2023 4E EMath Prelim
Monfort Secondary School

Answer **all** questions.

1 Given that $2^{-w}+2^{-w}+2^{-w}+2^{-w}=8^{w}$, find w.

Answer $w =$ ..... [2]

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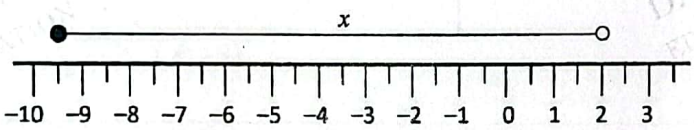
2 A range of values for x is represented on the number line below.



Write down inequalities that represent this range of values of x.

Answer ..... [1]

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3 A survey was done to find the number of hours each student spent on social media per day. The results are shown in the table below.

Number of hours (hrs)	5	6	7	8	9
Number of students	2	8	6	x	5

(a) Find the range.

Answer .....hrs [1]

(b) Calculate the smallest possible value of x when the median is 8.

Answer ..... [1]

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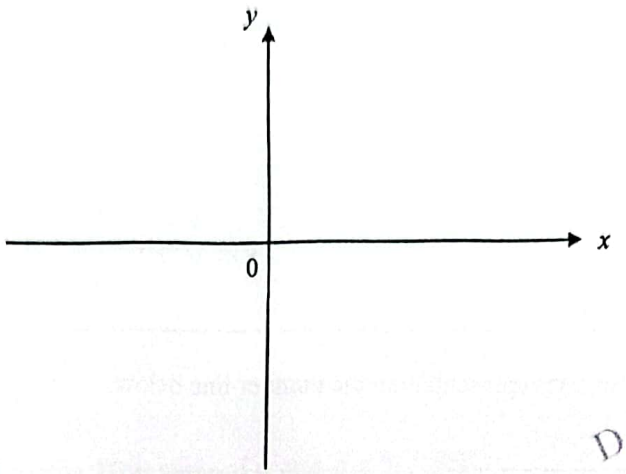
4 **Sketch** the graph of $y = (3-x)(x+10)$ on the axes below.

Indicate clearly the values where the graph crosses the x- and y- axes.



[2]

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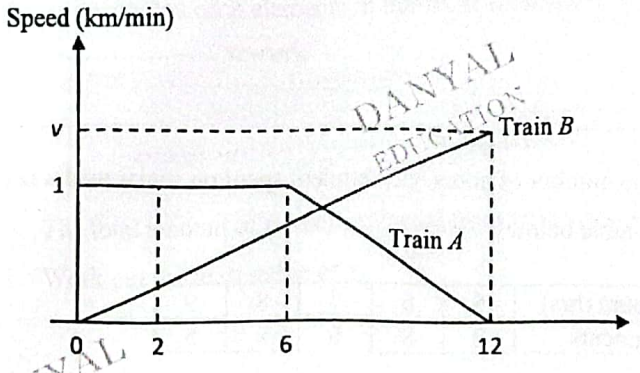
5 The diagram shows the speed-time graphs of both trains during a period of 12 minutes. Train A and B started from the same point at the same time and travel in the same direction.



(a) Calculate the retardation of Train A at 11 minutes.

Answer .....km/min^2 [1]

(b) Calculate the value of v, the speed of Train B at the end of 12 minutes, given that the two trains travelled the same distance during the period of12 minutes.

Answer ..... km/min [1]

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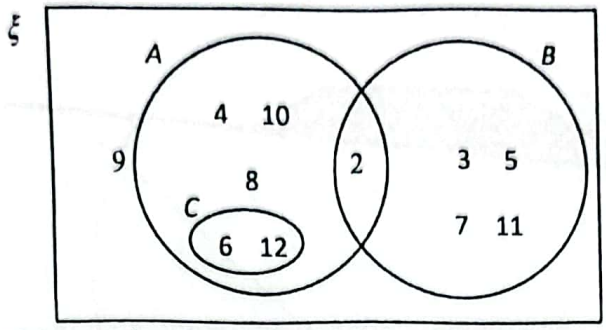
6 $\xi = \{\text{integers } x : 2 \le x \le 12\}$

The Venn diagram shows the elements of ξ and three sets A, B and C.



Use one of the notations below to complete each statement.

$\not\subset \quad \subset \quad \in \quad \phi \quad \notin$

(a) 9 ..... $(A \cup B)'$ [1]

(b) $\{3, 5\}$ ..... A [1]

(c) $B \cap C =$ ..... [1]

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7 A swimming pool is 60% full.
16% of the water in the swimming pool is removed.
There are 1260 litres of water in the pool.

Calculate the capacity of the swimming pool when full.

Answer ..... litres [2]

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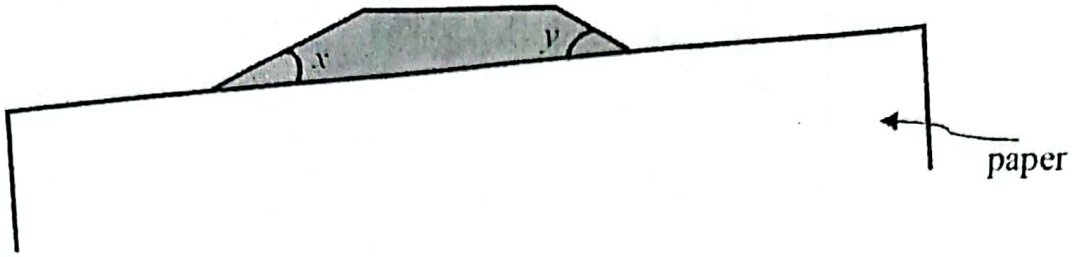
8 In the figure below, a regular shaded polygon is partially covered with a sheet of blank paper. Given that $x + y = 80°$, calculate the number of sides this polygon has.



Answer .....sides [3]

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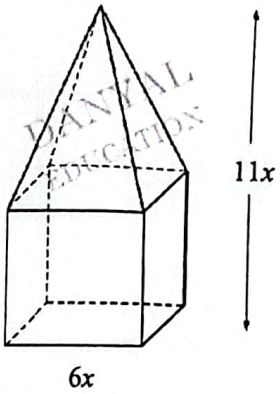
9 A solid shape consists of a cube with a pyramid on top has a total height of $11x$ cm. The pyramid sits perfectly on one surface of the cube.

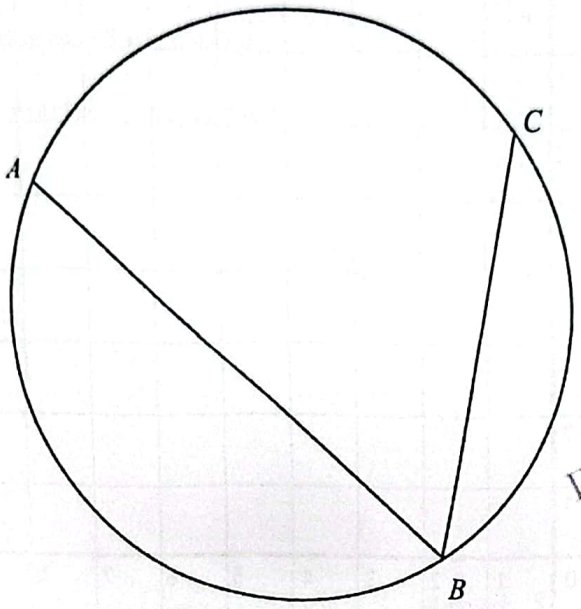
Each side of the cube is $6x$ cm.
Find an expression, in terms of x, for the surface area of the solid.



Answer .....cm^2 [3]

10 The diagram shows a circle with two chords AB and BC.

Answer



(a) Construct the perpendicular bisector of AB. [1]

(b) Construct the bisector of angle ABC. [1]

(c) Shade the region inside the circle that is closer to AB than to BC and closer to B than to A. [1]

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11 Solve $\frac{x-3}{4} - \frac{x+2}{3} = \frac{1}{2}$.

Answer $x =$ ..... [3]

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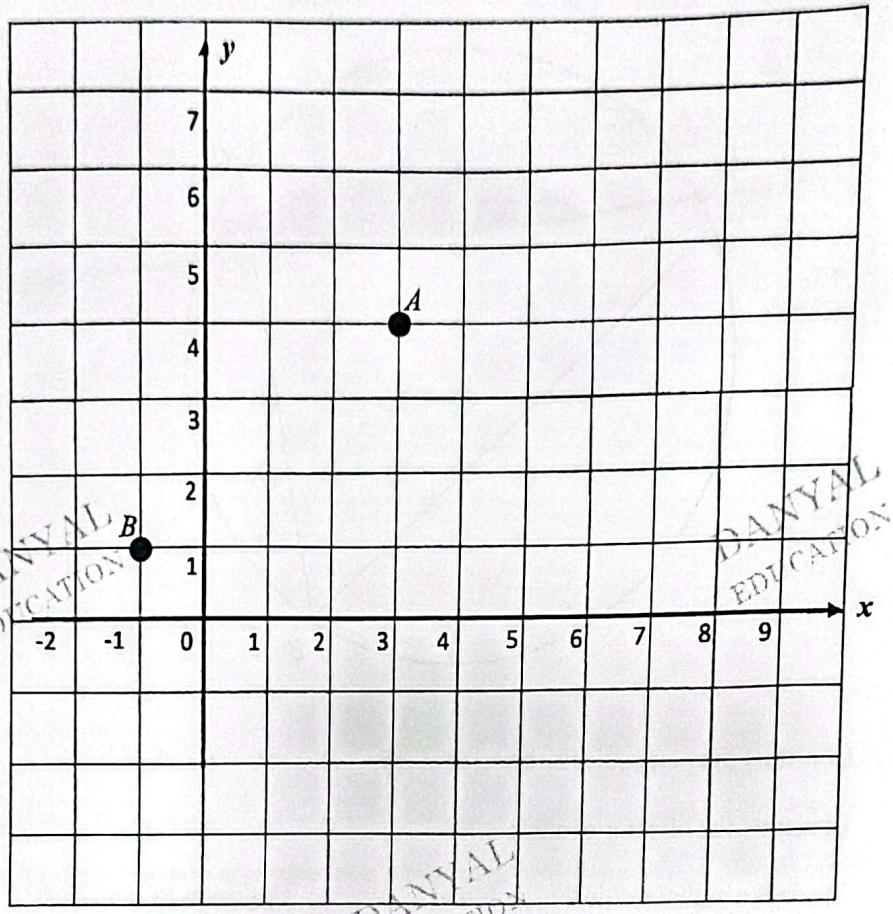
12 On the grid below, the point A is $(3, 4)$ and the point B is $(-1, 1)$.



(a) Mark out and label point C such that $\overrightarrow{BC} = \begin{pmatrix} 7 \\ -2 \end{pmatrix}$. [1]

(b) Find $|\overrightarrow{BC}|$.

Answer $|\overrightarrow{BC}| =$ ..... [1]

13 John deposits a sum of money in a bank that pays a compound interest of 3.8% per year. After 5 years, the money is expected to earn a total interest of $1619.50

Calculate the sum of money John deposits.

Give your answer correct to the nearest dollar.

Answer $ ..... [2]

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14 (a) Factorise $2x^2 - 7x - 15$.

Answer ..... [2]

(b) Hence, factorise $2(3y-1)^2 - 7(3y-1) - 15$.

Write your answer as simply as possible.

Answer ..... [2]

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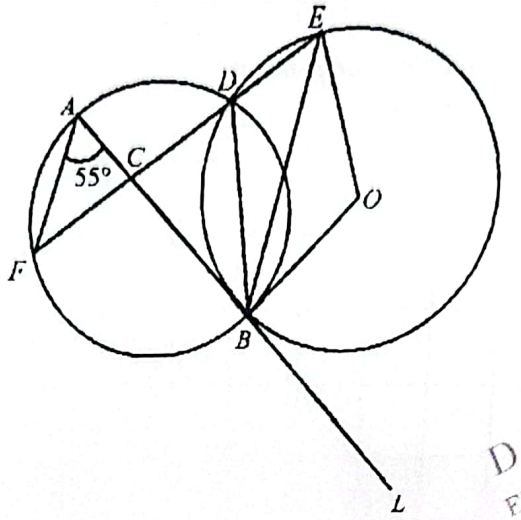
15 In the diagram, $ACBL$ is a tangent to the circle DEB with centre O, at B. $\angle CAF = 55°$ and $FCDE$ is a straight line.



Find, stating the reasons clearly,

(a) $\angle BDE$,

Answer $\angle BDE$ =.....° [2]

(b) $\angle ABE$,

Answer $\angle ABE$ =.....° [3]

16 Ethan observed that the queue at Stall *A* in his school's canteen on a particular day. He decided to do a survey to improve the current situation.

Queueing Time (t seconds)	$0 \le t < 40$	$40 \le t < 80$	$80 \le t < 120$	$120 \le t < 160$	$160 \le t < 200$	$200 \le t < 240$
Number of students	6	20	24	30	32	8

(a) Calculate an estimate of the mean queuing time.

Answer .....seconds [1]

(b) Calculate an estimate of the standard deviation of these times.

Answer .....seconds [1]

(c) Eddie claims that 80% of students queuing at Stall *A* had to wait at most 180 seconds.
Is Eddie's claim true?
Explain your answer.

Answer .....

.....

.....

..... [2]

17 The first four terms of a sequence are

$$278,\ 269,\ 260,\ 251,\ \ldots\ldots$$

(a) Write down the 8^{th} term of the sequence.

Answer ..... [1]

(b) Write down an expression, in terms of n, for the n^{th} term of the sequence.

Answer ..... [1]

(c) Find the first negative term of the sequence.

Answer ..... [2]

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18 An area of 5 cm^2 on a map represents an actual area of 32 000 m^2.
Find the linear scale of the map, giving your answer in the form 1 : n.

Answer 1 : ..... [2]

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19 The cash price of a laptop is $x.
John bought the laptop on hire purchase.
He paid a deposit of one-third of the cash price followed by 18 monthly instalments of $120.
Given that the total amount he paid for the laptop is $3300, find the value of x.

Answer $x =$ ..... [2]

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20 At a sale, all prices are reduced by 30%.
The price of a watch during the sale is $693.

(a) Find its original price.

Answer $..... [2]

(b) The sale price of the watch is exclusive of 8% Goods and Services Tax (GST).
Find the amount of GST payable.

Answer $..... [1]

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21 $a = \dfrac{b^2 + 44}{b^2 - c}$

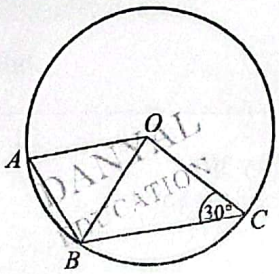
(a) Find a when $b = -8$ and $c = -11$.

Answer $a =$ ..... [1]

(b) Rearrange the formula to make b the subject.

Answer $b =$ ..... [3]

22



In the diagram, A, B and C are points on a circle, centre O.
Angle $OCB = 30°$ and angle OAB is 2.5 times of angle OCB.

(a) Find reflex angle of AOC.

Answer .....° [3]

(b) Explain why AO is parallel to BC.

.....

.....

..... [2]

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23 **(a)** Express 6300 as a product of its prime factors.

Answer 6300 = ... [1]

(b) Given that $6300 \times 15p = q^3$, where p and q are integers.
Find the smallest values of p and q.

Answer p = ...

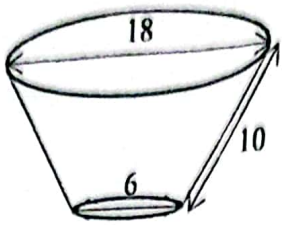
q = ... [2]

(c) The lowest common multiple of the two numbers is 6300.
The highest common factor of the two numbers is 6.
Both numbers are greater than 100.

Find the two numbers.

Answer ..... and ..... [2]

24



The figure shows a solid in the form of a frustum.
Its circular top and base have diameters 18 cm and 6 cm respectively.
The slant height is 10 cm long.

(a) Find the height of the frustum.

Answer .....cm [2]

(b) Find, in its simplest form, the ratio of the volume of the original cone to that of the frustum.

Answer ..... : ..... [1]

(c) Calculate the total surface area of the frustum, leaving your answer in terms of π.

Answer .....cm^2 [4]

25 (a) p is directly proportional to cube root of q.
Given that $q = 125$ and $p = 3$,

(i) find an expression for p in terms of q

Answer $p =$ ..... [2]

(ii) find the value of q when $p = 0.2$.

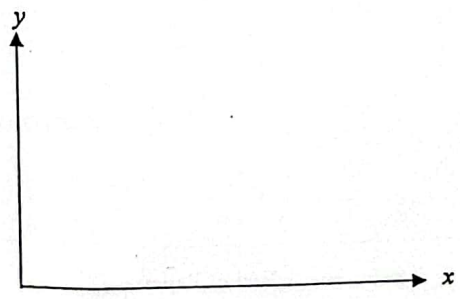
Answer $q =$ ..... [1]

(b) y is inversely proportional to x.

(i) When x has a certain value, $y = a$.
Find an expression of y, in terms of a, when x is halved.

Answer $y =$ ..... [2]

(ii) Sketch the graph of y against x.



[1]

26 Tickets to a carnival cost \$10 for adults (A), \$8 for senior citizens (S) and \$5 for children (C). This information can be represented by the matrix **Q** below.

$$\mathbf{Q} = \begin{pmatrix} 10 \\ 8 \\ 5 \end{pmatrix}$$

(a) 68 adults, 15 senior citizens and 70 children bought tickets through ticket counter.
x adults and 88 children bought tickets through online.
Represent this information in a 2×3 matrix **P**.

Answer **P** = A S C
( )
counter
online [1]

(b) Find the matrix **R**, in terms of x such that **R** = **PQ**.

Answer **R** = ( ) [2]

(c) Explain what each elements in matrix **R** represents.

Answer .....

..... [1]

(d) The total amount of money collected from ticket counter is less than online sales.
Work out the least value of x.

Answer x = ... [2]

(e) During a promotion, there is a 15% discount for adults, 25% discount for senior citizens and 20% discount for children.

Write down matrix **D** such that the elements in matrix multiplication of **PDQ** gives the amount of money collected from the sales of tickets through ticket counter and online respectively after discount.

Answer **D** = ( ) [1]

End of Paper