Types of Forces Tutorial – Solution

<table>
<tr><td>Q1</td><td colspan="2">Ans: A
Compression of air, the fall of water droplets and the spreading of petrol all have very little resistance to its motion.
But when applying paint to the surface of a wall it would actually take more effort to ‘push’ the paint across the wall.</td></tr>
<tr><td>Q2</td><td>(a) Friction is the force which opposes the relative motion of the two bodies in contact or tends to oppose one body from moving relative to the other.

(b) (i)
1. Friction is useful in braking when the brakes of a moving car is stepped.

2. Friction is a useful force to enable walking by the floor exerting a forward force on the shoe/foot.

3. Friction provides the driving force of a car through the frictional force exerted by the road surface on the tire in the forward direction of the car.

4. In unloading heavy wooden crates from a lorry on an inclined plank, friction slows down the crates as the crates slide down the plank.

(ii) Without friction during braking, the car will skid and may result in accident.

Without friction between the floor and the shoe/foot, a person may slip and falls.

Without friction to provide driving, it is difficult to control a vehicle on the road which will result in accident.

Without friction during unloading heavy wooden crates on the incline plank, the crates may slide down too fast and cause injury to workers.</td><td>1

2

1</td></tr>
<tr><td>Q3</td><td colspan="2">Ans: B
For parallel combinations, $k_{eff} = k_1 + k_2$ (make sure you know how to derive this)
$F = k_{eff}\, x$
$10 = 2k\,(0.2)$
$k = 25\ Nm^{-1}$</td></tr>
<tr><td>Q4</td><td colspan="2">When $W = 0$ N, the length L is the natural length (L_o) of the spring.

From the graph, when $W = 0$ N, $L = L_o = 6$ cm

Using Hooke’s Law, $F_s = ke$ where F_s = force on spring, e = extension of spring, k = spring constant
Therefore, $k = \frac{F_s}{e}$
Using the point (5 N, 10 cm), $k = \frac{W}{(L - L_o)} = \frac{5}{(10-6)\times 10^{-2}}\ N\,m^{-1}$ = 125 N m^{-1}</td></tr>
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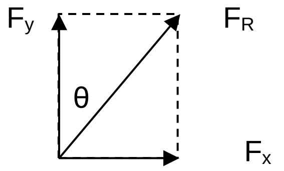
Q5 First, find the resultant force F_R of the 3 coplanar forces:



1

y-component of the resultant force, $F_y = 40$ N
x-component of the resultant force, $F_x = 50-20 = 30$ N

$\tan\theta = (30/40) \rightarrow \theta = 36.9°$

Resultant = $\sqrt{30^2 + 40^2} = 50$ N

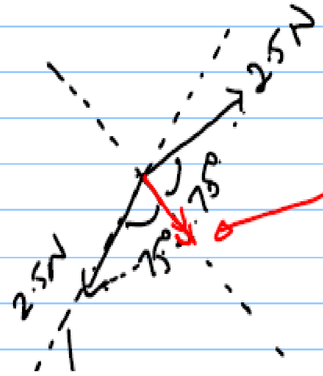
To maintain equilibrium, the counter force must be of same magnitude but in opposite direction to F_R. 1
Hence, **F = 50 N at bearing of 220° (2 s.f.)** 1

Q6 As this is an equilibrium system, resultant force is zero and resultant torque is zero.
Hence all three lines of force must intersect at a common point, and, when the force vectors are joined head-to-tail they should form a closed triangle with arrows pointing in the same direction (anticlockwise in this case).

Answer: B

Q7 Essential questions

1. What is this force that the question speaks of? → force of gum on tooth
2. What does it mean that the tooth is in equilibrium? → no net force acting on body, and no net moment about ANY POINT on the tooth.
3. Given the directions of the two 2.5 N forces, which is the only way that force can act to put the tooth in equilibrium? → Since the two 2.5 N forces give a resultant force that acts in the direction of X, the **3rd force must act opposite in direction (i.e. direction Z) and be of equal magnitude to the resultant of the two 2.5 N forces.**
4. For 3 coplanar forces that are in equilibrium, what other special result exists? → they form a closed triangle since no net force exists.



For equilibrium, the 3rd force must be equal & opp to this resultant force

⇒ Direction must be Z

∴ Answer (D)

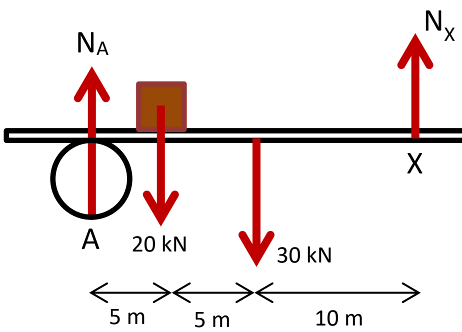
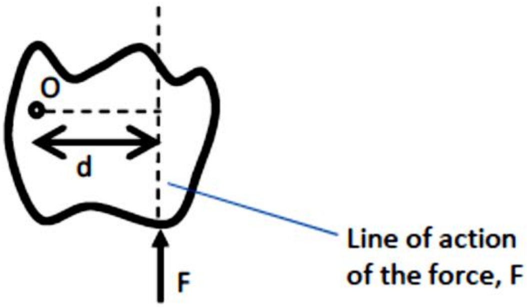
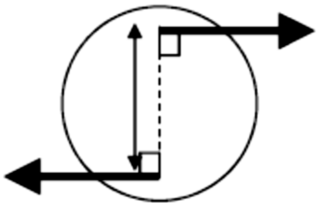
By elimination, answer is D.

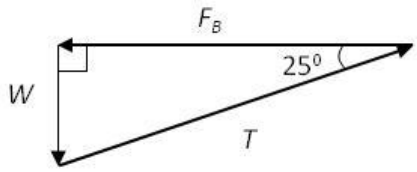
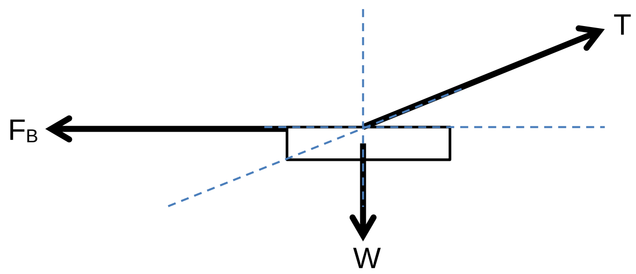
How to verify that magnitude is 1.3 N?

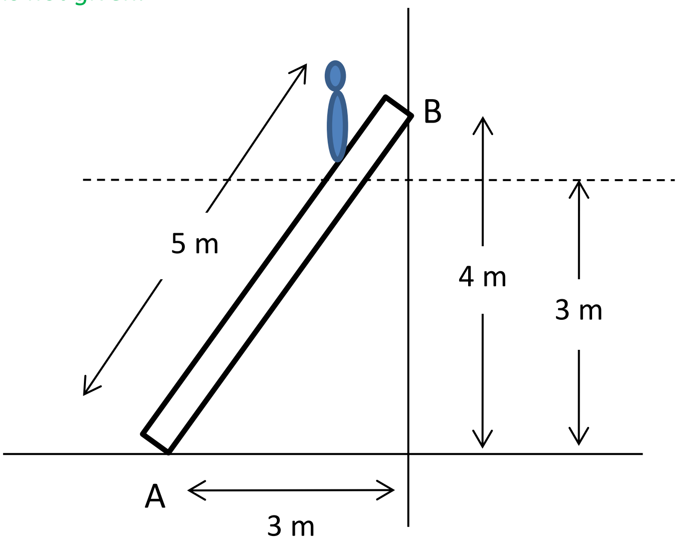
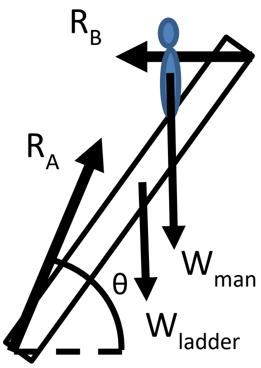
→ By sine rule, $\frac{F_{net\ of\ 2.5N\ forces}}{\sin 30°} = \frac{2.5\ N}{\sin 75°}$

$F_{net\ of\ 2.5N\ forces} = 1.294 = 1.3N$

Answer: D

Q8		Draw a **Free-body diagram** of **system comprising the Trailer & its load of 20 kN**, indicating all the external forces acting on the system and their respective perpendicular distances from the pivot. Let N_A and N_X be the upward forces exerted on the trailer at points A and X respectively.  Aim: To find N_X **By Principle of Moments**, **taking moments about point A**, **Sum o**f anticlockwise moments = **Sum of** clockwise moments N_X (10 + 5 + 5) = 20 (5) + 30 (5 + 5) N_X = 20 kN (1 s.f.)	 1 1
Q9	(a)	Moment of a force about any point is defined as the product of the force and the perpendicular distance from that point to the line of action of the force.  A couple are two forces of equal magnitude and acting in opposite directions whose lines of action are parallel but separate from one another. 	1 1 1 1
	(b)	For a body to be in equilibrium, the following two conditions must be met: 1. **No <u>net</u> force** acts on the body. 2. **No <u>net</u> moment of force <u>about any point</u>** acts on the body. OR 1. The vector sum of all forces acting on the body must be equal to zero 2. The vector sum of all moments of force acting on the body about any point must be equal to zero	 1 1

		Note: "Total clockwise moments = Total anticlockwise moments" is NOT acceptable because this only covers 2-D cases.		
	(c)	(i)	*2 key words : labelled and vector diagram* Let *T* be the pull exerted by the cable, W the weight of the section *S* and F_B the force exerted by B on S which is specified as a horizontal force in the question.  *Note:* • *The length of the arrows drawn must reflect the magnitude of the forces. Indicate the directions.* • *Vector diagram shows a closed polygon since forces are in equilibrium.* • *Question did not ask for a free-body diagram but a vector diagram. Thus a free-body diagram is not acceptable here.*	1 1
		(ii)	From the vector triangle, T = W / sin 25° = (3.0 x 10^5)/ sin 25º = 7.1 x 10^5 N (2 s.f.) F_B = W / tan 25° = (3.0 x 10^5)/ tan 25° = 6.4 x 10^5 N (2 s.f.)	1 1 1 1
		(iii)	**All three lines of force will meet at the same point.** If the free-body diagram is drawn, since locations of T and W are given by the question, location of F_B is deduced to act along the upper surface as shown: 	1

Q10	(a)	*It is always helpful to draw out the diagram of what you are dealing with if the diagram is not given.*  R_B is a normal contact force (the force is perpendicular to the wall) since the wall is smooth and there is no frictional force. Let R_A be the **resultant** contact force on ladder by ground. R_A consists of both vertical and horizontal components because the ground is rough and thus both normal contact force and friction act on end A. Free Body Diagram of Ladder with Man:  *** Check that you have drawn in the Man in the free-body diagram & showed clearly that the weight of the man acts on the man, not the ladder.* Since ladder is in equilibrium, Taking moments about A, by Principle of Moments, Sum of clockwise moments = Sum of anti-clockwise moments $(20)(9.81)[1.5]+(70)(9.81)\left[\frac{3}{4}(3)\right]=R_B(4)$ $R_B = 460 \quad N$	 1 1
	(b)	Balancing vertical forces, Vertical component of R_A = (20 x 9.81) + (70 x 9.81) = 883 N Balancing horizontal forces, Horizontal component of R_A = R_B = 460 N Magnitude:	1

		$R_A = \sqrt{460^2 + 883^2} = 1000N$ (2 s.f.) Direction: $\tan\theta = \frac{883}{460}$ $\rightarrow \theta = 63°$, *anticlockwise above the horizontal* OR For vertical equilibrium of forces, R_A sin θ = (20 x 9.81) + (70 x 9.81) R_A sin θ = 883 ---------(1) For horizontal equilibrium of forces, R_A cos θ = R_B R_A cos θ = 460 --------(2) (1)/(2): $\tan\theta = \frac{883}{460}$ $\rightarrow \theta = 63°$, *anticlockwise above the horizontal* Sub into (1) or (2): R_A = 1000 N (2 s.f.)	1 1

Q11	Ans: **D** $P_X = P_{atm} + \rho gh$ (h: **vertical** height of liquid column) = 100kPa + (1020)(9.81)(1.00 sin 30°) = 105 kPa
Q12	Ans: **D** Upthrust = Difference in forces acting on the top and bottom surfaces = $F_2 - F_1$ = $P_2A - P_1A$ since $pressure = \frac{force}{area}$

Q13	(a)	(i)	Point C shown at centre of rectangle.
		(ii)	Arrow vertically downwards, from C.
	(b)	(i)	Reaction / upwards / supporting / normal reaction force / friction force(s) at the rod.
		(ii)	Comes to rest with (line of action of) weight acting through rod so that weight does not have a moment about the pivot / rod.

Assignment Question

<table>
<tr><td>A1</td><td>

Ans: A

For X: $W = 2k(e)$

Therefore $e_X = \dfrac{W}{2k}$

For Y: $\dfrac{1}{k_{eff}} = \dfrac{1}{2k} + \dfrac{1}{2k} \Rightarrow k_{eff} = k$

$W = k(e)$

Therefore $e_Y = \dfrac{W}{k}$

For Z: $\dfrac{1}{k_{eff}} = \dfrac{1}{2k} + \dfrac{1}{k} \Rightarrow k_{eff} = \dfrac{2k}{3}$

$W = \dfrac{2k}{3}(e)$

Therefore $e_Z = \dfrac{3W}{2k}$

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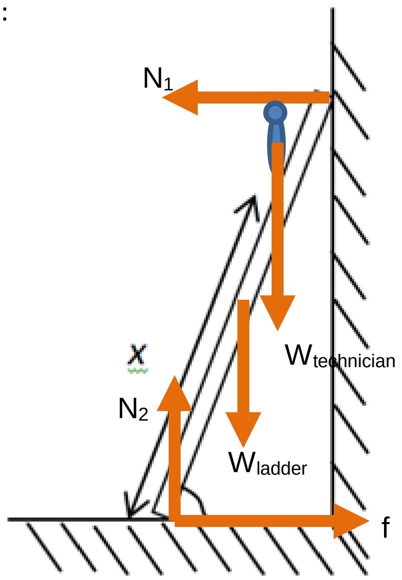
<tr><td>A2</td><td>

Free body diagram of ladder with technician:



N_1: Normal contact force exerted by wall *(* a common mistake is to call this a reaction force, which is too generic. All forces are part of an action-reaction force pair. Name the TYPE of force (refer back to Chapter 3 lecture notes)*

N_2: Normal contact force exerted by ground

f: friction exerted by ground

$W_{technician}$: weight of technician *(* a common mistake is to draw this vector as acting on the ladder rather than the man)*

W_{ladder}: weight of ladder

[Note: this is a free body diagram of the system comprising ladder with technician, so the contact forces between the ladder and technician are internal forces and not drawn on this free body diagram]

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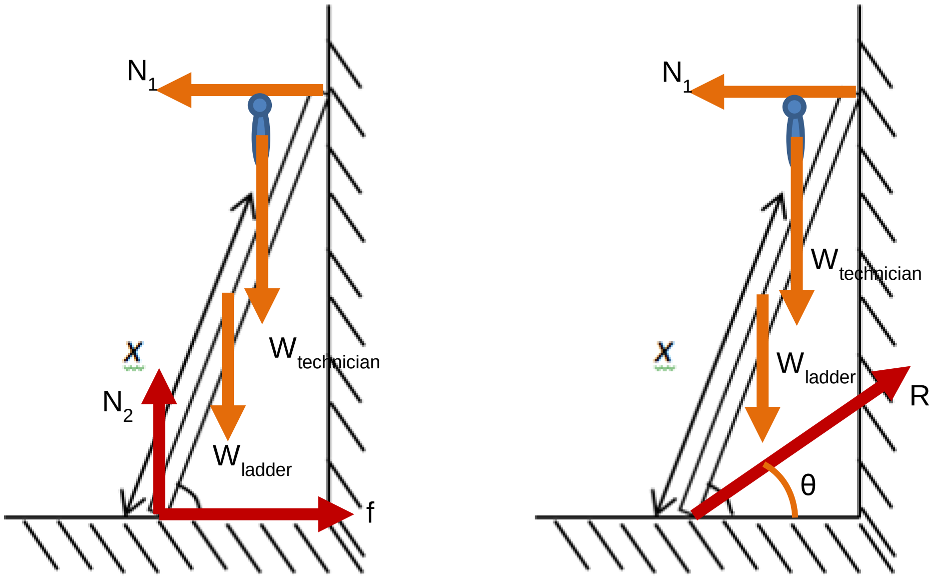
(a) When the ladder 'slips', it rotates about its C.G. The ladder will slip when it experiences a net torque. This happens when the total clockwise moments exceeds the total anticlockwise moments.

Before the ladder slips, it is still in *equilibrium*.
Additionally, at the instant JUST before it slips, friction is at its *maximum* value, i.e. 300 N. [1]

Applying the conditions for equilibrium:

- Balancing horizontal forces: $N_1 = maximum\ value\ of\ f = 300$ N
- And applying the Principle of Moments, taking moments about the ground-end of the ladder,

  Total clockwise moments = Total anticlockwise moments
  $20(9.81)(1.5\ cos60°) + 60(9.81)(x\ cos60°) = (300)(3\ sin60°)$ [1]
  $x = 2.15 = 2\ m\ (1\ s.f)$ [1]



Either of the above free body diagrams is acceptable.

Note: R is the resultant contact force exerted by the ground on the ladder.
R is the resultant force of N_2 and f.
$R\ cos\vartheta = f$
$R\ sin\vartheta = N_2$

(b)
Slipping occurs when the ladder experiences a net torque (because it starts rotating). The higher up the technician climbs, the greater the clockwise moment caused by his weight. [1]
If the wall is rough, an upward frictional force will act at the wall-end of the ladder, and can produce a greater total anticlockwise moment to balance the clockwise moment caused by the weight of the technician. Therefore the range of x for which equilibrium is possible is increased. Thus the answer is Yes, the technician would be able to climb higher if the wall was rough. [1]

Supplementary Questions

S1	Key Words: • Rest • Friction between ladder and ground, ladder and wall • Arrows that represent forces on ladder Essential Questions:

	1) What is the main physics concept this question is testing? • The main physics concept is the conditions for equilibrium system, where resultant moment and resultant forces are zero. 2) What are the essential skills required for this question? • As options A to D are pictorial representation, the skills needed here are visualization (model of drawings) and free body diagram. 3) Why are frictions given? Which directions are they acting on? • To prevent the ladder from sliding down, friction on ladder by wall should be upwards and friction on ladder by ground should be leftwards. 4) Are there other forces acting on the ladder, if yes, what are they and which directions are they acting on? • There are weight of ladder and normal contact force from wall on ladder and from ground on ladder. Weight is acting downwards and normal contact force acted by the wall on the ladder is rightwards and normal contact force acted by the ground on the ladder is upwards. 5) How do we link the arrows to all these forces? • The arrows at the point of contact with the wall and ground is a representation of resultant forces of friction and normal contact force acting on the ladder by the wall and ground respectively. Extension Question: 6) How do we link the arrows to the condition of equilibrium system? • The arrows will form a closed polygon with all arrows pointing in the same anti-clockwise or clockwise direction if the resultant force of the system is zero. • The line of action (arrows) shall pass through the same point if the resultant moment is zero, as long as there are no parallel arrows. Answer: C

S2	C is not correct because torque, just like force, is a **vector** quantity which has both magnitude and direction. The correct statement should be “The torque provided by the vertical forces is **equal and opposite** to the torque provided by the horizontal forces.” Answer: C

S3	(a)	The mass is not uniformly distributed throughout the boat. The mass of the boat is more concentrated on its left side	1
	(b)	No net force acting on the boat. No net moment of force about any point.	1 1
	(c)	The question requires ‘using principle of moments’ to find the 2 tensions in the rope.	

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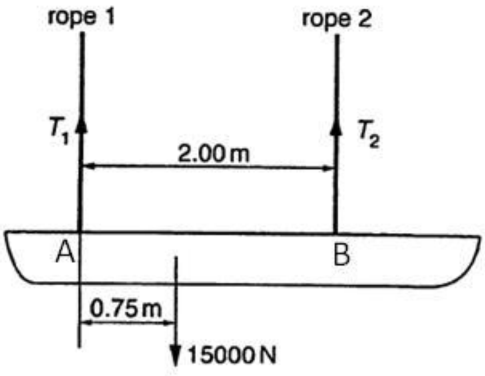

Applying the principle of moments:
Taking moment about B,

$$\sum clockwise\ moments = \sum anticlockwise\ moments$$
$$T_1(2.00) = 15000(2.00 - 0.25)$$
$$T_1 = 9375\ N = 9400\ N \quad (2.s.f)$$

Taking moment about A,

$$\sum anticlockwise\ moments = \sum clockwise\ moments$$
$$T_2(2.00) = 15000(0.75)$$
$$T_2 = 5625\ N = 5600\ N \quad (2.s.f)$$
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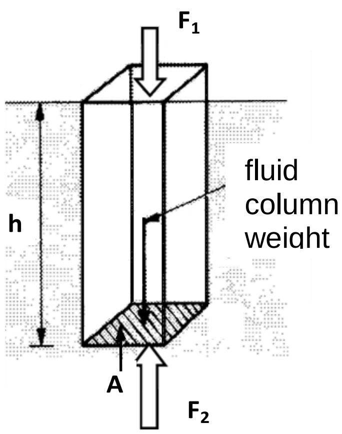
<tr>
<td>S4</td>
<td>
Pressure is defined as the <u>perpendicular force</u> acting per unit area of a surface. Density is defined as the mass of the object per unit volume of the object.

Consider a column of fluid as shown below.



h: height of the fluid column.
A: base area of the fluid column.
F_1 , F_2 : perpendicular forces on the upper and lower surfaces of the column respectively.
P_1 , P_2 : pressure on the upper and lower surfaces of the column respectively.
ρ : density of fluid
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<td>
Start from the basic definition of pressure and density to derive the expression ρgh.

You must explain your steps clearly.
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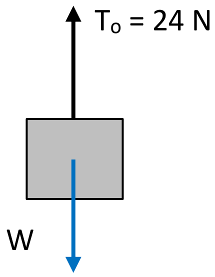
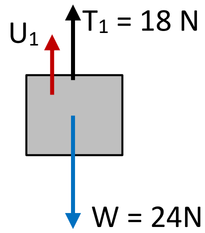
	V : volume of fluid column $\text{Density}, \rho = \dfrac{\text{mass}}{\text{Volume}} \rightarrow \text{mass} = \rho V = \rho Ah$ $\text{Pressure} = \dfrac{\text{Force}}{\text{Area}} \rightarrow F_1 = P_1A,\ F_2 = P_2A$ Fluid is static → Upward force = Downward force F_2 = Weight of fluid column + F_1 $P_2A = (\rho Ah)g + P_1A$ $P_2 = \rho hg + P_1$ Pressure difference (increase) down the fluid column, $\Delta P = P_2 - P_1 = \rho hg$ <u>Examiners' Report</u> Few explained clearly what they were doing and a sizeable minority thought they were proving Archimedes principle.	1 1	
S5	Ans: **A** Since pressure is independent of the shape of the container or the mass of the entire fluid, The pressure P exerted by the liquid on the base of the jar, $P = h\rho g = (0.200)(600)(9.81) = 1.18 \times 10^3$ Pa		
S6	Ans: **C** Upthrust is due to the pressure difference between the top and bottom surfaces of the immersed object.		
S7	Ans: **B** Upthrust is due to the pressure difference between the top and bottom surfaces of the immersed object. For a floating object which is in vertical equilibrium, by Newton's 1st law no net vertical force acts, and thus the upthrust must be equal to the weight of the object.		
S8	(a) $p = (9.8 \times 10^3)h$ $= 9.8 \times 10^3 \times 0.055$ $= 539$ Pa $F = p \times A = 539\ (1.3 \times 10^{-3})$ $= 0.7007 = 0.701$ N (3 s.f.) (b) Upwards (c) The magnitude of the force calculated in part (a) is the value of upthrust acting on the metal tube. Since vertical forces on the tube must be balanced for it to float, sum of upwards forces (upthrust in this case) must equal the sum of downward forces (weight in this case). Hence weight equals 0.701 N.	1 1 1 1	
S9	<u>Object suspended in the air</u>		



W = T_o = 24 N

<u>Object fully submerged in water</u>



1

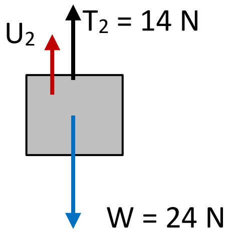
At equilibrium, $U_1 + T_1 = W$

$U_1 + 18 = 24$ 1

$U_1 = 24 - 18 = 6$ N

$V\rho_w g = 6$ ------- (1)

<u>Object fully submerged in unknown liquid</u>



At equilibrium, $U_2 + T_2 = W$

$U_2 + 14 = 24$

$U_2 = 24 - 14 = 10$ N

$V\rho_L g = 10$ ------- (2)

$$\frac{(2)}{(1)} \Rightarrow \frac{\rho_L}{\rho_W} = \frac{10}{6} = \frac{5}{3}$$ 1

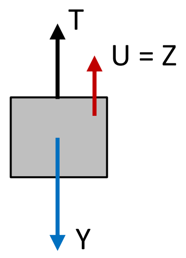
$$\rho_L = \frac{5}{3}\rho_W = \frac{5}{3}\times 1000 = 1.67\times 10^3 \ kg\, m^{-3}$$

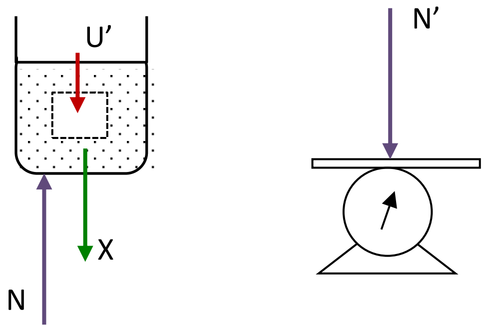
Weight of object, W = 24 N

$\Rightarrow V\rho_O g = 24 \quad ----$ (3)

$$\frac{(3)}{(1)} \Rightarrow \frac{\rho_O}{\rho_W} = \frac{24}{6} = 4$$ 1

$$\therefore \ \rho_O = 4\times 1000 = 4000 \ kg\, m^{-3}$$

S10	Ans: **A** As time progresses, air resistance increases with speed. By Newton's 2nd law of motion, resultant force decreases.	
S11	Ans: **A** *U* << *W* due to large differences in density of air compared to density of water.	
S12	Ans: **C** When under free-fall, $a = -g$. On hitting the water, it experiences viscous force upwards. The sphere hence either accelerates downwards with lower magnitude or it may start to deccelerate downwards	
S13	From the information given in the question: $W_{beaker+water} = X$, $W_{solid} = Y$, From Archimedes' Principle, Upthrust or <u>Upward</u> force that fluid exerts on the solid, U = Z <u>FBD (Free Body Diagram) of Solid</u> T: Tension in the string on the Solid  <u>FBD (Free Body Diagram) of Beaker of water (**Without the Solid**)</u> U': Downward force that the solid exerts on the fluid, U' = U = Z N: Normal contact force exerted by the balance on the beaker N': Normal contact force exerted by the beaker on the balance. N'=N	1 [Archimedes principle]

	 Therefore, from the above FBD, N = X + U' = X + Z Balance reading is given by N', N' = X + Z	1