RIVER VALLEY HIGH SCHOOL
2019 JC1 Common Test
Higher 2

NAME	

CLASS							INDEX NUMBER		

MATHEMATICS

9758/01

Paper 1

4 July 2019

2 hours 30 minutes

Candidates answer on the Question Paper
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.
Up to **2 marks may be deducted** for poor presentation in your answers.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 85.

For examiner's use only	
Question number	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

Calculator Model:

1 In the annual Best Infantry Unit Competition, 3 top units – Units A, B and C, are shortlisted for the eventual champion. The organizer decides to award x, y and z points for each NAPFA Gold Award, Shooting Marksmanship Badge and Army Proficiency Badge respectively and to declare the unit with the highest total points as the champion. With the emphasis of training of more marksman in the units, the organizer decides to have the points awarded for each Shooting Marksmanship Badge to be equal to the sum of points for an NAPFA Gold Award and an Army Proficiency Badge.

The number of NAPFA Gold Awards, Shooting Marksmanship Badges and Army Proficiency Badges achieved by each unit are summarized in the following table.

	NAPFA Gold Awards	Shooting Marksmanship Badges	Army Proficiency Badges
Unit A	24	27	118
Unit B	29	21	124
Unit C	26	25	121

Upon computation of points, it is known that the unit with the most NAPFA Gold awards scores a total of 295 points and it falls behind the unit with the most Marksmanship Badges by 3 points.

By solving a system of equations involving x, y and z, determine the eventual champion of the competition. [5]

2 **(i)** Show that $2x^2 - 2x + 1$ is always positive for all real *x*. [1]

(ii) Hence, without using a calculator, solve $\dfrac{2x^3 - 2x^2 + x}{(2x+1)(x-1)^2} \geq 0$. [3]

(iii) Using your result in part **(ii)**, find the exact range of values of x for which $\dfrac{2(\ln x)^3 - 2(\ln x)^2 + \ln x}{(2\ln x + 1)(\ln x - 1)^2} \geq 0$. [3]

3 The functions f and g are defined below.

$$\mathrm{f}(x) = \begin{cases} \dfrac{2}{2-x} & \text{for } 0 \le x < 2, \\ \ln(x-2) & \text{for } 2 < x \le 4 \end{cases}$$

$$\mathrm{g}(x) = -2x^2 + x + 1 \qquad \text{for } 0 \le x < 1$$

(i) Sketch the graph of $y = \mathrm{f}(x)$. [3]

(ii) State the range of g and show that the composite function fg exist. [2]

(iii) Find the range of the composite function fg. [2]

4 The curve C has the equation $y=\dfrac{ax^2-3x+b}{x-c}$ where a, b, and c are real constants. It is given that the equations of asymptotes of C are $x=2$ and $y=x-1$. Also, C passes through the point $P\left(0,-\dfrac{3}{2}\right)$.

(i) Explain why the value of a is 1. State the value of c and show that b = 3. [3]

(ii) Sketch the curve C, indicating clearly the equations of asymptotes, axial intercepts and stationary points. [2]

(iii) Find the exact equation of the normal to C at the point P. [3]

5 The triangle ABC is such that $AB = 3$, $AC = 4$ and angle $BAC = \theta$ radians. Given that θ is sufficiently small, show that $BC \approx \sqrt{1+12\theta^2} \approx 1 + p\theta^2 + q\theta^4$ for constants p and q to be determined. [4]

By using the above result and substituting $\theta = \frac{1}{4}$, find an approximate value of $\sqrt{7}$, leaving your answer as a single fraction. [2]

Without further calculations, state with a reason whether the approximate value obtained by the substitution $\theta = \frac{1}{10}$ would be a better estimate for $\sqrt{7}$. [1]

6 The function f is defined as $f : x \mapsto 6x - x^2$.

(i) The domain of f is given by $[a, \infty)$.

By sketching the graph of $y = f(x)$ when $a = 3$, give a reason to explain why the least value of a is 3 for f^{-1} to exist. [4]

For the rest of this question, the domain of f is assumed to be $[3, \infty)$.

(ii) Find f^{-1} in similar form. [3]

(iii) On the same axes as part **(i)**, sketch the graphs of $y = \mathrm{f}^{-1}(x)$ and $y = \mathrm{f}\,\mathrm{f}^{-1}(x)$, showing clearly their relationship with the graph of $y = \mathrm{f}(x)$.
(you are to display your graphs to this part on the diagram in part **(i)**) [3]

7 It is given that $u_n = \dfrac{2n-1}{n(n-1)}$, where $n \in \mathbb{Z}^+$.

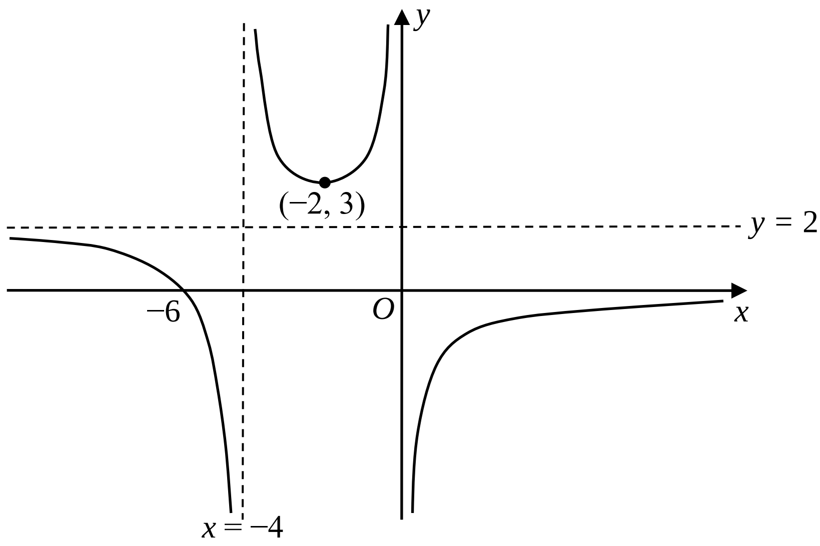
Show that $u_n - u_{n+1} = \dfrac{2}{(n+1)(n-1)}$. [1]

(i) Hence, find $\sum_{r=2}^{N} \frac{1}{(r+1)(r-1)}$, leaving your answer in terms of N. [3]

(ii) Give a reason why the series in part **(i)** is convergent and find the sum to infinity. [2]

(iii) Using your result in part **(i)**, show that $\sum_{r=2}^{N-1} \frac{1}{(r+2)(r)} = \frac{5}{12} - \frac{2N+1}{2N(N+1)}$. [3]

8 **(a)** The graph of $y = \mathrm{f}(x)$ is given below.



On separate diagrams, sketch the graphs of

(i) $y = \dfrac{1}{\mathrm{f}(x)}$, [3]

(ii) $y = \mathrm{f}'(x)$. [3]

(b) A curve C has parametric equations

$$x = t, \quad y = 2t^2 .$$

Describe a sequence of transformations to obtain the curve

$$x = 2t + 1, \; y = t^2$$

from C. [2]

9 *A* ***first order recurrence relation*** *for a sequence* $u_0, u_1, u_2, \ldots$ *is an equation that relates the general term* u_n *with its preceding term* u_{n-1}.

A student takes up a university study loan of \$20 000 from a bank. The bank charges a 0.5% interest rate per month. The interest is added on at the first day of each month. The student decides to make repayments monthly on the last day of each month. Each repayment is \$450 except for the final repayment which will ultimately pay off the loan.

Denoting the amount that he owes the bank at the end of the *n*th month by u_n, the above scenario can be represented by the following first order recurrence relation, $u_n = 1.005u_{n-1} - 450$, $n \geq 1$.

(i) Given that $u_0 = 20\,000$, use the recurrence relation to find u_1, u_2 and u_3. [3]

(ii) The student attempts to rewrite the recurrence relation in the following way:

$$\boxed{\begin{aligned} u_n &= 1.005u_{n-1} - 450 \\ &= 1.005\left(1.005u_{n-2} - 450\right) - 450 \\ &= \ldots \end{aligned}}$$

By applying the recurrence relation repeatedly and observing the pattern, show that u_n can be expressed as $90\,000 - 1.005^n\left(70\,000\right)$. [5]

(iii) Determine the number of months required to repay the loan and find the final repayment. [4]

10 Max Min & rate Of Change Problem not tested 2022 H2 MA CT.

In an experiment to test the design theory of launching a new space telescope into space, scientists monitor the movement of a charged particle in an experimental system with the help of a computer analytical programme that presents its path in the form of a curve on an *x*-*y* plane. The *x*- and *y*-coordinates of the particle, *t* minutes from launch are described by the parametric equations:

$$x = t - \frac{1}{t} + 2 \; ; \; y = t + \frac{1}{t} \quad \text{for } t \geq 0.5 .$$

(i) For the curve representing the path of the charged particle, show that its gradient is $\dfrac{t^2 - 1}{t^2 + 1}$. [2]

(ii) Find the stationary point of the curve and determine its nature. [5]

(iii) Describe the behaviour of the gradient of the path of the particle for large values of t and hence, provide a sketch of the graph of the path of the particle on the x-y plane for $t \geq 0.5$. [3]

(iv) Given that the rate of change of the y-coordinate of the particle is 0.5 units per minute, find the corresponding rate of change of the x-coordinate of the particle. [2]

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