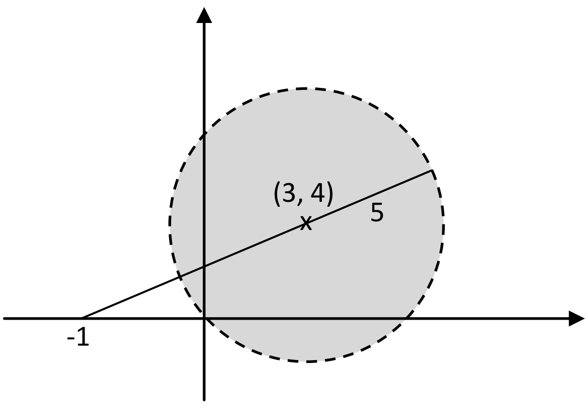
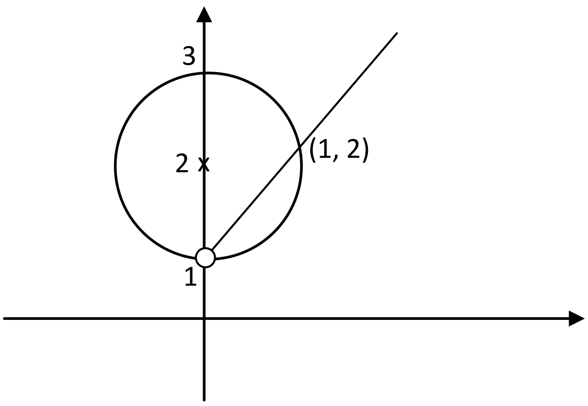
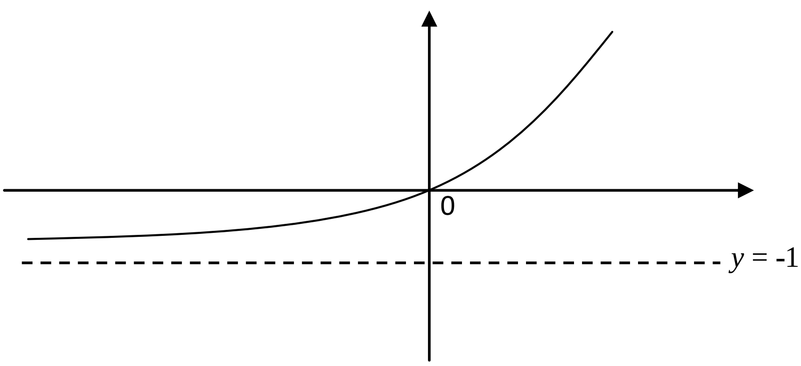
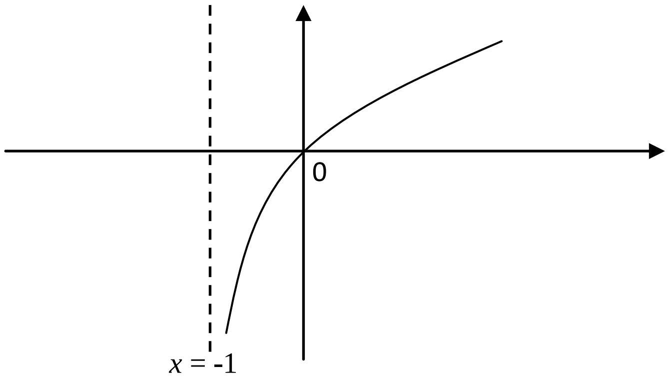
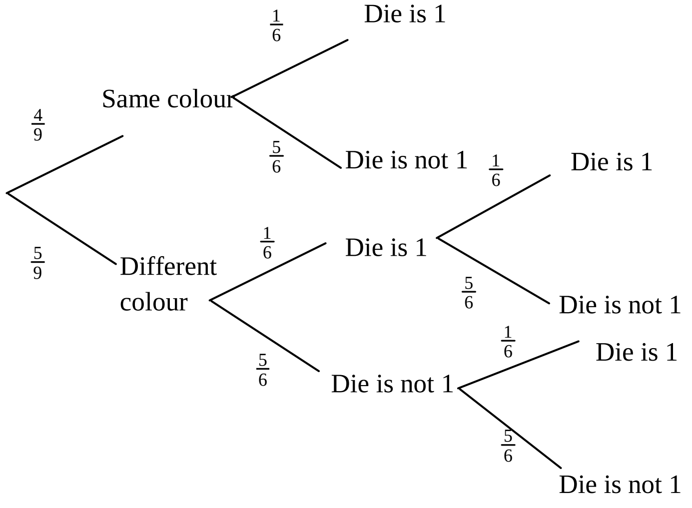
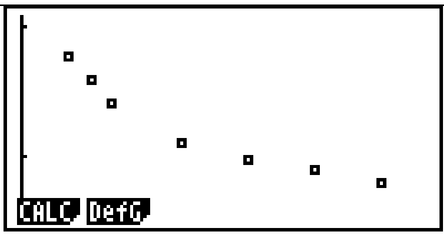
PU3 H2 Mathematics Paper 2 2012 Prelim II

1(a)(i)	
1(a)(ii)	$4\sqrt{2}-5<\|z+1\|<4\sqrt{2}+5$
1(b)(i)	
1(b)(ii)	$z=1+2i$
2(i)	

2(ii)	$y=e^{2x}-1$ $\ln(y+1)=2x$ $x=\frac{\ln(y+1)}{2}$ $f^{-1}:x\mapsto\frac{1}{2}\ln(x+1),\ x>-1$
2(iii)	
2(iv)	$e^{2x}=x+1$ $e^{2x}-1=x$ $x=-0.797$ $x=0$
3(i)	$\frac{dx}{dt}=2t$ $\frac{dy}{dt}=-\frac{1}{(t-2)^2}$ $\frac{dy}{dx}=-\frac{1}{(t-2)^2}\div 2t$ $\frac{dy}{dx}=-\frac{1}{2t(t-2)^2}$
3(ii)	$x=2\Rightarrow t^2+1=2\Rightarrow t=\pm 1$ $y=-1\Rightarrow \frac{1}{t-2}=-1\Rightarrow t=1$ At $t=1$, $\frac{dy}{dx}=-\frac{1}{2}$ Gradient of normal $=2$ $y+1=2(x-2)$ $y=2x-5$

3(iii)	$\frac{1}{t-2}=\frac{1}{2}\left(t^2+1\right)-2$ $2t^3-4t^2-3t+5=0$ $t=-1.158$ $t=1$ $t=2.158$
4(a)	$\sum_{r=1}^{n} r(r-4)$ $=\sum_{r=1}^{n} r^2-4\sum_{r=1}^{n} r$ $=\frac{n}{6}(n+1)(2n+1)-4\left(\frac{n}{2}\right)(n+1)$ $=\frac{n}{6}(n+1)\left[(2n+1)-12\right]$ $=\frac{n}{6}(n+1)(2n-11)$
4(b)(i)	$\sum_{r=1}^{n}\frac{1}{r(r+1)}=\sum_{r=1}^{n}\left(\frac{1}{r}-\frac{1}{r+1}\right)$ $=\left(\frac{1}{1}-\frac{1}{2}\right)+$ $\left(\frac{1}{2}-\frac{1}{3}\right)+$ $\ldots+$ $\left(\frac{1}{n}-\frac{1}{n+1}\right)$ $=1-\frac{1}{n+1}$
4(b)(ii)	$\sum_{r=1}^{\infty}\frac{2}{r(r+1)}=2\sum_{r=1}^{\infty}\frac{1}{r(r+1)}$ As $n\to\infty,\ 1-\frac{1}{n+1}\to 1$ $\therefore \sum_{r=1}^{\infty}\frac{2}{r(r+1)}\to 2$
5(a)	(i) Consider the green, yellow and purple tiles as 1 unit. Number of ways to arrange the green, yellow and purple tiles within the unit $=\frac{5!}{3!}$ Arranging the unit with the rest of the tiles $=\frac{7!}{3!3!}$ Thus number of possible arrangements for the tiles $=\frac{5!}{3!}\times\frac{7!}{3!3!}$ =2800

	(ii)Number of ways to arrange the 3 red, 3 green, 1 yellow and 1 purple tile = $\frac{8!}{3!3!}$ Number of ways to slot in the blue tiles = $\binom{9}{3}$ Number of possible arrangements such that no blue tiles are placed next to another = $\frac{8!}{3!3!} \times \binom{9}{3}$ =94080 (iii) Number of possible arrangements such that a red tile at the beginning and another red tile at the end of the line = $\frac{(3+3+3)!}{3!3!}$ = 10080
5(b)	(i) Number of ways = $(10-1)! = 362880$ (ii) <u>Insertion method</u> Number of ways $= (7-1)! \times {}^{7}C_{2} \times 2! \times 2! = 60480$ (iii) Number of ways $= (9-1)! \times 10 \times 2! = 806400$
6(i)	P(same colour) $= \frac{{}^{4}C_{2} + {}^{5}C_{2}}{{}^{9}C_{2}} = \frac{4}{9}$ 

	P(wins exactly one DVD) $=\frac{4}{9}\times\frac{1}{6}+\frac{5}{9}\times\frac{5}{6}\times\frac{1}{6}\times 2=\frac{37}{162}$ (0.228 (3 s.f.))
6(ii)	P(different colour|did not win any DVD) $=\frac{\text{P(different colour and did not win any DVD)}}{\text{P(did not win any DVD)}}$ $=\frac{\frac{5}{9}\times\frac{5}{6}\times\frac{5}{6}}{\frac{4}{9}\times\frac{5}{6}+\frac{5}{9}\times\frac{5}{6}\times\frac{5}{6}}=\frac{25}{49}$ (0.510 (3 s.f.))
6(iii)	P(wins 2 DVDs| different colour) $=\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$
6(iv)	Let X be the number of DVDs John can win, out of 2 dice throws. Then X ~ B(2, $\frac{1}{6}$). Mean number of DVDs = $2\times\frac{1}{6}=\frac{1}{3}$
7(i)	$r=-0.946$
7(ii)	 The scatter diagram shows that the relationship between t and x is non-linear.
7(iii)	From the scatter diagram, we see that *y* decreases as *x* increases, which is the case for model **A** ($x=a+\frac{b}{t},\ b>0$). Hence model **A** is appropriate.
7(iv)	$a=0.75046151$ $b=11.0423697$ $x=0.75046151+\frac{11.0423697}{t}$ $x=0.75046151+\frac{11.0423697}{45}=0.9958475033\approx 0.996$ This estimate is reliable since $t=45$ is between the range of values of *t* from 5 to 80, and the product moment correlation coefficient between *x* and *t* is 0.9754 (3 s.f.), which suggests a strong positive linear correlation between *x* and *t*.
8(a)	Let I be the r.v. 'no. of packets of macademia nuts sold in a week'. $I \sim \text{Po}(10)$ $\text{P}(I>11)=1-\text{P}(I\le 11)=1-0.69677=0.30323$ Let W be the r.v. 'no. of weeks, out of 52, of which more than 11 packets of macademia nuts

	are sold per week'. $W \sim \mathrm{B}\left(52,\ \mathrm{P}\left(I > 11\right)\right)$, i.e., $W \sim \mathrm{B}\left(52, 0.30323\right)$ $\mathrm{P}\left(W > k\right) < 0.08$ $\mathrm{P}\left(W \le k\right) > 0.92$ Using GC, $\mathrm{P}\left(W \le 20\right) = 0.869 < 0.92$, $\mathrm{P}\left(W \le 21\right) = 0.9209 > 0.92$ Least value of k = 21. Let I be the r.v. 'no. of packets of macademia nuts sold in a week'. $I \sim \mathrm{Po}\left(10\right)$ $\mathrm{P}\left(I > 11\right) = 1 - \mathrm{P}\left(I \le 11\right) = 1 - 0.69677 = 0.30323$ Let W be the r.v. 'no. of weeks, out of 52, of which more than 11 packets of macademia nuts are sold per week'. $W \sim \mathrm{B}\left(52,\ \mathrm{P}\left(I > 11\right)\right)$, i.e., $W \sim \mathrm{B}\left(52, 0.30323\right)$ $\mathrm{P}\left(W > k\right) < 0.08$ $\mathrm{P}\left(W \le k\right) > 0.92$ Using GC, $\mathrm{P}\left(W \le 20\right) = 0.869 < 0.92$, $\mathrm{P}\left(W \le 21\right) = 0.9209 > 0.92$ Least value of k = 21.
	Required probability = P(exactly 3 bonuses within the first 10 wks & bonus in the 11th wk) = P(exactly 3 bonuses within the first 10 wks) × P(bonus in the 11th wk) $= {}^{10}C_3 \left[\mathrm{P}\left(I > 11\right)\right]^3 \left[1 - \mathrm{P}\left(I > 11\right)\right]^7 \times \mathrm{P}\left(I > 11\right) = 0.00668$ Or, Let X = number of weeks (out of 10) where bonus is paid. $X \sim \mathrm{B}(10, \mathrm{P}(I > 11))$ P(exactly 3 bonuses within the first 10 wks & bonus in the 11th wk) = P(exactly 3 bonuses within the first 10 wks) × P(bonus in the 11th wk) = $\mathrm{P}(X = 3) \times \mathrm{P}(I > 11) = 0.00668$
8b(i)	Let $X \sim \mathrm{B}(n, p)$ Given $np = 3, \quad np(1-p) = 2.85 \quad \Rightarrow n = 60,\ p = 0.05$ <u>For X</u> $n = 60$ is large (>50) and p is small, $np = 3 < 5$, $\therefore X \sim \mathrm{P_o}\left(3\right)$ approximately <u>Similarly for Y</u> $n = 80$ is large (>50) and $p = 0.02$ is small, $np = 1.6 < 5$, $\therefore Y \sim \mathrm{P_o}\left(1.6\right)$ approximately

	By <u>additive property</u> of Poisson distribution, hence $X+Y \sim \mathrm{Po}(4.6)$
8(b)(ii)	From GC: $\mathrm{P}(X+Y=3)=0.163$ (3 s.f.)
9(i)	Let X and Y be random variables for the amount of time Singaporean youths and American youths spend at an ice skating rink per month respectively $X \sim N\left(10.1, 3.2^2\right)$ and $Y \sim N\left(9.3, 2.3^2\right)$ $P(X<5)=0.0555$
9(ii)	$X_1+X_2-2Y \sim N\left(2\times10.1-2\times9.3, 2\times3.2^2+2^2\times2.3^2\right)$ $=N(1.6, 41.64)$ $P(X_1+X_2>2Y)$ $=P(X_1+X_2-2Y>0)$ =0.5979 =0.598
9(iii)	Let W be the random variable for the cost, in \$, spent by a Singaporean youth per month. $W=7X \sim N\left(7\times10.1, 7^2\times3.2^2\right)=N(70.7, 501.76)$ $P(W>120)=0.0139$
10	For each of the classes from Arts and Business faculty, P(a student being selected) = $\frac{6}{20}$. For each of the classes for Science faculty, P(a student being selected) = $\frac{6}{30}$. Since the probabilities of selection is not common, the sample is not random.
	<u>Line up</u> the 240 students in some order (eg alphabetical order of name). From the first $\frac{240}{60}=4$ students, **<u>select one</u> <u>randomly</u>**. Thereafter, select the next 4th student till 60 are selected.
11(i)	Let X be the volume of coffee dispensed in a cup (in ml) with population mean μ $H_0: \mu=100$ $H_1: \mu>100$ Assumption: $X \sim N\left(\mu, \sigma^2\right)$ Test Statistic: $T=\frac{\bar{X}-\mu}{s/\sqrt{n}}$ Level of significance: 5%

	Reject H_0 if *p*-value < 0.05 Under H_0, using GC, *p*-value = 0.0550537= 0.0551 (3 s.f) Since *p*-value=0.0551>0.05, we do not reject H_0 and conclude that there is insufficient evidence at 5% level of significance that the machine is dispensing too much coffee
	To conclude that the machine is dispensing too much coffee i.e. $p - value < \frac{\alpha}{100}$ $\alpha > 5.51$