ANDERSON SERANGOON JUNIOR COLLEGE

H2 MATHEMATICS

9758/02

3 hours

JC2 Preliminary Examination Paper 2
(100 marks)

Additional Material(s): List of Formulae (MF26)

CANDIDATE NAME	
CLASS	/

READ THESE INSTRUCTIONS FIRST

Write your name and class in the boxes above.
Please write clearly and use capital letters.
Write in dark blue or black pen. HB pencil may be used for graphs and diagrams only.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions and write your answers in this booklet.
Do not tear out any part of this booklet.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

All work must be handed in at the end of the examination. If you have used any additional paper, please insert them inside this booklet.
The number of marks is given in brackets [ ] at the end of each question or part question.

Question number	Marks
1	
2	
3	
4	
5	
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7	
8	
9	
10	
11	
Total	

Section A: Pure Mathematics [40 marks]

1 Relative to the origin O, two fixed points, A and B, have position vectors **a** and **b** respectively. The point P has position vector **p** given by $\mathrm{p} = \lambda \mathrm{a} + (1-\lambda)\mathbf{b}$ where λ is a parameter such that $0 < \lambda < 1$.

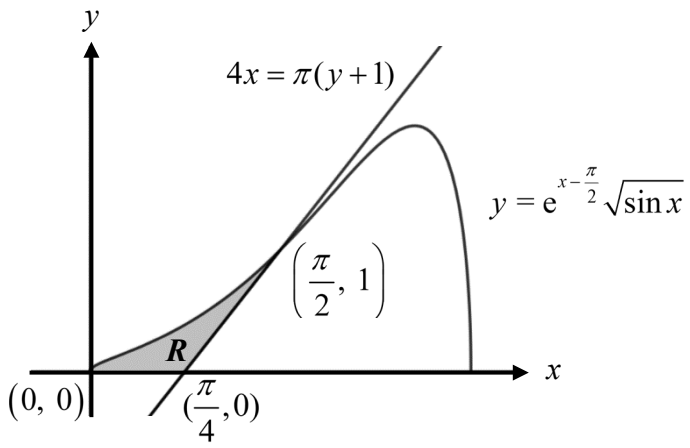
(a) Show that for all real values of λ, the point P is collinear with A and B. [2]

(b) If angle AOB is 90^0, show that the position vector of the foot of perpendicular from O to AB is $\mathbf{a} + \dfrac{|\mathbf{a}|^2}{|\mathbf{a}|^2 + |\mathbf{b}|^2}(\mathbf{b} - \mathbf{a})$. [3]

2 One of the roots of the equation $z^4 - 4z^3 + az^2 + bz + 78 = 0$, where a and b are real, is $3 + 2\mathrm{i}$. Find the values of a and b. Hence find the other roots of the equation, giving your answers in exact form. [6]

3 (a) Integrate by parts twice to show that $\int \mathrm{e}^{2x} \sin x \, \mathrm{d}x = \dfrac{1}{5}\mathrm{e}^{2x}(2\sin x - \cos x) + c$. [4]

(b) The diagram below shows the shaded region R that is bounded by part of the curve $y = \mathrm{e}^{x-\frac{\pi}{2}}\sqrt{\sin x}$, the line $4x = \pi(y+1)$ and the x – axis. The line $4x = \pi(y+1)$ cuts the curve and the x – axis at $\left(\dfrac{\pi}{2}, 1\right)$ and $\left(\dfrac{\pi}{4}, 0\right)$ respectively.



Show that the volume generated when region R is rotated 2π radians about the x – axis is given by $A\pi\left(B + \mathrm{e}^{-\pi}\right) + C\pi^2$, where A, B and C are constants to be determined. [4]

4 The functions f and g are defined as follows:

$$\mathrm{f}: x \mapsto \left|\mathrm{e}^{x-3} - 1\right|, \quad x \leq 3$$

$$\mathrm{g}: x \mapsto a^2 - x^2, \quad x \in \mathbb{R}, \quad x < 0, \text{ where } 0 < a < 1.$$

(a) Find f^{-1} in similar form. [3]

(b) Sketch the graphs of $y = \mathrm{g}(x)$, $y = \mathrm{g}^{-1}(x)$ and $y = \mathrm{g}^{-1}\mathrm{g}(x)$ on the same diagram. [3]

(c) Find the exact solution of $\mathrm{g}(x) = \mathrm{g}^{-1}(x)$, leaving your answer in terms of a. [2]

(d) Show that the composite function fg exists and find the range of fg, leaving your answer in terms of a. [2]

5 The curve C is given by the equation $\dfrac{y^2}{x^2} = \dfrac{3a - x}{a + x}$, $x \neq 0$, where a is a positive constant.

(a) Use an algebraic method to determine the range of values for which x can take, leaving your answer in term of a. [3]

For the rest of this question, take $a = 1$.

(b) Find the coordinates of the point(s) at which the tangent is parallel to the x-axis. [4]

(c) The normals to C at the points where $x = 1$ intersect each other at the point N. Find the coordinates of N. [4]

Section B: Probability and Statistics [60 marks]

6 In a chemical reaction, a chemical mixture is left to cool down to room temperature. The temperature of the chemical mixture, θ °C at time t minutes is given in this table.

Time (t minutes)	1	6	12	18	24	30
Temperature (θ °C)	97	62	45	37	31	27

(a) Sketch a scatter diagram for the data given in the table. [2]

A student attempts to model the relationship between θ and t using two models:

(A) $\theta = a + bt$ or (B) $\theta = c + \frac{d}{t}$.

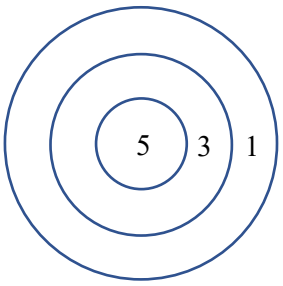
(b) By calculating the product moment correlation coefficients and referring to the scatter diagram, explain clearly which of the two models is a more appropriate model for this set of data. [2]

(c) Use the model you identified in **(b)** to find the equation of a suitable regression line and use your equation to estimate to predict the temperature when the time is 45 minutes. Comment on the reliability of the estimate. [3]

A temperature of T Kelvins is equivalent to a temperature of θ degrees Celsius, where $T = \theta + 273.15$.

(d) Re-write your equation in **(c)** so that it can be used when the temperature of the chemical, θ, is given in Kelvins. [1]

7 A circular target board is made up of 3 concentric circles as shown below.



The circles have the same centre and their radii are 10 cm, 20 cm and 30 cm respectively. A player plays a game of darts using this target board. The probability of the player missing the target board completely is $\frac{1}{5}$ and the player scores no point if he misses the target board. It can be assumed that he is equally likely to land his dart anywhere on the target board if he does not miss the target board.

The player scores 5 points if the dart lands on the inner most circle, 3 points if the dart lands between the inner most and the middle circles, and 1 point if the dart lands between the middle and the outer most circles. It can be assumed that the dart will not land on the circumference of the circles.

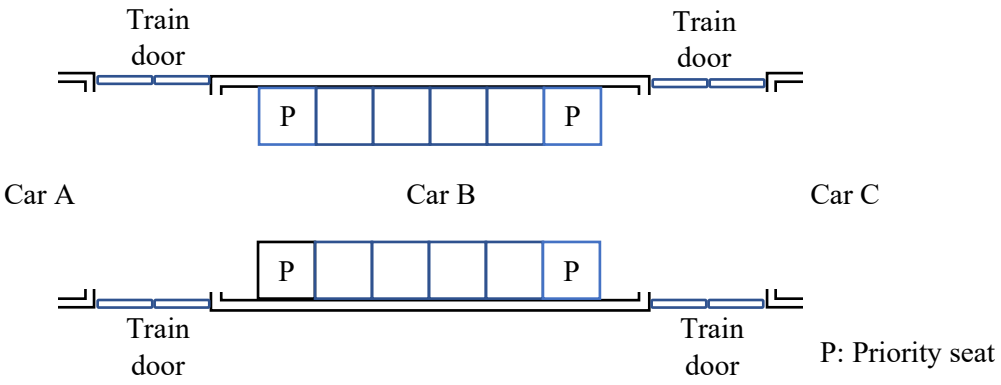
(a) Show that the probabilities that the player scores 5 points, 3 points and 1 point with 1 throw are $\frac{4}{45}$, $\frac{4}{15}$ and $\frac{4}{9}$ respectively. [3]

The player throws the dart twice and the random variable X denotes the higher score out of the 2 throws.

(b) Tabulate the probability distribution of X. [4]

(c) Find the exact value of $\mathrm{E}(X)$. [2]

8 Some MRT trains have cars featuring 12 seats each. Passengers take the trains to travel from station to station each day.



Seats located nearest to the train doors are labelled as priority seats reserved for vulnerable commuters, namely those who are pregnant, elderly, with infants, or the differently-abled. Anyone can sit at the priority seats if there are sufficient seats for all vulnerable commuters in the car.

For one trip, 12 passengers, of which 4 are vulnerable commuters, got onto Car B at the first station.

(a) Given that all the vulnerable commuters are seated at priority seats, find the total number of ways all the 12 passengers can all sit in Car B. [2]

(b) Find the number of ways the 12 passengers can be seated in Car B if

(i) two particular girls are seated in the same row but not beside each other. [3]

(ii) exactly 4 of the 5 girls are seated together. [4]

9 **In this question you should state clearly the values of the parameters of any normal distributions that you use.**

The masses, in grams, of apples and pears have the independent distributions $N(260, 15^2)$ and $N(210, 10^2)$ respectively.

(a) Find the probability that the mass of a randomly chosen apple is at least 250 grams.

(b) Find the probability that the mass of a randomly chosen apple differs from the mass of a randomly chosen pear by more than 55 grams. [2]

A certain recipe requires 5 apples and 6 pears. The recipe requires the apples and pears to be prepared by removing the cores. This process reduces the mass of each apple by 5% and the mass of each pear by 15%.

(c) Find the probability that the total mass, after preparation, of 5 randomly chosen apples and 6 randomly chosen pears is at least 2.3 kilograms. [3]

(d) Find $P(3(0.7W) - 0.8(V_1 + V_2) > 200)$. Explain, in the context of this question, what your answer represents. [4]

10 Epple Company manufactures a large number of E-phones. It is known that 1% of the E-phones manufactured are defective.

(a) A sample of 10 E-phones was randomly selected. Find the probability that at least 1 but less than 3 of them are defective. [2]

(b) Another sample of 10 E-phones was randomly selected. Find the probability that the tenth E-phone selected was the eighth E-phone that was not defective. [3]

A carton, which consists of 24 E-phones, will be rejected if there exists at least two defective E-phones.

(c) Find the probability that a randomly chosen carton is rejected. [2]

Samsong Company manufactures Galexy phones. It is known that 1.5% of Galexy phones are defective. Galexy phones are packed into cartons of 20.

(d) Moon-hub retailer ordered a total of 250 cartons of E-phones and 500 cartons of Galexy phones. By using an approximation, estimate the probability that the total number of defective Galexy phones exceed the total number of defective E-phones by at least 100. [5]

11 Korean dramas, or Kdramas, have been immensely popular among students, captivating their attention during leisure hours. Mr. Kim, a high school Math teacher, wants to investigate the average amount of time that students from junior colleges (JC) in Singapore spend watching Kdrama per day.

(a) To obtain a sample of 100 JC students, he decided to post a survey on his social media and collected the first 100 responses. Explain why the sample collected will not be a random sample. [1]

Ms. Shin, a teacher at KJC, claims that, on average, the students in her school watch 37 minutes of Kdrama per day. However, a student feels that Ms. Shin has overstated the average time spent. To investigate this, the student recorded the time spent per day watching Kdrama x, in minutes, of a random sample of 50 students. The summarized data is provided below.

$$\sum x = 1707 \qquad \sum x^2 = 71695$$

(b) Calculate unbiased estimates of the population mean and variance for the number of minutes spent watching Kdrama. [2]

(c) State the hypotheses that can be used to test if Ms Shin has overstated the average time spent, defining any symbols you use. Carry out the test at the 5% level of significance, giving your conclusion in the context of the question. [5]

(d) Explain "5% level of significance" in the context of the question. [1]

After the end-of-year exam, the principal of KJC conducted a whole school census survey in which every student participated. It was found that the variance of the time spent watching Kdrama was 285 minutes.

(e) Another random sample of 50 students is collected, and the mean time is labeled as k. Assuming that the time spent by a student watching Kdrama in a day is normally distributed, find the set of values of k for which there is sufficient evidence to conclude that the average time spent watching Kdrama is different from what Ms. Shin claimed at the 5% level of significance. [3]