn</th><th>Solution</th></tr>
<tr><td>1</td><td>Let the amount invested in Banks A, B and C be x, y and z respectively.
We have
$$x+y+z=25000---(1)$$
$$0.06x+0.07y+0.08z=1620----(2)$$
$$y-z=6000--(3)$$
By GC, $x=15000,\ \ y=8000,\ \ z=2000$.
$\therefore$ the amount invested in Banks A, B and C are $15000, $8000 and $2000 respectively.</td></tr>
<tr><td>2(a)</td><td>$\int \frac{3x^2+1}{2\sqrt{x}}\mathrm{d}x$
$=\int \frac{3}{2}x^{\frac{3}{2}}+\frac{1}{2}x^{-\frac{1}{2}}\mathrm{d}x$
$=\frac{3}{5}x^{\frac{5}{2}}+\sqrt{x}+c$
where c is an arbitrary constant</td></tr>
<tr><td>2(b)</td><td>$\frac{\mathrm{d}}{\mathrm{d}x}\ln\left(\frac{\sqrt{4x-3}}{x^2-1}\right)$
$=\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{1}{2}\ln(4x-3)-\ln\left(x^2-1\right)\right]$
$=\frac{2}{4x-3}-\frac{2x}{x^2-1}$</td></tr>
<tr><td>3i</td><td>$\frac{dy}{dx}=1-\frac{4}{x^2}$
$1-\frac{4}{x^2}>0$
$1>\frac{4}{x^2}$
$x^2>4$
$x^2-4>0$
$(x-2)(x+2)>0$
$x<-2$ or $x>2$
(rejected since $x>0$)</td></tr>
</table>

3ii	
3iii	$x^3+5x\le 4+x^2$ $x\left(x^2+5\right)\le 4+x^2$ since x is positive, $x^2+5\le \frac{4}{x}+x$ Add $y=x^2+5$ to C, intersection pt is (0.842,5.68) $\therefore\ 0<\mathrm{x}\le 0.824$
4(i)	
4(ii)	$10-\frac{15}{2x+3}=mx+5$ $\frac{20x+30-15}{2x+3}=mx+5$ $20x+15=\left(mx+5\right)\left(2x+3\right)$ $2mx^2+\left(3m-10\right)x=0$ discriminant > 0 $\left(3m-10\right)^2>0$ $m\in\mathbb{R},\ m\ne \frac{10}{3},0$

4(iii)	$\int_{-\frac{3}{4}}^{0} 10-\frac{15}{2x+3}\ \mathrm{d}x+\frac{1}{2}\left(\frac{5}{2}\right)\ 5$ $=\left[10x-\frac{15}{2}\ln\|2x+3\|\right]_{-\frac{3}{4}}^{0}+\frac{25}{4}$ $=\left(0-\frac{15}{2}\ln 3\right)-\left(-\frac{15}{2}-\frac{15}{2}\ln\frac{3}{2}\right)+\frac{25}{4}$ $=-\frac{15}{2}\ln 3+\frac{15}{2}+\frac{15}{2}\ln 3-\frac{15}{2}\ln 2+\frac{25}{4}$ $=-\frac{15}{2}\ln 2+\frac{55}{4}$ $=\frac{1}{4}\ 55-30\ln 2$ $p=55,\ q=30$
5i	$C=20x-5\sqrt{x}+0.01x^2+500$
5ii	$\left(x,S\right)=(0,420)$ and $(45,60)$ $m=\frac{420-60}{0-45}=-8$ sub $(0,420)$ and $m=-8$ into straight line: $S=mx+c$ $420=-8(0)+c$ $c=420$ $\therefore S=-8x+420$
5iii	Revenue = $\left(-8x+420\right)x$ Profit = revenue - cost = $-8x^2+420x-20x+5\sqrt{x}-0.01x^2-500$ = $-8.01x^2+400x-500+5\sqrt{x}$ $\therefore a=-500$
5iv	$\frac{\mathrm{d}P}{\mathrm{d}x}=\frac{5}{2\sqrt{x}}+400-16.02x$ $\frac{\mathrm{d}P}{\mathrm{d}x}=0$ $\frac{5}{2\sqrt{x}}+400-16.02x=0$ using GC graph, find x-intercept $\Rightarrow\ x=25$ $\therefore S=-8(25)+420=220$ million dollars =\$220000,000 Method 1: $\frac{\mathrm{d}^2P}{\mathrm{d}x^2}=-\frac{5}{4}x^{-\frac{3}{2}}-16.02=-16.03<0\ \therefore\ P$ is maximum. Method 2:

x	24.99	25	25.01
$\frac{dP}{dx}$	0.160	0	-0.160
shape	/	—	\

$\therefore$ P is maximum.

5v

when $x = 40$,

$\frac{dP}{dx} = -240.40 = -240$ million dollars/plane

When the production level increased from 40 to 41 planes,
the profit decreases by approximately $240 million dollars.

The production level should be decreased in this case.

6(i)

The probability of an employee is working from home is constant at 0.37.
Whether an employee is working from home is independent of any other employee.

(ii)

Let X be the number of employees from the 30 who are working from home.

$X \sim B(30, 0.37)$

$P(X > 18) = 1 - P(X \leq 18)$

$= 0.0030511 = 0.00305$ (3sf)

(iii)

$X \sim B(30, 0.37)$

Mean $= 30 \times 0.37 = 11.1$ (exact)

Variance $= 30 \times 0.37 \times (1 - 0.37) = 6.993$ (exact)

6(iv)

Method 1:
Probability of a company getting the monetary incentive
$= P(X > 18) = 0.0030511$

Answer: $0.0030511 \times (1 - 0.003511) \times 2 = 0.00608$

Method 2:
Let Y be the no. of companies that will get the monetary incentive out of 2.

$Y \sim B(2, 0.0030511)$

$P(Y = 1) = 0.00608$

7a (i)

$N_1N_2N_3N_4N_5N_6N_7$ S_1S_2 = 3 object to arrange

Number of ways $= 8! \times 3! = 241\,920$

7a (ii)

Considering only the 8 north zone and 2 south zone schools
Method 1:
There are 3 cases:

Case 1: S _ _ _ _ _ _ _ S _ _ $2! \times 8! = 80640$

Case 2: _ S _ _ _ _ _ _ _ S _ $2! \times 8! = 80640$

Case 3: _ _ S _ _ _ _ _ _ _ S $2! \times 8! = 80640$

Number of ways = $3 \times 80640 = 241920$

$\times 6!$

Method 2:

S_1_ _ _ _ _ _ _ S_2 N_1 N_2 = 3 object to arrange

Number of ways $= {}^{8}C_6 \times 6! \times 2! \times 3! = 241\ 920$

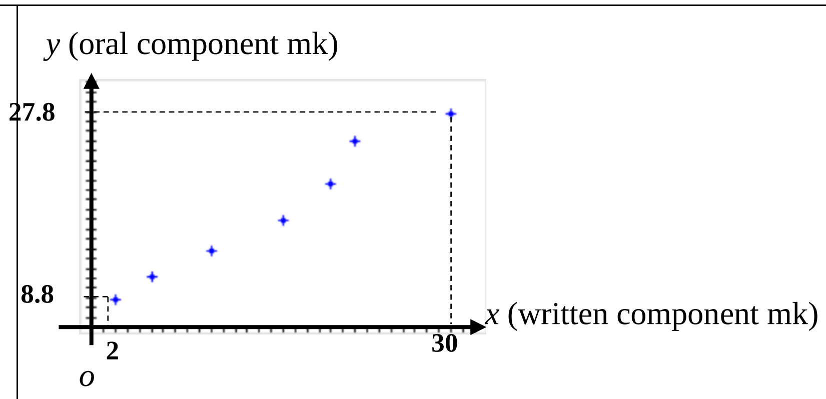
7b

	East (out of 14)	West (out of 6)
Case 1	1	6
Case 2	2	5
Case 3	3	4

Number of ways $= {}^{14}C_1 \times {}^{6}C_6 + {}^{14}C_2 \times {}^{6}C_5 + {}^{14}C_3 \times {}^{6}C_4 = 6020$

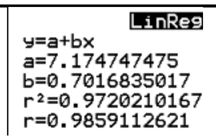
8(i)	unbiased estimate of the population mean $= \frac{\sum x}{60} = \frac{231}{60} = 3.85$ (exact) unbiased estimates of the population variance $= \frac{60}{59}\left(\frac{\sum\left(x-\bar{x}\right)^2}{60}\right) = \frac{12.733}{59} = 0.215813 = 0.216$ (3sf)
(ii)	Let μ cm be the average body length of the cockroach. Let X be the body length (in cm) of a randomly chosen cockroach. Test $H_o : \mu = 4$ (claim) Against $H_1 : \mu \neq 4$ (researcher's suspicion) at 1% sig. level Under H_0, $\bar{X} \sim N\left(4, \frac{0.21581}{60}\right)$ approx. by Central limit theorem since $n = 60$ large, Test Stats value = – 2.501 p – value = 0.0124 > 0.01 (do not reject H_0) There is insufficient evidence at 1% significance level to conclude that the mean body length of the cockroach is not 4 cm.
(iii)	Test $H_o : \mu = 4$ (claim) Against $H_1 : \mu \neq 4$ (researcher's suspicion) at 5% sig. level Under H_0, $\bar{X} \sim N\left(4, \frac{0.22}{60}\right)$ approx. by Central limit theorem since $n = 60$ large, Test statistics $z = \frac{k-4}{\sqrt{0.22/60}}$ 0 The given conclusion is to reject H_0, therefore the test stats z lies in the critical/rejection region. $\frac{k-4}{\sqrt{0.22/60}} < -1.96$ or $\frac{k-4}{\sqrt{0.22/60}} > 1.96$ $\Rightarrow k < 3.88$(3sf) or $k > 4.12$ (3sf)

9(i)



(ii)

r = 0.986 (3sf)
There is a **strong** and **positive** linear correlation between the marks for the written component and the oral component.
As the marks for the written component increase, the oral component marks also increases.



(iii)

$y = 0.702x + 7.17$
$a = 7.17$ (2sf)   and $b = 0.702$ (3sf)

<u>Significance of gradient b:</u>
For each increase of 1 mark in the written component (x), the oral component (y) increases by 0.702 marks.

<u>Significance of y-intercept a:</u>
The oral component (y) mark is expected to be 7.17
($x = 0$, $y = 7.17$) if a student scores zero mark for the written component (x).

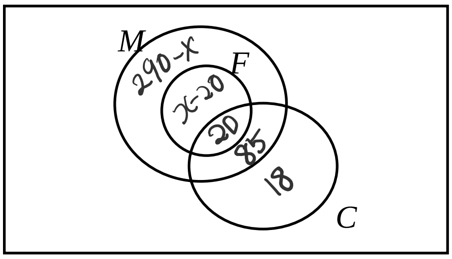
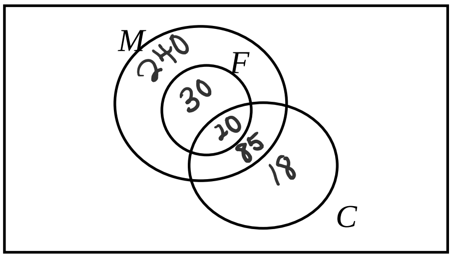
(iv)

Estimated oral component mark
$y = 0.70168(17) + 7.1747 = 19.1$ marks

r = 0.986 is **very close to +1**, therefore there is a **strong positive linear correlation** between the written component marks and the oral component marks.

$x = 17$ is within the data range of x (i.e. $2 \le x \le 30$). This is an interpolation.

Hence the estimate for the oral component mark is reliable.

10(i)	 no. of pupils who study Mathematics only $= 393-(x-20)-20-85-18=290-x$ no. of pupils who study Physics but not Chemistry $=(x-20)$ $290-x=8(x-20)$ $\therefore\ x=50$ 
10(ii)	$P(M\cap C)=\frac{105}{393}=\frac{35}{131}=0.267(3sf)$
10(iii)	$\mathrm{P}(M)=\frac{375}{393}=\frac{125}{131}$ $\mathrm{P}(C)=\frac{123}{393}$ $\mathrm{P}(M)\mathrm{P}(C)=\frac{5125}{17161}=0.299(3sf)\neq 0.267=\mathrm{P}(M\cap C)$ $\therefore\ M$ and C are not independent events.
10(iv)	The probability of a student taking Mathematics given that the student also takes Chemistry. $\mathrm{P}(M \mid C)=\frac{\mathrm{P}(M\cap C)}{\mathrm{P}(C)}=\frac{105/393}{123/393}=\frac{35}{41}$
10(v)	Number of students taking exactly two subjects=30+85=115 Method 1: $\frac{{}^{115}C_2\,{}^{278}C_1}{{}^{393}C_3}=0.182(3s.f)$ Method 2: $\frac{115}{393}\cdot\frac{114}{392}\cdot\frac{278}{391}\cdot 3=0.182(3s.f)$

11 (i)

Let M be mass of a medium size egg, $M \sim \text{N}\left(49.6\ ,\ \sigma^2\right)$

$$\text{P}(M > 55) = 0.17173$$

$$\text{P}\left(Z > \frac{55-49.6}{\sigma}\right) = 0.17173 \qquad Z \sim \text{N}(0,1)$$

By GC,

$$\frac{55-49.6}{\sigma} = 0.94735 \rightarrow \sigma = 5.70$$

Variance = $\sigma^2 = (5.7001)^2 = 32.5$

(ii)

Let L be the mass of a large size egg, $L \sim \text{N}\left(56.8\ ,\ 6.1^2\right)$

$L_1 + L_2 + L_3 + L_4 \sim \text{N}\left(4\times 56.8\ ,\ 4\times 37.21\right)$ i.e. $\text{N}\left(227.2\ ,\ 148.84\right)$

$$P\left(L_1 + L_2 + L_3 + L_4 > 250\right) = 0.030822 = 0.0308 \text{ (3sf)}$$

(iii)

Let X be no. of packs of large egg with mass > 250grams

Distribution $X \sim \text{B}\left(n\ ,\ 0.030822\right)$

$$P\left(X \geq 1\right) < 0.9$$

$$1 - P\left(X = 0\right) < 0.9$$

$$P\left(X = 0\right) > 0.1$$

$${}^{n}C_0\left(0.030822\right)^0\left(1-0.030822\right)^n > 0.1$$

$$\left(0.96918\right)^n > 0.1$$

Method 1:	Method 2:
By GC, let $y = \left(0.96918\right)^n$	$\left(0.96918\right)^n > 0.1$
n = 72, y = 0.105	$n < \dfrac{\ln(0.1)}{\ln(0.96918)}$
n = 73, y = 0.1017 > 0.1	$n < 73.55$
n = 74, y = 0.0986	

Greatest value of n is 73.

(iv)

Let $T = \left(M_1 + M_2 + M_3 + M_4 + M_5 + M_6\right) - 1.5\left(L_1 + L_2 + L_3 + L_4\right)$ where

$\text{T} \sim \text{N}\left(-42.2\ ,\ 167.21\right)$

$$E(T) = 49.6\times 6 - 1.5\left(56.8\times 4\right) = -43.2$$

$$\text{Var}(T) = 6\times 32.5 + (1.5)^2\left(37.21\times 4\right) = 529.89$$

$$P\left(M_1 + M_2 + M_3 + M_4 + M_5 + M_6 > 1.5\left(L_1 + L_2 + L_3 + L_4\right)\right)$$

$$= P\left(\left(M_1 + M_2 + M_3 + M_4 + M_5 + M_6\right) - 1.5\left(L_1 + L_2 + L_3 + L_4\right) > 0\right)$$

$= 0.0303$ (3sf)

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normalcdf
lower:0
upper:E99
μ:-43.2
σ:√(529.89)
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