RAFFLES INSTITUTION
H2 Mathematics (9758)
2023 Year 6

2023 Year 6 H2 Mathematics Timed Practice: Solutions with Comments

1 **(a)** **(i)** Find $\int \sin^2 x \, dx$. [2]

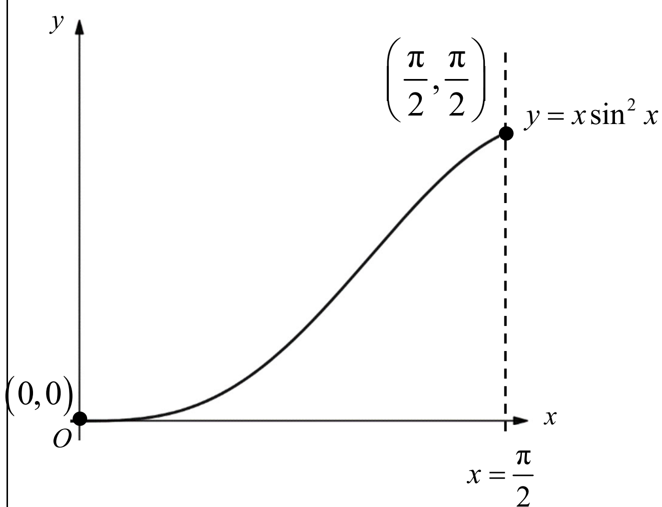
(ii) Hence or otherwise, show that

$$\int x\sin^2 x \, dx = \frac{1}{4}x^2 - \frac{1}{4}x\sin 2x - \frac{1}{8}\cos 2x + c.$$ [3]

(b) **(i)** Sketch the graph of $y = x\sin^2 x$ for $0 \le x \le \frac{1}{2}\pi$, labelling the end points clearly. [2]

(ii) Hence, find the exact area of the region bounded by the graph of $y = x\sin^2 x$, the x-axis and the line $x = \frac{1}{2}\pi$. [3]

Solutions		**Comments**
(a)(i) **[2]**	$\int \sin^2 x \, dx = \int \left(\frac{1-\cos 2x}{2}\right) dx$ $= \frac{1}{2}\int (1-\cos 2x) \, dx$ $= \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + c$ $= \frac{1}{2}x - \frac{\sin 2x}{4} + c$	
(a)(ii) **[3]**	$\int x\sin^2 x \, dx = x\left(\frac{1}{2}x - \frac{1}{4}\sin 2x\right) - \int \left(\frac{1}{2}x - \frac{1}{4}\sin 2x\right) dx$ $= x\left(\frac{1}{2}x - \frac{1}{4}\sin 2x\right) - \left[\frac{1}{2}\left(\frac{x^2}{2}\right) - \frac{1}{4}\left(\frac{-\cos 2x}{2}\right)\right] + c$ $= \frac{1}{2}x^2 - \frac{1}{4}x\sin 2x - \frac{1}{4}x^2 - \frac{1}{8}\cos 2x + c$ $= \frac{1}{4}x^2 - \frac{1}{4}x\sin 2x - \frac{1}{8}\cos 2x + c$ (shown)	"Hence" method: Integration by parts: $u = x$; $\frac{dv}{dx} = \sin^2 x$ $\frac{du}{dx} = 1$; $v = \frac{1}{2}x - \frac{\sin 2x}{4}$

	Alternatively, $\int x\sin^2 x \, dx = \int x\left(\frac{1-\cos 2x}{2}\right) dx$ $= \frac{x^2}{4} - \frac{1}{2}\int x\cos 2x \, dx$ $= \frac{x^2}{4} - \frac{1}{2}\left[x\left(\frac{\sin 2x}{2}\right) - \int\left(\frac{\sin 2x}{2}\right) dx\right]$ $= \frac{x^2}{4} - \frac{x\sin 2x}{4} + \frac{1}{4}\left(\frac{-\cos 2x}{2}\right) + c$ $= \frac{1}{4}x^2 - \frac{1}{4}x\sin 2x - \frac{1}{8}\cos 2x + c$ (shown)	
(b)(i) **[2]**	 	Students needs to zoom to the appropriate scale to see the shape of the graph. Do observe the gradient as well.

(b)(ii) **[3]**	Required area $= \int_0^{\frac{\pi}{2}} x \sin^2 x \, dx$ $= \left[\frac{1}{4}x^2 - \frac{1}{4}x\sin 2x - \frac{1}{8}\cos 2x\right]_0^{\frac{\pi}{2}}$ $= \left[\frac{1}{4}\left(\frac{\pi}{2}\right)^2 - \frac{1}{4}\left(\frac{\pi}{2}\right)\sin\left(2\left(\frac{\pi}{2}\right)\right) - \frac{1}{8}\cos\left(2\left(\frac{\pi}{2}\right)\right)\right]$ $-\left[\frac{1}{4}(0)^2 - \frac{1}{4}(0)\sin(2(0)) - \frac{1}{8}\cos(2(0))\right]$ $= \frac{\pi^2}{16} - \frac{1}{8}(-1) + \frac{1}{8}$ $= \left(\frac{\pi^2}{16} + \frac{1}{4}\right)\text{units}^2$	Some students made careless mistakes with the signs.

2 **(a)** A point R has position vector $\mathbf{r} = \begin{pmatrix} a \\ 2 \\ 3 \end{pmatrix} + a\begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix}$, where a is a real number.

Describe geometrically the set of all possible positions of R as a varies. [2]

(b) The angle between the vectors **a** and **b** is $\frac{\pi}{3}$ radians.

(i) Given that the angle between the vectors **b** and $\mathbf{a} - 2\mathbf{b}$ is a right angle, show that $|\mathbf{a}| = 4|\mathbf{b}|$. [3]

(ii) Given also that $|\mathbf{b}| = 2$, find the exact length of projection of $(2\mathbf{a} + \mathbf{b})$ onto $(\mathbf{a} + 2\mathbf{b})$. [5]

Solutions		**Comments**
(a) **[2]**	$\mathbf{r} = \begin{pmatrix} a \\ 2 \\ 3 \end{pmatrix} + a\begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + a\begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix}$, $a \in \mathbb{R}$ R lies on the line passing through the point with coordinates $(0,2,3)$, and the line is parallel to the vector $2\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$.	Many students did not manipulate the expression to have a as a parameter. Equation of a line must take the form: $\mathbf{r} = \mathbf{c} + a\,\mathbf{d}$, where **c** and **d** are <u>constant</u> vectors.
(b)(i) **[3]**	Angle between **a** and **b** is $\frac{\pi}{3}$ $\Rightarrow \quad \mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\frac{\pi}{3} = \frac{1}{2}|\mathbf{a}||\mathbf{b}| \quad \ldots (1)$ Angle between **b** and **a** – 2**b** is a right angle $\Rightarrow \quad \mathbf{b} \cdot (\mathbf{a} - 2\mathbf{b}) = 0$ $\Rightarrow \quad \mathbf{b} \cdot \mathbf{a} - 2\mathbf{b} \cdot \mathbf{b} = 0$ $\Rightarrow \quad 2|\mathbf{b}|^2 = \mathbf{b} \cdot \mathbf{a} \quad \ldots (2)$ Substitute (1) into (2) : $|\mathbf{b}|^2 = \frac{1}{4}|\mathbf{a}||\mathbf{b}|$ Divide throughout by $|\mathbf{b}|$, we have $|\mathbf{a}| = 4|\mathbf{b}|$. (shown)	

(b)(ii) **[5]**	From (bi), since $\lvert\mathbf{b}\rvert = 2$, $\lvert\mathbf{a}\rvert = 4\lvert\mathbf{b}\rvert = 8$ and $\mathbf{a}\cdot\mathbf{b} = 8$ Length of projection of $(2\mathbf{a}+\mathbf{b})$ onto $(\mathbf{a}+2\mathbf{b})$ $= \dfrac{\lvert(2\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}+2\mathbf{b})\rvert}{\lvert\mathbf{a}+2\mathbf{b}\rvert}$ $= \dfrac{\left\lvert 2\lvert\mathbf{a}\rvert^2 + 2\lvert\mathbf{b}\rvert^2 + 5\mathbf{a}\cdot\mathbf{b}\right\rvert}{\sqrt{(\mathbf{a}+2\mathbf{b})\cdot(\mathbf{a}+2\mathbf{b})}}$ $= \dfrac{176}{\sqrt{\lvert\mathbf{a}\rvert^2 + 4\mathbf{a}\cdot\mathbf{b} + 4\lvert\mathbf{b}\rvert^2}}$ $= \dfrac{176}{\sqrt{112}}$ $= \dfrac{44}{\sqrt{7}}$ units (or $\dfrac{44\sqrt{7}}{7}$ units)	Reminder: $\lvert\mathbf{a}\rvert = 4\lvert\mathbf{b}\rvert$ DOES NOT mean $\mathbf{a} = 4\mathbf{b}$ The modulus sign is necessary to ensure a positive value is obtained for length of projection. Many students did not consider the dot product to find $\lvert\mathbf{a}+2\mathbf{b}\rvert$. Note that $\lvert\mathbf{a}+2\mathbf{b}\rvert \neq \lvert\mathbf{a}\rvert + \lvert 2\mathbf{b}\rvert$ in general.

3 **Do not use a calculator in answering this question.**

Two complex numbers p and q are given by $p=\frac{3}{\sqrt{2}}+\mathrm{i}\frac{3}{\sqrt{2}}$ and $q=1+\mathrm{i}\sqrt{3}$ respectively.

(a) Find p^*q^2 in the form $a+\mathrm{i}b$, where a and b are exact real values, and p^* denotes the conjugate of p. [3]

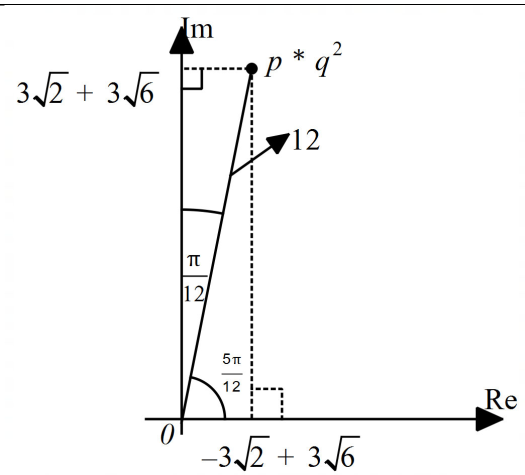
(b) **(i)** Find the modulus and argument of p and q. [2]

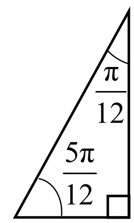
(ii) Write down the value of p^*q^2 in the form $r(\cos\theta+\mathrm{i}\sin\theta)$, where $r>0$ and $-\pi<\theta\le\pi$. [2]

(c) **(i)** Represent p^*q^2 on an Argand diagram, labelling clearly the information found in parts **(a)** and **(bii)**. [1]

(ii) Hence, find the exact value of $\sin\frac{1}{12}\pi$. [2]

Solutions		**Comments**
(a) **[3]**	$p^*q^2=\left(\frac{3}{\sqrt{2}}-\mathrm{i}\frac{3}{\sqrt{2}}\right)\left(1+\mathrm{i}\sqrt{3}\right)^2$ $=\frac{3}{\sqrt{2}}(1-\mathrm{i})\left(1+\mathrm{i}2\sqrt{3}-3\right)$ $=\frac{3}{\sqrt{2}}(1-\mathrm{i})\left(-2+\mathrm{i}2\sqrt{3}\right)$ $=\frac{3}{\sqrt{2}}\left(-2+\mathrm{i}2\sqrt{3}+2\mathrm{i}+2\sqrt{3}\right)$ $=3\sqrt{2}\left[\left(-1+\sqrt{3}\right)+\mathrm{i}\left(1+\sqrt{3}\right)\right]$ $=\left(-3\sqrt{2}+3\sqrt{6}\right)+\mathrm{i}\left(3\sqrt{2}+3\sqrt{6}\right)$ $a=-3\sqrt{2}+3\sqrt{6}$ and $b=3\sqrt{2}+3\sqrt{6}$	Many students did not simplify the answer.
(b)(i) **[2]**	$\lvert p\rvert=\sqrt{\left(\frac{3}{\sqrt{2}}\right)^2+\left(\frac{3}{\sqrt{2}}\right)^2}=3$, $\arg(p)=\frac{\pi}{4}$ $\lvert q\rvert=\sqrt{(1)^2+\left(\sqrt{3}\right)^2}=2$, $\arg(q)=\frac{\pi}{3}$	
(b)(ii) **[2]**	From (bi), $p=3\mathrm{e}^{\mathrm{i}\frac{\pi}{4}}$ and $q=2\mathrm{e}^{\mathrm{i}\frac{\pi}{3}}$	Many students did not use the exponential form/properties to find the argument and modulus. It is very difficult to

	$p^*q^2=\left(3e^{-i\frac{\pi}{4}}\right)\left(4e^{i\frac{2\pi}{3}}\right)$ $=12e^{i\left(-\frac{\pi}{4}+\frac{2\pi}{3}\right)}$ $=12e^{i\frac{5\pi}{12}}$ $=12\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$ Alternatively, $\lvert p^*q^2\rvert=\lvert p\rvert\lvert q\rvert^2=3(2^2)=12$ $\arg\left(p^*q^2\right)=2\arg q-\arg p=\frac{2\pi}{3}-\frac{\pi}{4}=\frac{5\pi}{12}$ $\therefore p^*q^2=12\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$	work from the cartesian form since students are not supposed to use GC for this question.
(c)(i) **[1]**		Many students did not illustrate the argument proportionally. Note that $\frac{5\pi}{12}$ is 75°.
(c)(ii) **[2]**	$\sin\left(\frac{\pi}{12}\right)=\frac{\text{Re}\left(p^*q^2\right)}{\lvert p^*q^2\rvert}$ $=\frac{-3\sqrt{2}+3\sqrt{6}}{12}=\frac{\sqrt{6}-\sqrt{2}}{4}$	Do refer to the diagram in **(c)(i).**

	Alternatively, $\sin\left(\frac{\pi}{12}\right)=\sin\left(\frac{\pi}{2}-\frac{5\pi}{12}\right)$ $=\cos\left(\frac{5\pi}{12}\right)$ $=\frac{\text{Re}\left(p^*q^2\right)}{\left\|p^*q^2\right\|}$ $=\frac{-3\sqrt{2}+3\sqrt{6}}{12}=\frac{\sqrt{6}-\sqrt{2}}{4}$  	

4 Two planes p_1 and p_2 are perpendicular. Plane p_1 has equation

$$\mathbf{r}=\begin{pmatrix}2\\1\\3\end{pmatrix}+\lambda\begin{pmatrix}1\\a\\2\end{pmatrix}+\mu\begin{pmatrix}2\\0\\1\end{pmatrix},$$

where a is a constant, and λ and μ are parameters.

Plane p_2 contains the line l with equation $x-3=z-4,\ y=1$.

(a) Given that the equation of p_1 may also be expressed as

$$\mathbf{r}\cdot\begin{pmatrix}1\\3\\-2\end{pmatrix}=d,$$

where d is a constant, find the values of a and d. [2]

(b) Find a cartesian equation of p_2. [3]

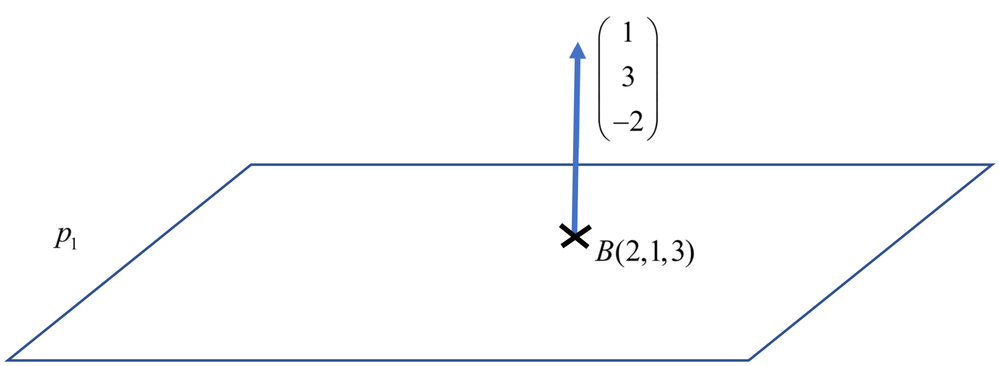
(c) Show that a vector equation of m, the line of intersection of p_1 and p_2, can be expressed as

$$\mathbf{r}=\begin{pmatrix}2\\1\\3\end{pmatrix}+s\begin{pmatrix}5\\1\\4\end{pmatrix}, \text{ where } s \text{ is a parameter.}$$ [3]

(d) Find the acute angle between l and m. [2]

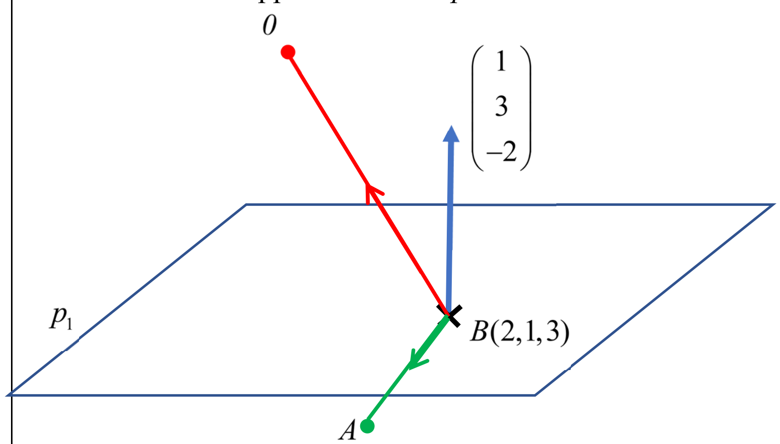
(e) The points O, A and B have coordinates $(0,0,0)$, $(3,1,4)$ and $(2,1,3)$ respectively. By considering the dot products of vectors $\overrightarrow{BO}$ and $\overrightarrow{BA}$ with the normal of p_1, determine whether O and A are on the same side or opposite sides of p_1.

You should indicate clearly the vectors $\overrightarrow{BO}$ and $\overrightarrow{BA}$ on the diagram below. [2]



Solutions		Comments
(a) **[2]**	$\begin{pmatrix}1\\a\\2\end{pmatrix}\cdot\begin{pmatrix}1\\3\\-2\end{pmatrix}=0\Rightarrow 1+3a-4=0\ \Rightarrow a=1$ $\begin{pmatrix}2\\1\\3\end{pmatrix}\cdot\begin{pmatrix}1\\3\\-2\end{pmatrix}=2+3-6=-1\quad\Rightarrow d=-1$ **<u>Alternative Method</u>** A vector perpendicular to p_1 is $\begin{pmatrix}1\\a\\2\end{pmatrix}\times\begin{pmatrix}2\\0\\1\end{pmatrix}$ $=\begin{pmatrix}a\\3\\-2a\end{pmatrix}$ which is parallel to $\begin{pmatrix}1\\3\\-2\end{pmatrix}$ $\Rightarrow a=1$ $\begin{pmatrix}2\\1\\3\end{pmatrix}\cdot\begin{pmatrix}1\\3\\-2\end{pmatrix}=2+3-6=-1\quad\Rightarrow d=-1$	
(b) **[3]**	Direction vector for line l : $\begin{pmatrix}1\\0\\1\end{pmatrix}$ A vector perpendicular to $p_2=\begin{pmatrix}1\\0\\1\end{pmatrix}\times\begin{pmatrix}1\\3\\-2\end{pmatrix}=\begin{pmatrix}-3\\3\\3\end{pmatrix}=3\begin{pmatrix}-1\\1\\1\end{pmatrix}$ Normal of $p_2=\begin{pmatrix}-1\\1\\1\end{pmatrix}$. Note that $\begin{pmatrix}3\\1\\4\end{pmatrix}\cdot\begin{pmatrix}-1\\1\\1\end{pmatrix}=-3+1+4=2$ Hence, a cartesian equation of p_2 is $-x+y+z=2$.	Do revise how to convert the form of a line from cartesian to vector form. Some students obtained $\begin{pmatrix}1\\1\\0\end{pmatrix}$ as direction vector of l instead, which is incorrect.

(c) **[3]**	Direction vector is $\begin{pmatrix}1\\3\\-2\end{pmatrix}\times\begin{pmatrix}-1\\1\\1\end{pmatrix}=\begin{pmatrix}5\\1\\4\end{pmatrix}$ Note that (2,1,3) lies on p_1 as observed from its equation $\mathbf{r}=\begin{pmatrix}2\\1\\3\end{pmatrix}+\lambda\begin{pmatrix}1\\a\\2\end{pmatrix}+\mu\begin{pmatrix}2\\0\\1\end{pmatrix}$, when $\lambda=\mu=0$. Since $-2+1+3=2$, (2,1,3) also lies on p_2. Equation of m : $\mathbf{r}=\begin{pmatrix}2\\1\\3\end{pmatrix}+s\begin{pmatrix}5\\1\\4\end{pmatrix}$, $s\in\mathbb{R}$.	Students need to show clearly: 1. the point $(2,1,3)$ lies on both planes 2. find a direction vector that is parallel to $\begin{pmatrix}5\\1\\4\end{pmatrix}$.
(d) **[2]**	Let θ be the acute angle between l and m. $\cos\theta=\dfrac{\left\lvert\begin{pmatrix}1\\0\\1\end{pmatrix}\cdot\begin{pmatrix}5\\1\\4\end{pmatrix}\right\rvert}{\sqrt{2}\sqrt{42}}=\dfrac{9}{\sqrt{84}}$ $\theta=10.9°$ (to 1 d.p.)	The modulus sign is necessary to ensure that the angle obtained is acute.

(e) **[2]**	$\overrightarrow{BO}\cdot\begin{pmatrix}1\\3\\-2\end{pmatrix}=-\begin{pmatrix}2\\1\\3\end{pmatrix}\cdot\begin{pmatrix}1\\3\\-2\end{pmatrix}=-(2+3-6)=1>0$ $\overrightarrow{BA}\cdot\begin{pmatrix}1\\3\\-2\end{pmatrix}=\begin{pmatrix}1\\0\\1\end{pmatrix}\cdot\begin{pmatrix}1\\3\\-2\end{pmatrix}=1-2=-1<0$ $\Rightarrow$Angle between $\overrightarrow{BO}$ and $\begin{pmatrix}1\\3\\-2\end{pmatrix}$ is acute while angle between $\overrightarrow{BA}$ and $\begin{pmatrix}1\\3\\-2\end{pmatrix}$ is obtuse. $\Rightarrow O$ and A are on opposite sides of p_1. 	The values of the dot products indicate one angle is acute while the other is obtuse, leading to the conclusion that the points are on opposite sides of the plane. Also, do note that vector BO is longer than vector BA.

5 John models the nitrate level, N mg/L, present in the aquarium at a time t days after setting up his fish tank. The model assumes that, at any time t, the rate of increase of nitrate level due to ammonia and bacteria in the tank is kN mg/L per day, for some positive constant k. The initial nitrate level is 10 mg/L and it is found to be 20 mg/L after 1 day.

(a) **(i)** Write down a differential equation involving N, t and k. [1]

(ii) Solve this differential equation to find an expression for N in terms of t and show that $k = \ln 2$. [3]

(iii) Find the nitrate level after 3 days. [1]

Hornwort is an aquatic plant that can help to reduce nitrate levels in aquariums. John adds s stalks of hornwort to his fish tank after 3 days. He then models the rate of decrease of nitrate level by the hornwort, x days after it is added, by h mg/L per stalk per day. The rate of increase of nitrate level due to ammonia and bacteria in the tank remains at $N\ln 2$ mg/L per day.

(b) **(i)** Explain why the differential equation in this case is now $\frac{\mathrm{d}N}{\mathrm{d}x} = N\ln 2 - sh$. [1]

(ii) Solve this differential equation, giving N in terms of x, s and h. [3]

(c) In the case where $h = 18.5$, find the least number of stalks of hornwort needed in order for the nitrate level to decrease to 0 mg/L at $x = 7$. [3]

Solutions		**Comments**
(ai) **[1]**	$\frac{\mathrm{d}N}{\mathrm{d}t} = kN$	
(aii) **[3]**	$\int \frac{1}{kN}\,\mathrm{d}N = \int 1\,\mathrm{d}t$ Since $N > 0$, $\frac{1}{k}\ln N = t + c'$ $\ln N = kt + c$, where $c = kc'$ $N = \mathrm{e}^{kt+c}$ $N = A\mathrm{e}^{kt}$, where $A = \mathrm{e}^{c}$ When $t = 0, N = 10$. Hence $A = 10$. When $t = 1, N = 20$. Hence, $20 = 10\mathrm{e}^{k}$ $k = \ln 2$ (shown) $\therefore N = 10\mathrm{e}^{t\ln 2}$ or $N = 10(2^{t})$	Note that the modulus sign in $\ln\lvert N\rvert$ should be included if there is no mention of N being positive. Question states that we need to find an expression for N in terms of t.

(aiii) **[1]**	$N=10\mathrm{e}^{t\ln 2}$ or $N=10(2^t)$ When $t=3$, $N=10\mathrm{e}^{3\ln 2}=80$ The nitrate level after 3 days is 80 mg/L.	
(bi) **[1]**	Since h is the rate of decrease of nitrate level by one stalk of hornwort per day, sh would be the rate of decrease of nitrate level by s stalks of hornwort in one day. Total rate of change = (rate of increase due to ammonia and bacteria) – (rate of decrease due to s stalks of hornwort) hence $\frac{\mathrm{d}N}{\mathrm{d}x}=N\ln 2-sh$.	
(bii) **[3]**	$\frac{\mathrm{d}N}{\mathrm{d}x}=N\ln 2-sh$ $\int \frac{1}{N\ln 2-sh}\,\mathrm{d}N=\int 1\,\mathrm{d}x$ $\frac{1}{\ln 2}\ln\lvert N\ln 2-sh\rvert=x+d'$ $\ln\lvert N\ln 2-sh\rvert=x\ln 2+d$, where $d=d'\ln 2$ $N\ln 2-sh=\pm\mathrm{e}^{x\ln 2+d}$ $N\ln 2-sh=B\mathrm{e}^{x\ln 2}$, where $B=\pm\mathrm{e}^{d}$ $N=\frac{B\mathrm{e}^{x\ln 2}+sh}{\ln 2}$ ------- (*) When $x=0$, $N=80$. $80=\frac{B+sh}{\ln 2}$ $B=80\ln 2-sh$ Substituting $B=80\ln 2-sh$ into (*), $N=\frac{(80\ln 2-sh)\mathrm{e}^{x\ln 2}+sh}{\ln 2}$ $N=\frac{(80\ln 2-sh)2^{x}+sh}{\ln 2}$	Note that the modulus sign for $\ln\lvert N\ln 2-sh\rvert$ is required. The initial condition found in part **a(iii)** should be used to find the value of the arbitrary constant, B.
(c) **[3]**	**Method 1:** Given $h=18.5$. When $x=7$, $N=0$. $0=\frac{(80\ln 2-18.5s)2^{7}+18.5s}{\ln 2}$ $128\times 80\ln 2-127\times 18.5s=0$ $s=\frac{128\times 80\ln 2}{127\times 18.5}=3.02$ (to 3s.f.) The least number of stalks needed is 4.	

	Method 2: Given $h=18.5$. When $x=7$, $N\le 0$. $0\ge\dfrac{\left(80\ln 2-18.5s\right)2^7+18.5s}{\ln 2}$ $128\times 80\ln 2-127\times 18.5s\le 0$ $s\ge\dfrac{128\times 80\ln 2}{127\times 18.5}=3.02$ (to 3s.f.) The least number of stalks needed is 4.	

6 **(a)** Find the number of arrangements of all ten letters of the word ENGAGEMENT with no restrictions. [1]

(b) Find the number of arrangements of all ten letters of the word ENGAGEMENT in which the first and last letters are vowels. [3]

(c) Find the number of arrangements of all ten letters of the word ENGAGEMENT in which no two vowels are next to each other. [3]

(d) It is now given that 4 letters are randomly selected from the ten letters in the word ENGAGEMENT.

Find the probability that the 4 letters selected contains exactly 2 distinct letters. [4]

Solutions		**Comments**
(a) [1]	Number of ways $=\frac{10!}{3!2!2!}=151200$	
(b) [3]	*Case 1: Start with E and end with E* Number of ways $=\frac{8!}{2!2!}=10080$ *Case 2: Start with E and end with A* Number of ways $=\frac{8!}{2!2!2!}=5040$ *Case 3: Start with A and end with E* Number of ways $=\frac{8!}{2!2!2!}=5040$ (same as case 2) Total number of ways $=10080+5040+5040=20160$	The three E's are identical, hence there is only 1 way to choose an E to begin and another E to end.
(c) [3]	The vowels are 3Es and 1A. The consonants are 2Ns, 2Gs, 1M and 1T. Number of ways to arrange the consonants $=\frac{6!}{2!2!}=180$ $\uparrow C_1 \uparrow C_2 \uparrow C_3 \uparrow C_4 \uparrow C_5 \uparrow C_6 \uparrow$ Number of ways to choose 4 slots to insert the vowels $={}^7C_4=35$ Number of ways to arrange the vowels $=\frac{4!}{3!}=4$ $\therefore$ The number of different arrangements $=180\times35\times4=25200$	Slotting method is efficient in this context to ensure no 2 vowels are next to each other. Avoid the complement method for this question.

(d) **[4]**	Case 1: E E E E ' Req probability is $\frac{3}{10}\times\frac{2}{9}\times\frac{1}{8}\times\frac{4!}{3!}=\frac{1}{30}$ Case 2: EENN or EEGG or NNGG Req probability is $\left\{2\left[\frac{3}{10}\times\frac{2}{9}\times\frac{2}{8}\times\frac{1}{7}\right]+\frac{2}{10}\times\frac{1}{9}\times\frac{2}{8}\times\frac{1}{7}\right\}\times\frac{4!}{2!2!}=\frac{1}{30}$ $\therefore$P(4 letters selected contains exactly 2 distinct letters) $=\frac{1}{30}+\frac{1}{30}=\frac{1}{15}$ Alternative: Case 1: E E E E ' Req probability is $\frac{{}^{7}C_{1}}{{}^{10}C_{4}}=\frac{1}{30}$ Case 2: EENN or EEGG or NNGG Req probability is $\frac{{}^{3}C_{2}\times 2+1}{{}^{10}C_{4}}=\frac{1}{30}$ $\therefore$P(4 letters selected contains exactly 2 distinct letters) $=\frac{1}{30}+\frac{1}{30}=\frac{1}{15}$	Note the interpretation of "two distinct letters" in this context. Some missed out case 1 when there are 3Es and one other letter.

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