RAFFLES INSTITUTION
2024 YEAR 6 TIMED PRACTICE

CANDIDATE NAME

CLASS 24

MATHEMATICS 9758

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiner's Use Only							
Sect A: Pure Math	Q1	Q2	Q3	Q4	Q5	Q6	Q7
	/ 7	/ 6	/ 9	/ 10	/ 10	/ 12	/ 11
Sect B: Prob & Stats	Q8	Q9	Q10	Q11		TOTAL	
	/ 6	/ 8	/ 10	/ 11		/ 100	

This document consists of **22** printed pages.

Section A: Pure Mathematics [65 marks]

1 Consider the second order differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+a\frac{\mathrm{d}x}{\mathrm{d}t}=b\,,$$

where a and b are non-zero constants and $\frac{\mathrm{d}x}{\mathrm{d}t}\neq\frac{b}{a}$.

(a) By substituting $y=\frac{\mathrm{d}x}{\mathrm{d}t}$, show that the above second order differential equation can be written as $\frac{\mathrm{d}y}{\mathrm{d}t}=b-ay$. [2]

(b) Find y in terms of t, and hence find x in terms of t. [5]

1 [Continued]

2 Referred to an origin O, the position vectors of three points A, B and C are $3\mathbf{i}-2\mathbf{j}+5\mathbf{k}$, $\mathbf{i}+4\mathbf{j}-2\mathbf{k}$ and $\alpha\mathbf{i}+2\mathbf{j}+4\mathbf{k}$ respectively.

(a) Determine the value of α for which points O, A, B and C are coplanar. [2]

(b) Given that $\overrightarrow{OD} = \overrightarrow{OA} + \lambda \overrightarrow{OB}$, find the value of λ for which $\overrightarrow{OD}$ is perpendicular to the y-axis. [2]

(c) Hence, find the exact area of triangle OAD. [2]

3 **(a)** Write down constants A and B such that for all values of x, $x-1=A(2x-4)+B$. [1]

(b) By expressing x^2-4x+5 in completed square form, find

$$\int \frac{x-1}{x^2-4x+5}\,\mathrm{d}x.$$ [4]

(c) Hence, without using a calculator, show that

$$\int_{-2}^{2} \frac{|x-1|}{x^2-4x+5}\,dx = \frac{\pi}{2} + \frac{1}{2}\ln\frac{p}{q} - \tan^{-1} q\,,$$

where p and q are exact real constants to be determined. [4]

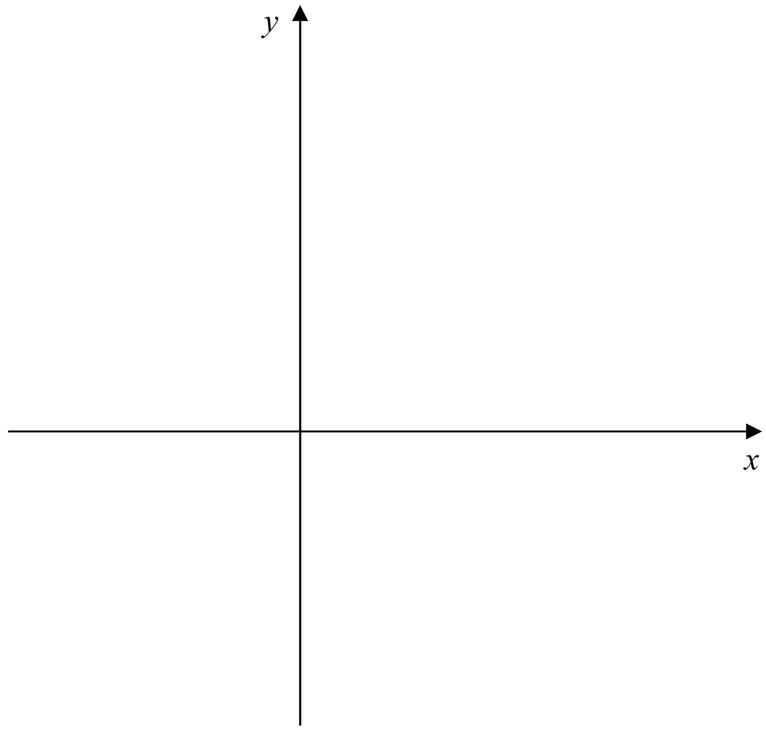
4 The functions f and g are defined by

$$\mathrm{f}: x \mapsto \frac{2x+14}{3x-2}, \ x \in \mathbb{R}, \ x > \frac{2}{3},$$
$$\mathrm{g}: x \mapsto \frac{\sin x}{x}, \ x \in \mathbb{R}, x > 0.$$

(a) Find $\mathrm{f}^{-1}(x)$ and state its domain. [3]

(b) Explain why $\mathrm{f}^{2024}(x) = x$ for $x > \frac{2}{3}$. [1]

(c) On the same diagram, sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{2024}(x)$, giving the equations of any asymptotes. [3]

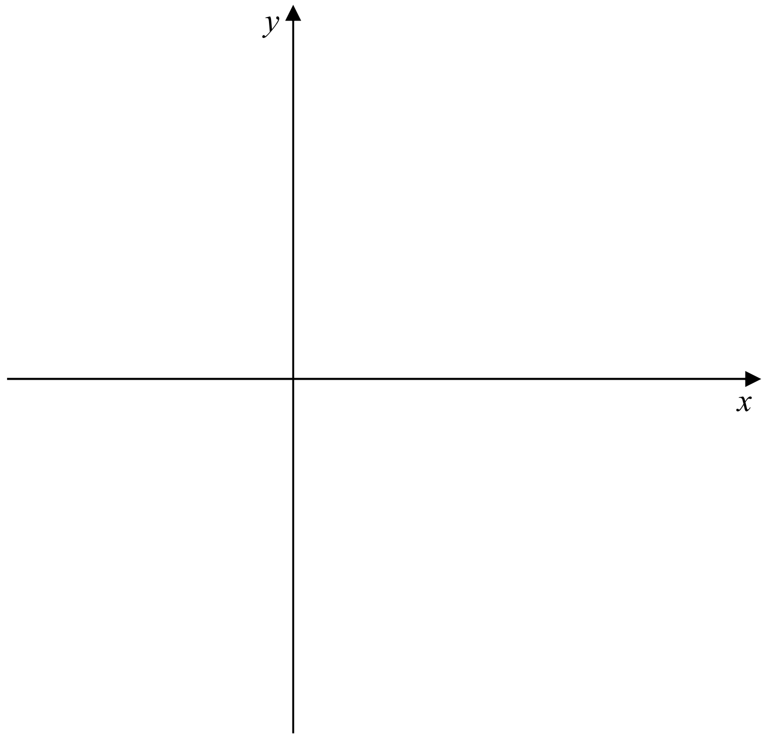


(d) Explain why the composite function gf exists. [1]

(e) Find the range of gf . [2]

5 The curve C is defined by the parametric equations $x = t^2$ and $y = 2t + t^3$, where $-1 \leq t \leq 1$.

(a) Sketch the graph of C. [2]



(b) Without using a calculator, find the equation of the normal to C at the point where $t = \dfrac{1}{2}$. [4]

(c) Find the exact area bounded by C, the positive y-axis and the normal found in part **(b)**. [4]

6 The point A has coordinates $(1.5,\ 0,\ 2)$, and the planes π_1 and π_2 have equations $x+y=4$ and $3x+2y-5z=7$ respectively.

(a) π_1 and π_2 intersect in a line l. Find a vector equation of l. [2]

(b) Find the coordinates of the foot of perpendicular from A to π_1. [4]

(c) The point B is the mirror image of A in π_1. Find the coordinates of B and determine if it lies on π_2. [3]

(d) Find the cartesian equation of the plane π_3, which is the reflection of π_2 in π_1. [3]

7 **(a)** Nathan is saving for a car that he intends to buy in the future, and he would like to save a minimum of \$130 000. He saves regularly in an account which offers no interest. He makes an initial deposit of \$$A$ on 31 January 2022. Each subsequent month, he deposits \$150 more than he deposited in the previous month. His final deposit is made on 30 June 2024. Find, to the nearest dollar, the smallest value of A so that he can save at least \$130 000. [2]

Nathan comes across a savings plan offered by a bank. Interest is added to the account at the end of each month at a fixed rate of 1.25% of the amount in the account at the beginning of that month under this savings plan.

(b) Nathan opened an account under this savings plan. He decides to deposit \$$X$ at the beginning of the first month and then a further \$$X$ at the beginning of the second and each subsequent month. He also decides that he will not draw any money out of the account, but just leave the money in the account for the interest to build up.

(i) Write down the amount in the bank account, including the interest, at the end of 1 month. [1]

(ii) Show that, at the end of n months, when the interest for the last month has been added, he will have a total of \$ $mX\left(1.0125^n - 1\right)$ in his bank account, where m is an integer to be determined. [3]

(iii) After how many complete months will he have, for the first time, at least \$$13X$ in his bank account? [2]

7 [Continued]

(c) Nathan decides that, to assist him in his everyday expenses, he will withdraw the interest as soon as it has been added. With this decision, he deposits $\$Y$ at the beginning of each month. Show that, at the end of N months, he will have received a total of $\$kYN(N+1)$ in interest, where k is an exact constant to be determined. [3]

Section B: Probability and Statistics [35 marks]

8 From past data, it is known that the mean time taken by students at a school to complete a H2 Mathematics A-level examination paper was 170 minutes. In June 2024, Ms Tan believes that this cohort of students is not ready for the examination and will take more than 170 minutes to complete the paper. She gives the same examination paper to 50 students, assuming that these students have not seen the paper before. She notes the time taken by each student and carries out a test.

(a) State null and alternative hypotheses for the teacher's belief, defining any parameters you use. [2]

(b) Given that the sample of 50 students took an average of 172.1 minutes and the sample variance is 53.8 minutes^2, test, at the 5% significance level, whether the teacher's belief is supported. [4]

9 The letters from the word PERIMETER are arranged in a row.

(a) Find the number of different arrangements of the nine letters. [1]

(b) Find the number of different arrangements if there are at least 6 letters between the two R's. [3]

One of the E's is removed and the remaining letters are arranged randomly in a row.

(c) Find the probability that no adjacent letters are the same. [4]

10 A biased die in the form of a tetrahedron has its four faces labelled 1 to 4, with one number printed on each face. The die is tossed and X is the random variable representing the number on the face on which the die lands. The probability distribution of X is shown in the table below.

x	1	2	3	4
$\mathrm{P}(X = x)$	p	q	q	p

(a) State the numerical value of $\mathrm{E}(X)$. [1]

(b) Given that $\mathrm{Var}(X) = 1.65$, find the values of p and q. [4]

A game is played by a player tossing this die once. If the die lands on the face with the number 4 printed on it, the player wins, otherwise the player loses. The random variable Y denotes the number of wins out of 50 games the player plays.

(c) State two assumptions needed for Y to be well modelled by a binomial distribution. [2]

Assume that Y follows a binomial distribution.

(d) Find the probability the player wins more than the expected number of games won. [3]

11 In this question you should state clearly the values of the parameters of any normal distributions you use.

A fruit seller sells two kinds of fruits, namely durians and soursops. The masses, in kg, of a durian and a soursop are modelled as having normal distributions with means and standard deviations as shown in the following table.

	Mean mass (kg)	Standard deviation (kg)
Durian	2.2	0.4
Soursop	1.3	0.2

(a) Find the mass that is exceeded by 95% of the soursops. [1]

(b) Three durians are randomly chosen. Find the probability that two of these durians each has mass less than 2.3 kg and one of the durians has mass more than 2.5 kg. [3]

(c) Find the probability that the mass of a randomly chosen durian differs from twice the mass of a randomly chosen soursop by more than 300 grams. [3]

(d) Write down an assumption that you have used in part **(c)**. [1]

(e) Durians are sold at $20 per kg. Find the probability that 5 randomly chosen durians cost at least $225. [3]